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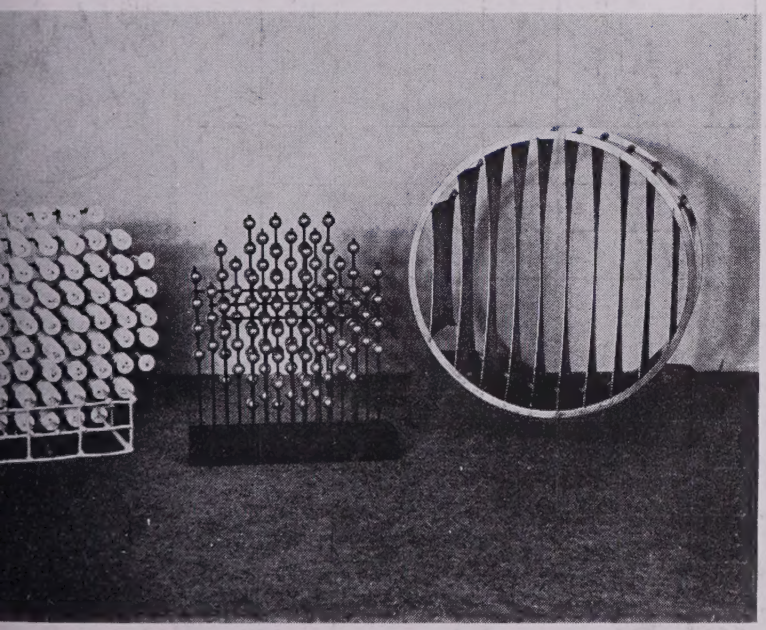

Bell Telephone Laboratories

MACRO-OPTICAL ELEMENTS

The two structures at the left are vastly magnified models of the atomic arrangements which enable glass lenses to focus visible light. These structures, however, are effective as refractive elements for microwaves, and cover a considerable group of frequencies in each case. (At the right is an older "delay lens," incapable of focusing so wide a band of frequencies.)

PROCEEDINGS OF THE I.R.E.

Atomic Structure

Theory of FM Noise

New Ionospheric-Meteorological Correlation

Wide-Range-Tuning Phase-Shift Oscillator

Radiation Patterns of Horn Antennas

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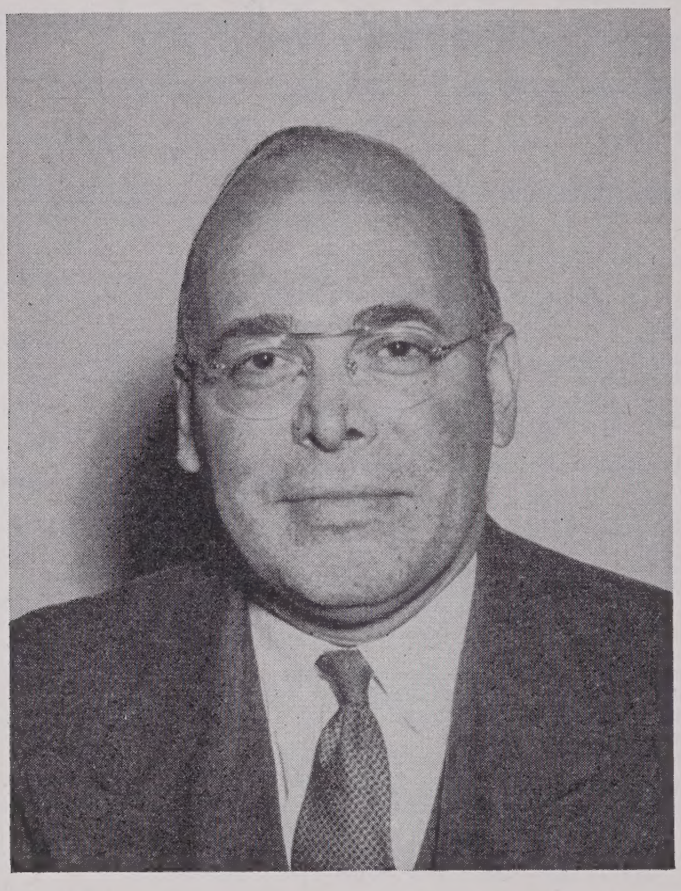
The Institute of Radio Engineers

John B. Coleman

Regional Director, 1948–1949



John B. Coleman was born in Indiana County, Pa., on August 29, 1899. He started in radio work as an amateur in 1914. For a short time in 1917 he served as a radio operator for the Marconi Company, and in 1918 was an instructor in the Signal Corps Air Service School for Radio Mechanics at the Carnegie Institute of Technology. He was also active in the Westinghouse Company's early development of broadcasting.

He was graduated from Carnegie Institute of Technology in 1923 with the B.S. degree in electrical engineering and joined the radio division of the Westinghouse Electric and Manufacturing Company, where he was engineer in charge of radio station WBZ from 1923 to 1925; from 1925 to 1927 he conducted development and design work on transmitters. From 1927 to 1930 he was section engineer in charge of high-power transmitter design, still with the Westinghouse Company.

In 1930 Mr. Coleman transferred to the engineering department of the RCA Manufacturing Company at Camden, N. J. He was engaged, between 1930 and 1939, in the development and design of transmitters for broadcasting and commercial and Navy communications. Through the years 1939 to 1945 he was chief engineer of the Special Apparatus Engineering Department, and in 1945 was appointed assistant director of engineering for the RCA Victor Division of RCA.

Mr. Coleman joined the IRE as an Associate in 1925, became a Member in 1938, and Senior Member in 1945. He was Chairman of the Philadelphia Section in 1942. He is a member of the AIEE, the American Society of Naval Engineers, the Army Ordnance Association, Tau Beta Pi, and Eta Kappa Nu.

Older concepts rigidly restricted the thoughts and activities of the engineer to strictly technical matters. The mode of utilization of the products of his skill, their suitability to social needs, and what might be termed the engineer's adaptation to social environments, were matters usually neglected.

In the following guest editorial, there are presented thoughts based on engineering skill, long industrial and governmental experience, and a seasoned philosophy of life. The author urges engineers to integrate their environmental, psychological, and technical aspects into a more useful and humanly satisfying whole. The author is himself a Past President of The Institute of Radio Engineers, its present Secretary, and vice-president and chief engineer of the American Cable and Radio Corporation. His thoughtful presentation is earnestly recommended to the readers of this journal.—*The Editor.*

Engineering Thinking and Human Progress

HARADEN PRATT

During these troubled days, when people's thoughts are perplexed by the ideologic confusion brought about by meaningless propaganda, diverse economic theories and historical teachings, enlightened yet venal agencies of government, and scientific facts as opposed to spiritual concepts, a special need arises for some means whereby all these seemingly antagonistic forces can be woven together into a consistent pattern. For youth, with its limited experience, it must be particularly difficult to judge the accuracy of the vast amount of information available today. Unless young people are able to separate the wheat from the chaff, they may be guided more by emotion than reason, and their outlook correspondingly narrowed.

The basic objective of the scientist is an unending search for truth. He constantly seeks the facts that will explain that which he cannot understand. The present-day emphasis on factual knowledge plus the brilliant achievements of science and technology during the past few decades have tended to further the view that all truth must be susceptible of concrete proof. This viewpoint is shortsighted, limiting belief to the material, and allowing no scope for any concepts beyond the immediate reach of hard scientific facts.

The engineer is also an ardent searcher for truth, and the most important substance with which he works is scientific fact. To accomplish his purpose of serving the public, however, he must compound an alloy, and draw from fields outside his own—law, finance, politics, business administration—to secure the necessary ingredients for his crucible. He succeeds in proportion to his ability to appraise and select from these fields, and his success can, of course, be attributed to skill in analysis. On the other hand, this approach does not debar empirical methods *per se*, for the engineer should recognize that results are not measured as much by the means by which they are accomplished as by their eventual benefit to society. Civilization has, after all, been largely the result of learning by trial and error, and this age of industrial and technical development has made extensive use of that method. The engineer thus supplements reasoning with imagination. His methods of attack reject no suitable tools: only the nonessential is excluded. In the heat-treating and molding of the alloy he makes, his final conclusions are reached, and, in arriving at them, he does not neglect to weigh the moral aspect along with the material.

Because of the enormous increase not only in technology but in public awareness of it brought about by the last war, an increasing number of laymen without scientific or technical education are unconsciously adopting the engineer's outlook in many of their attitudes. This trend will grow, and the engineering profession must help it by recognizing its responsibility to contribute actively toward better thinking. Only through such increased perception can we build the better leadership and firmer moral and spiritual attitudes needed to guide us away from the quicksands that can so easily impede our progress today.

Atomic Structure

It is the policy of The Institute of Radio Engineers, as approved by its Board of Directors, to further the scientific and technical development of the communications and electronic field. One division of this field, and accordingly a matter within the scope of interest of the Institute, is the use of electronic instrumentation and controls in the realm of atomic and nucleonic phenomena. The IRE membership have already contributed substantially to that division of their field, and will doubtless continue to do so. It is therefore appropriate that these PROCEEDINGS shall, from time to time, contain the following:

1. Papers dealing with methods and equipment for electronic instrumentation and controls in the atomic and nucleonic field.
2. Explanatory papers of general and tutorial nature, dealing with the nature of the outer and inner portions of the atom and the various interactions between atoms and portions thereof, as well as the natural and man-induced phenomena in that segment of matter or energy.
3. More specialized papers, also of the tutorial nature, dealing with scientific, technical, and industrial applications of available knowledge of the atom and its parts.

It is the purpose of papers of Type 1 above to communicate knowledge and to stimulate progress in the field of the IRE. It is the purpose of papers of Types 2 and 3 above to give the readers of the PROCEEDINGS such background knowledge in the nuclear field as will enable them more effectively to understand and apply the information contained in papers of Type 1.

The constructive presentation which begins on page 1070 is the first of a series of papers of Type 2, as described above. Succeeding papers will accordingly form a basis for the necessary knowledge of engineers working in the nucleonic field. Papers of Types 1 and 3 above will appear in later issues of the PROCEEDINGS.

The following paper and others by the same authors have been made available through the co-operation of the authors, Ralph E. Lapp and H. L. Andrews, and of Prentice-Hall, Incorporated, publishers of a book entitled "Nuclear Radiation Physics" by the authors of these papers. This book, which has just been published, contains the material of these papers in chapter form. The co-operation of the authors and the publisher is gratefully acknowledged.—*The Editor.*

INTRODUCTION

THE CONCEPT of the atom, accepted by scientists from the days of ancient Greece until almost the twentieth century, was that of a unitary indivisible entity. More simply, it was the so-called "billiard-ball atom." For instance, atoms of fluorine or copper, according to this viewpoint, were hard spherical minute units of these respective elements, incapable of separation into anything else, and permanently fixed as to their status in the universe.

Communications and electronic engineers interested in modern atomic theory, and in the instrumentation and control of nuclear processes, are well aware that the simple atomic theory outlined above has been completely discarded. Instead, the splitting of the atom into its component portions has led to the more or less adequate identification and description of numerous new "fundamental" particles. It has also led to a fair general knowledge of the structure of the outer portions of the atom, but a far less complete though suggestive knowledge of the construction of the nucleus or central portion of the atom.

The subject of atomic structure is nevertheless still in a state of partial flux. Thus, some of the information given in this Introduction must be regarded as subject to some later revisions. The number of "fundamental" particles grows apace, as will be judged from the list appearing further in this discussion. Hence the catalog of such particles may steadily expand. Nuclear chemistry and physics, so to speak, promise to be complex but fruitful branches of science and technology.

Later in this discussion references are made in some cases to the mass of certain elementary particles, their size, their electrical charges, and the length or brevity of their lives. It must at once be admitted that the "size" of these particles is a somewhat indeterminate quantity, subject in effect to the application of the curious Heisenberg principle of uncertainty or indeterminacy. Further, under certain conditions, a particle of matter may become intrinsically a packet of waves. It is planned that these topics shall be discussed further in later articles of this series.

Further, in speaking of the "mass" of an elementary particle, it should be noted that the extent to which such mass is purely in the nature of energy, in the electromagnetic field surrounding the particle or otherwise, is still a subject for further investigation. It is also well known that moving charges or bodies acquire an increased energy as the result of their motion, and that the magnitude of this energy increase is such that the conclusion is reached that the "mass" of such particles increases very rapidly at high velocities, and indeed reaches infinity for any ponderous particle at a limiting velocity, namely, that of light. Conversely, energy itself may be regarded as simply one aspect or exemplification of mass.

It is beyond the scope of this Introduction to do more than mention that certain elementary particles are also found to possess a quality designated as "spin" which, in a very general fashion, is analogous to rotary motion and therefore gives such particles corresponding and definite angular momenta. Further, charged particles with finite spins may have corresponding magnetic moments. This is perhaps to be expected, but it is more puzzling to find that certain uncharged particles, like the neutron, also have a magnetic moment.

For convenient reference, there may be briefly mentioned in this Introduction such facts as the following

Elementary particles, or their aggregations, can exist only in configurations having certain definite energy levels. Such particles or aggregations do not acquire or lose energy continuously and smoothly; on the contrary, they gain or lose energy only in finite amounts, known as "quanta." The magnitudes of these quanta are calculable according to well-demonstrated laws. Further, within the configurations made up of elementary particles there exist what are termed "potential wells." These are, to all intents and purposes, representations in graphical and conveniently usable form of the energy levels encountered in passage into or across an assembly of elementary particles.

In addition, in the confined central portion of the atom—that is, its nucleus—there is evidently some sort of "nuclear glue" which is responsible for holding together what would otherwise be an unstable arrangement of particles. This stabilizing agency is known as an "exchange force." The exchange forces between elementary particles in the nucleus of an atom act in manners that seem strange to the classical physicist, or even to his modern successors. Yet they seem to hold together the nucleus of the atom and, in addition, to cause rapid and recurrent charges in the very nature or identity of the elementary particles within the nucleus which are thus held together as a stable assembly.

For convenience of reference there follows a partial list of elementary units or particles, as at present known. It must be pointed out that some of these (e.g., the neutrino) have not been detected as yet. Yet the existence of the neutrino has a high degree of probability, on the basis of energy analyses of certain nuclear processes. There has also been included in the following the photon, although this is usually regarded as a unit of electromagnetic radiation (light), rather than a material or electrical particle. Also mentioned are deuterons, and alpha particles, even though these are well known to consist of more than one elementary particle. Deuterons and alpha particles do, however, play an important part in modern atomic investigations, usually as disruptive missiles. The constants are given in rationalized mks units.

1. Electrons

   Charge: 1.602×10^{-19} coulomb (negative)
   Rest Mass: 9.11×10^{-31} kilogram
   Diameter: 5.6×10^{-15} meter
   Life: Permanent.

2. Protons

   Charge: 1.602×10^{-19} coulomb (positive)
   Rest Mass: 1.673×10^{-27} kilogram (nearly the same as that of monatomic hydrogen)
   Diameter: 2×10^{-15} meter
   Life: Permanent outside of nucleus; possibly transformable into neutrons within nucleus.

3. Alpha particles

   These are helium nuclei, unaccompanied by the usual two planetary electrons. They accordingly consist of two protons and two neutrons in close association.

4. Deuterons

   These are heavy-hydrogen (deuterium) nuclei, unaccompanied by the usual single planetary electron. They therefore consist of one proton and one neutron.

5. Neutrons

   Charge: Zero
   Rest Mass: 1.675×10^{-27} kilogram (nearly the same as that of monatomic hydrogen)
   Diameter: 2×10^{-15} meter
   Life: Approximately one-half hour outside of nucleus; possibly transformable to proton within nucleus.

6. Positrons

   Charge: Same as that of electron but positive
   Mass: Same as that of electron
   Diameter: 5.6×10^{-15} meter
   Life: Indefinite in free space; usually combines speedily with free electron, both electron and positron being "annihilated" with simultaneous emission of flash of intense radiation. The reverse transformation of radiation (high-energy photons) into electron-positron pairs also occurs.

7. Neutrinos

   Charge: Zero
   Rest Mass: Negligible
   Diameter: Less than 0.1 that of electron
   Life: Unknown.

8. Mesons (or mesotrons) of small mass

   Charge: Positive or negative, assumed equal in value to that of electron
   Rest Mass: 1.8×10^{-28} kilogram (about 200 times that of the electron)
   Size: Unknown
   Life: About 2 microseconds.

9. Mesons of large mass

   Charge: Positive or negative, assumed equal in value to that of electron
   Rest Mass: 2.9×10^{-28} kilogram (about 300 times that of the electron)
   Size: Unknown
   Life: Probably 10^{-8} second.

10. Photons
    Charge: Zero
    Rest Mass: Zero (traveling at the velocity of light in all cases)
    Size: Indeterminate
    Frequency: The frequency of the photon is its energy multiplied by a constant quantity h, which is 6.62 $(10)^{-34}$ joule-second.

The authors of the following paper have kindly prepared a convenient tabulation of the above (and certain added) material (Table IA). (This Table uses units not of the rationalized mks system.)

As indicated above, there is some likelihood that additional "elementary" particles will be discovered from time to time. Further, the above list may be found to contain what are now regarded as fundamental particles, but which will later be found to be, in fact, combinations of new "elementary" particles.

Even now, some 500 or more isotopes of the chemical elements are known which are made up of some of the above particles. Some of these elements are found in nature, others are man-made. Some of them are essentially stable, while others are radioactive.

What the future holds in the way of information concerning the structure of particles is uncertain—except that the information will doubtless be complex and challenging. There is thus reason for much encouragement in our present limited knowledge of nuclear phenomena and elementary particles. Our present limitations offer, as in the past, the possibility of great progress and major scientific and technological discoveries and accomplishments in the years to come.

—*The Editor*

TABLE IA

Data About Nuclear "Particles"*

Name	Symbol	Mass[1]	Charge[2]	Remarks
Electron	$_{-1}e^0$	0.0005	$-e$	Also called beta particles (β)
Positron	$_{+1}e^0$	0.0005	$+e$	Also symbolized as (β^+)
Proton	$_1H^1$	1.0076	$+e$	Symbol p is often used
Neutron	$_0n^1$	1.0089	0	Unstable with half life of the order of $\frac{1}{2}$ hour.
Deuteron	$_1H^2$	2.0142	$+e$	Heavy hydrogen, also written $_1D^2$
Triton	$_1H^3$	3.0171	$+e$	The nucleus a heavy heavy hydrogen-tritium ($_1T^3$) Half life = 12 yrs.
Alpha Particle	$_2He^4$	4.0028	$+2e$	The nucleus of a helium atom, symbol α particle.
Gamma Ray	$h\nu$	Zero Rest Mass	0	Also called quantum or a photon or X-ray.
Neutrino	$_0\eta^0$	Almost Zero Rest Mass	0	Rest mass is probably a few per cent that of the electron
Meson	π?	0.150	$\pm e$	Artificially produced by nuclear bombardment. Half life very short $\sim 10^{-8}$ seconds
Mesotron	μ	0.100	$\pm e$	Cosmic ray produced. Half life $= 2.1 \times 10^{-6}$ seconds

* Adapted from Table 12-1, "Nuclear Radiation Physics," R. E. Lapp and H. L. Andrews, Prentice-Hall, Inc. 1948.
[1] Expressed in atomic mass units. 1 Mass unit $= 1.6603 \times 10^{-24}$ gm $= 931.04$ Mev.
[2] Units are given in esu. The charge on an electron (e) is 4.80×10^{-10} esu.

Atomic Structure*

R. E. LAPP† AND H. L. ANDREWS‡

Summary—Data obtained from the Rutherford scattering experiment are used to present a simple picture of the structure of the atom. This picture is shown to be in agreement with the experimental findings on the electrical nature of matter. On the assumption that the particles which make up the nucleus (neucleons) are neutrons and protons, a "building-up" of the various atoms is explained and a system of nuclear nomenclature is developed. The model of an atom as a central nucleus of neutrons and protons surrounded by outer shells of electrons is found to fit in with the results of many physical and chemical experiments. In particular, the results of experimental spectroscopy yield quantitative agreement with certain calculations made on this atomic model. Bohr's theory of the hydrogen atom is developed in simple terms and the equation for energy level in the H-atom is derived. Excitation and ionization of atoms are described in terms of the Bohr model of the atom.

* Decimal classification: 539. Original manuscript received by the Institute, April 26, 1948. Reprinted by special permission of, and copyright by, Prentice-Hall, Inc., New York, N. Y., 1948.
† Research and Development Board, Washington, D. C.
‡ National Institute of Health, Bethesda, Md.

The Rutherford Scattering Experiment

THE RESULTS of many experiments throughout the years have agreed in showing that atoms have dimensions of the order of 10^{-8} cm, but no real progress in determining the internal structure of atoms was made until Rutherford carried out his scattering studies in 1911. The experimental arrangement used by Rutherford is sketched in Fig. 1. A stream of alpha particles (helium atoms from which all the electrons have

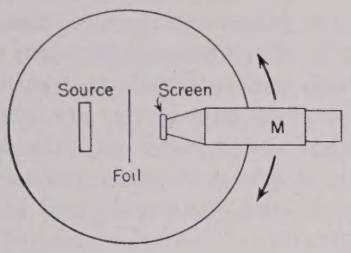


Fig. 1—The Rutherford scattering experiment. The source and foil are fixed while the microscope and fluorescent screen rotate about the foil.

been removed) was allowed to fall on a thin foil of gold, silver, or other metal. The angular distribution of the alpha particles scattered by the atoms in the foil was determined by counting the scintillations on a zinc-sulfide screen with a microscope.

This method of detection is based on the fact that an alpha particle striking a material such as zinc sulfide produces a short flash of visible light, which can be observed visually by a dark-adapted eye. Luminous instrument dials, which appear to emit a steady uniform light, consist of a mixture of radium salt which emits alpha particles and a phosphor in quantities sufficient to produce enormous numbers of scintillations.

The angular distribution of the scattered alpha particles would be expected to depend strongly on the structure of the scattering atoms. If matter consists of closely spaced atoms, each a sphere of about 10^{-8}-cm radius, no alpha particles would be expected to penetrate even thin metal foils. If each atom consists essentially of empty space with sharply localized masses and charges, the alpha particles should pass through thin foils without difficulty.

The results of scattering experiments depend, then, very strongly upon the structure of the scattering material. Before 1910, the atom was conceived to be a sphere of about 10^{-8}-cm radius, with the mass and charges distributed rather uniformly throughout the volume.

The Rutherford Model

Rutherford assumed that the positive electric charge in an atom is equal to an integral number Z times the electronic charge, and is concentrated in a small volume which contains most of the atomic mass. This relatively massive nucleus mass was assumed to be surrounded by Z electrons, so that the atom as a whole would be electrically neutral. The positively charged nucleus will repel and deflect the incident alpha particles, the exact amount of deflection depending on the initial path of the particle (see Fig. 2).

On the assumption that the repulsive forces obeyed the coulomb inverse-square law, Rutherford deduced

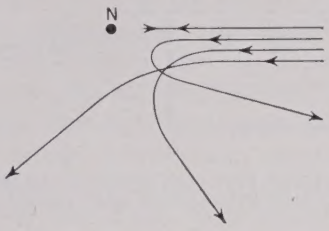


Fig. 2—The scattering of alpha particles by a nucleus.

an expression for the number of particles scattered as a function of the scattering angle. Experiments verified this relationship accurately, except for particles making almost head-on collisions with the nucleus. The experiments not only confirmed the original assumptions made by Rutherford, but also permitted an evaluation of the *atomic number Z*.

For the lighter elements, Z was found to have a value of about one-half the atomic weight, and when the elements were arranged in order of ascending Z it was found that the arrangement almost coincided with the periodic table proposed by Mendelyeev.

Rutherford's work definitely shows that matter consists mostly of empty space. The experimental results indicate that the relatively massive nucleus has a diameter of the order of 10^{-13} cm. Each electron surrounding this nucleus has a diameter of the same order of magnitude. The results from experiments on the macroscopic properties of materials agree well, and assign to the atom as a whole a diameter of about 10^{-8} cm. This value is one hundred thousand times the nuclear diameter, so that the atom must be as empty, on a minute scale, as our solar system.

The failure of the Rutherford scattering law at large angles can be explained by a failure of the inverse-square law at very small distances, since these alpha particles approach nearer to the nucleus than those less strongly scattered.

The Electrical Nature of Matter

Electrons can be liberated from matter in a number of ways. Thermionic emission and the photoelectric effect have already been described. Electrons always have the same properties, regardless of their origin. For example, the same value of specific charge e/m results when the measured values of m are corrected for variations in m with velocity. These negatively charged particles appear, therefore, to be elementary constituents of matter. They are fundamental particles, because no smaller

quantity of negative electricity has ever been observed.

When an electric current passes through a gas at low pressure, the free electrons move with high velocities toward the positively charged anode. Because most of these electrons originate near the cathode, they are frequently called cathode rays. In addition, a stream of heavy, positively charged particles will move toward the cathode (see Fig. 3). If a small hole or "canal" is

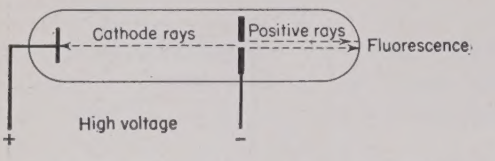


Fig. 3—A gas discharge tube. The canal rays or positive rays pass through a small hole in the cathode and impinge on the end of the glass tube.

bored in the cathode, some of these particles will stream through the aperture and will impinge on the end of the glass tube. Their presence can be demonstrated by the visible fluorescence of the glass. In a dark room the path of the *positive rays*, or *canal rays*, as they are sometimes called, is faintly visible behind the cathode as a glow whose color is characteristic of the gas in the tube. This stream of positive rays can be deflected by either electric or magnetic fields and hence must carry an electric charge.

It is evident that the originally neutral gas molecules are capable of separation into positively and negatively charged components, which are then accelerated in opposite direction by the electric field, thus forming the canal rays and cathode rays. When the discharge tube is filled with hydrogen gas, which has the lowest molecular weight of any element, the hydrogen molecule (H_2) is found to break up into two positively charged particles (H^+), which are called *protons*. The specific charge, e/m, can be measured from electric and magnetic deflections, and, since the charge e can be measured independently, the mass m can be determined. The mass is found to be 1.681×10^{-24} grams, or about 1840 times the mass of the electron. The charge of 1.60×10^{-19} coulombs is equal in magnitude and opposite in sign to the charge carried by an electron.

Although many exhaustive experiments have been carried out with hydrogen, no more than one electron has been found associated with one proton. This evidence shows that the hydrogen atom contains only two components—one proton and one electron. The following discussion of the atomic structure of hydrogen is based upon this fact.

The Simplest Atoms

Until the early 1930's, helium, about four times heavier than hydrogen, was thought to be the next lightest atom. However, Urey and his collaborators discovered a rare form of hydrogen that proved to be twice as heavy as ordinary hydrogen. Since it was discovered in the form of water, the discoverers called it "heavy water." Ordinary water has the chemical formula H_2O, whereas heavy water is often written as D_2O, which indicates that the heavy-water molecule is made up of one oxygen atom and two heavy hydrogen atoms. The symbol D stands for *deuterium*, which is the technical name given to heavy hydrogen. The deuterium atom is chemically identical with the hydrogen atom.

Since the H-atom is composed of one proton and one electron, it might be logical to assume that the D-atom, which is twice as heavy, would be made up of two protons and two electrons. However, it is known experimentally that the deuterium atom contains only 1 electron and 1 proton. Furthermore, the exact weight of the deuterium atom is not quite consistent with the assumption that the nucleus contains two protons. This inconsistency perplexed the physicist until the discovery of a neutral particle of almost the same mass as the proton. This new particle, the neutron, formed the key to the riddle of the structure of deuterium, as well as to that of all other elements.

Prior to the discovery of the neutron, physicists conceived of the deuterium nucleus, or atomic core, as being made up of 2 protons and 1 electron; the electron inside the nucleus was postulated in order to leave a single positive charge on the nucleus, for the negative electron would neutralize the charge of one of the protons. However, conclusive evidence showed that it is not possible for electrons to exist as separate enties within the nucleus.

With the discovery of the neutron, the deuterium atom was assumed to consist of a nucleus of 1 neutron and 1 proton. This assumption has been justified by the corrections of the results predicted for many nuclear experiments. These two elementary particles can be used to build up the nuclei of all heavier elements, such as helium, lithium, beryllium, iron, lead, and uranium.

Before proceeding to a discussion of the heavier elements, we will greatly simplify our discussion by introducing a consistent set of definitions and symbols, as follows:

Z = atomic number, the number of protons in the nucleus of an atom. An atom of atomic number Z has Z elementary positive charges in its nucleus and Z electrons outside the nucleus

A = mass number, the sum of all neutrons and protons in the nucleus of an atom

N = neutron number, the number of neutrons contained in the nucleus of an atom

n = the symbol for the neutron

p = the symbol for the proton

e = the symbol for the electron

Nucleon = a word that refers to either nuclear constituent, the neutron or the proton

Isotopes = two nuclei having the same number of protons but different numbers of neutrons.

In writing the formula for any isotope, the following form will be followed:

(symbol of element)^A or $_Z$(symbol of element)^A. Thus the form would be: for normal hydrogen, $_1H^1_3$; for heavy hydrogen, $_1H^2$ or $_1D^2$; for normal helium, $_2He^4$; for uranium-235, $_{92}U^{235}$; and so on. Strictly speaking, we need not include the subscript, since its value is fixed when the name of the element is given. The subscript is convenient for balancing nuclear transformation equations, and for this reason will be given frequently.

The Periodic System

Among the many elements, there are certain ones that bear remarkable relationships to one another in their physical and chemical characteristics. This relationship is particularly striking in the case of the alkaline-earth metals, Be, Mg, Ca, Sr, Ba, and Ra, all of which show remarkable similarities. All have the same chemical valence, are metallic, have low density, exhibit similar atomic spectra, and possess other similar physical characteristics.

Mendelyeev arranged the elements in a series of groups as shown in Table I. Such an arrangement is known as a *periodic table of elements*. Elements in any one of the vertical groups are chemically related. In the table, the number following the chemical symbol for the element gives the atomic weight of the element, and the lowest number in each box indicates the atomic number of the element.

Examination of any group in the periodic table reveals that the differences between the atomic numbers of consecutive members of a group follow a pattern—8, 8, 18, 18, 32. The electrons in an atom determine its chemical characteristics; hence, this regularity suggests that there is a building-up pattern followed by the electrons in the elements. The building-up principle requires that the electrons group themselves in discrete shells about the nucleus. Each shell has an upper limit to the number of electrons that it can hold. The elements having complete electron shells form the series of noble gases, which are characterized by their failure to enter into any chemical combinations.

All matter is composed of the three elementary particles: neutrons, protons, and electrons. Different chem-

TABLE I

Periodic Arrangement of the Elements

Series	Period	Zero Group	Group I R_2O	Group II RO	Group III R_2O_3	Group IV RH_4 RO_2	Group V RH_3 R_2O_5	Group VI RH_2 RO_3	Group VII RH R_2O_7	Group VIII		
0												
1			Hydrogen H = 1.0078 No. 1									
2	1	Helium He. = 4.002 No. 2	Lithium Li = 6.940 No. 3	Beryllium Be. = 9.02 No. 4	Boron B = 10.82 No. 5	Carbon C = 12.00 No. 6	Nitrogen N = 14.008 No. 7	Oxygen O = 16.000 No. 8	Fluorine F = 19.00 No. 9			
3	2	Neon Ne. = 20.183 No. 10	Sodium Na = 22.997 No. 11	Magnesium Mg. = 24.32 No. 12	Aluminum Al = 26.97 No. 13	Silicon Si = 28.06 No. 14	Phosphorus P = 31.02 No. 15	Sulfur S = 32.06 No. 16	Chlorine Cl = 35.457 No. 17			
4	3	Argon A. = 39.944 No. 18	Potassium K. = 39.10 No. 19	Calcium Ca. = 40.08 No. 20	Scandium Sc. = 45.10 No. 21	Titanium Ti = 47.90 No. 22	Vanadium V = 50.95 No. 23	Chromium Cr = 52.01 No. 24	Manganese Mn = 54.93 No. 25	Iron Fe = 55.84 No. 26	Cobalt Co = 58.94 No. 27	Nickel Ni = 58.69 No. 28
5			Copper Cu = 63.57 No. 29	Zinc Zn. = 65.38 No. 30	Gallium Ga = 69.72 No. 31	Germanium Ge. = 72.60 No. 32	Arsenic As = 74.93 No. 33	Selenium Se = 79.2 No. 34	Bromine Br = 79.916 No. 35			
6	4	Krypton Kr. = 82.9 No. 36	Rubidium Rb. = 85.44 No. 37	Strontium Sr. = 87.63 No. 38	Yttrium Y = 88.92 No. 39	Zirconium Zr. = 91.22 No. 40	Columbium Cb = 93.3 No. 41	Molybdenum Mo. = 96.0 No. 42	Masurium Ma = ? No. 43	Ruthenium Ru = 101.7 No. 44	Rhodium Rh = 102.91 No. 45	Palladium Pd = 106.7 No. 46
7			Silver Ag. = 107.880 No. 47	Cadmium Cd. = 112.41 No. 48	Indium In = 114.8 No. 49	Tin Sn. = 118.70 No. 50	Antimony Sb = 121.76 No. 51	Tellurium Te = 127.5 No. 52	Iodine I = 126.932 No. 53			
8	5	Xenon Xe. = 130.2 No. 54	Caesium Cs = 132.81 No. 55	Barium Ba. = 137.36 No. 56	Lanthanum La = 138.90 No. 57	Cerium Ce = 140.13 No. 58						
9												
10	6					Hafnium Hf = 178.6 No. 72	Tantalum Ta. = 181.4 No. 73	Tungsten W = 184.0 No. 74	Rhenium Re. = 186.31 No. 75	Osmium Os. = 190.8 No. 76	Iridium Ir. = 193.1 No. 77	Platinum Pt. = 195.23 No. 78
11			Gold Au = 197.2 No. 79	Mercury Hg = 200.61 No. 80	Thallium Tl = 204.39 No. 81	Lead Pb = 207.22 No. 82	Bismuth Bi. = 209.00 No. 83	Polonium Po. = 209.99 No. 84	Alabamine Am. = ? No. 85			
12	7	Radon Rn = 222 No. 86	Virginium Va = ? No. 87	Radium Ra = 225.97 No. 88	Actinium Ac = 227.02 No. 89	Thorium Th = 232.12 No. 90	Protoactinium Pa. = 231.03 No. 91	Uranium U = 238.14 No. 92	No. 93			

Elements Not Classified in the Table Above

raseodymium Pr. = 140.92 No. 59	Neodymium Nd. = 144.27 No. 60	Illinium Il. = 146.(?) No. 61	Samarium Sm. = 150.43 No. 62	Europium Eu. = 152.0 No. 63	Gadolinium Gd. = 157.3 No. 64	Terbium Tb. = 195.2 No. 65
Dysprosium Dy. = 162.46 No. 66	Holmium Ho. = 163.5 No. 67	Erbium Er. = 167.84 No. 68	Thulium Tm. = 169.4 No. 69	Ytterbium Yb. = 173.5 No. 70	Lutecium Lu. = 175.0 No. 71	

New Elements

Note: The following elements have been discovered in conjunction with the work of the Manhattan District:

Neptunium, Np, At. No. 93, At. Wt. 239
Plutonium, Pu, At. No. 94, At. Wt. 239
Americium, Am., At. No. 95
Curium, Cm, At. No. 96

ical elements result from different numbers of protons in the nucleus because there must be an equal number of electrons outside the nucleus to make an electrically neutral atom. With a constant number of nuclear protons the chemical behavior will be independent of the number of nuclear neutrons, since each of these isotopes will possess the same number of extranuclear electrons.

Atomic Spectra and Energy Levels

Each element emits a characteristic optical spectrum which is so specific that one can use it as a means of identifying the elements. For example, hydrogen gas, when electrically excited in a discharge tube, will emit radiation that can be dispersed by a prism or diffraction grating into a number of sharp lines (see Fig. 4). These lines form the atomic or line spectrum of hydrogen.

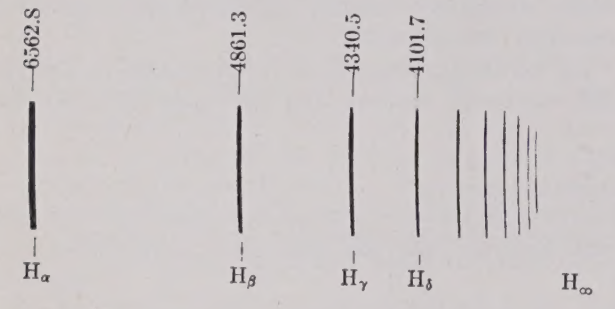


Fig. 4—Emission spectrum of the hydrogen atom in the visible and near ultraviolet region (Balmer series, Herzberg (41)). H_∞ gives the theoretical position of the series limit.

It was found empirically that some of the spectrum lines formed a series whose wavelength is given by the relation

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{M^2}\right] \quad \text{where} \quad M = 3, 4, 5, \cdots . \qquad (1)$$

The quantity $1/\lambda$ is the wave number τ (§1.03). R is known as the Rydberg constant and has a value of 109,677 cm^{-1}. Most of the lines described by (1) lie in the visible region of the spectrum and are known as the Balmer series.

In the ultraviolet region of the hydrogen spectrum there is a second group of lines called the Lyman series. The wavelength of the components of this series are given by

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{M^2}\right] \quad \text{where} \quad M = 2, 3, 4, \cdots . \qquad (2)$$

Other series of lines, all in the infrared, are recognized, and in each case the wavelengths are given by expressions having the form of (1) and (2) with different values of the denominators inside the brackets.

The wavelengths and wave numbers of the lines in the various series have been measured with great accuracy. Some of the values for the Balmer and Lyman series are given in Table II.

TABLE II

Balmer and Lyman Series Wavelengths and Wave Numbers

Balmar Series		Lyman Series	
λ (A)	τ (cm^{-1})	λ (A)	τ (cm^{-1})
6562.2 (H_3)	15,233	1215	82,258
4861.3	20,264	1026	97,482
4340.5	23,032	972	102,823
4101.7	24,373	950	105,290
Series Limit			
3636.0	27,419	911	109,677

If we substitute $M=3$ in (1), we find that the wave number of H_α is 15,233, in agreement with the value listed in column 2 of Table II. When $M=\infty$ is substituted in (1), the wave number of the limit of the Balmer series is obtained.

Instead of using the empirical equations, a new type of scheme can be employed to illustrate a fundamental principle of atomic spectra. The wave numbers of the series limits given in Table II have been plotted in Fig. 5, putting the Lyman-series limit value of 109,677

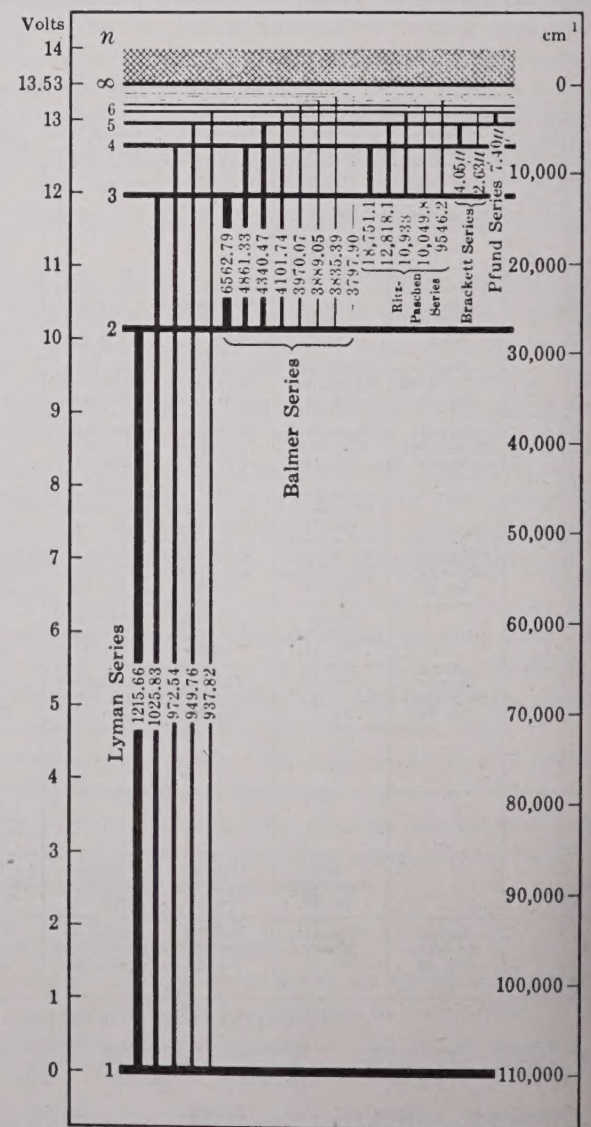


Fig. 5—Energy-level diagram of the H atom (Grotrian (8)).

cm^{-1} as the lowest line and the Balmer-series limit as the next line. Similarly, the series limits for the succeeding series in the hydrogen spectrum are located on succeeding lines in order of decreasing magnitude upward from the lowest level. The difference between wave numbers corresponding to the level $M'=1$ and $M'=2$ is $109,677-27,419=82,258$, which is the wave number of the first line in the Lyman series. Note that the value of M' corresponds to the constants in the first term of (1) and (2). In Fig. 5 the vertical lines indicate transitions from one level to another, and represent graphically the arithmetical calculation just carried out. In words, this principle can be stated as follows: For the spectrum of any atom, there always exists a series of wave numbers (term values), differences between which yield the wave number of the observed spectrum lines. This rule is known as the Ritz Combination Law. Tables used in connection with it, such as Table II, are called *term tables*. The principle signifies that every atom has definite energy levels. Transitions within the atom from one energy level to another result in the emission or absorption of a spectrum line of a characteristic wave number determined by the difference between the energy levels.

Not all atoms emit a spectrum as simple as that of hydrogen; nor, indeed, is the hydrogen spectrum really as simple as has been described. Fig. 6 shows a sodium

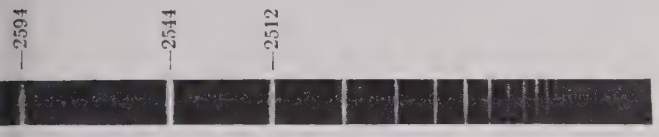


Fig. 6—Absorption spectrum of the Na atom (Kuhn (42)). The spectrogram gives only the short wavelength part, starting with the fifth line of the principal series. The lines appear as bright lines on a dark, continuous background, just as on the photographic plate.

absorption spectrum, each line of which is really a doublet instead of a single line. In general, the heavier elements emit correspondingly more complex spectra. Iron, for example, emits over 10,000 separate lines, many of which have been carefully measured and cataloged.

The Bohr Atom

The Rutherford concept of the hydrogen atom as a proton-electron system serves to explain the scattering of alpha particles, but it does not account for the stability of the system or for the emission of sharp and discrete spectral lines. Between the proton and the electron there is an attractive force given by

$$F = \frac{q_1 q_2}{r^2}$$

where in this case $q_1=q_2=e$. If the electron were at rest, it would be pulled into the proton by this coulomb force. The electron must therefore be in a continuous state of motion, and must experience a central acceleration. From classical electrodynamics, an accelerated electric charge should radiate energy, and hence the electron would quickly spiral into the nucleus after producing a continuum of protons. Thus in the classical picture there was no explanation of the stability of the hydrogen atom or of the emission of discrete line spectra.

In 1913 the Danish scientist, Niels Bohr, proposed a solution to this difficult problem by applying Planck's quantum hypothesis to an atomic system. Planck assumed that electromagnetic energy is emitted in quanta of energy E given by the relation

$$E = h\nu. \tag{3}$$

Bohr saw the relation between the Ritz combination principle and Planck's equation, and proposed the relation

$$h\nu = E_2 - E_1 \tag{4}$$

where

ν = is the frequency of the emitted electromagnetic radiation

E_2 = is the initial energy of the atom (prior to emission)

E_1 = is the final energy of the atom (after emission).

Since $\lambda\nu=c$, where c is the velocity of light, this equation can be written

$$\tau = 1/\lambda = \frac{E_2}{hc} - \frac{E_1}{hc}. \tag{5}$$

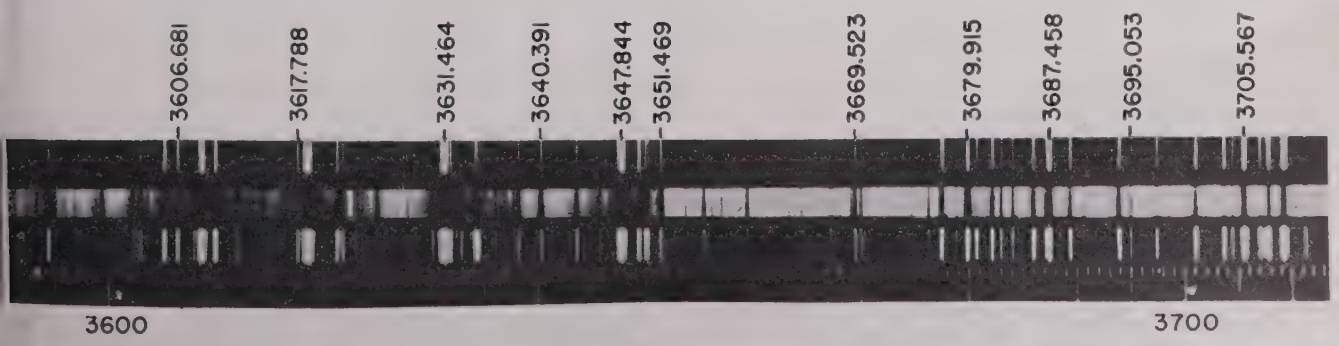


Fig. 7—More complex atomic spectra shown above are two emission spectra (bright line) taken of an iron arc. Bracketed by the ion comparison spectra is the absorption (dark line) spectrum of the sun. Only a portion (100 angstrom units) of the spectrum is illustrated.

From (5) it is clear that the values of a term in an atomic-term table are equal to the energy of a state of an atom multiplied by the constant factor $1/hc$. Analytically, the energy of an atomic state E is given by the relation

$$E_1 = \frac{hc}{\lambda}. \tag{6}$$

Equation (6) is equivalent to stating that there exists, for every atom, a set of discrete stationary states, each having a definite energy value. Radiation is emitted from an atom only when it changes from one stationary state to another; this transition results in radiation of wave number given by (5). We should emphasize that, while the atom is in a *stationary state*, no radiation is emitted. This result is, however, contrary to the predictions of classical theory.

According to the Bohr picture, the simple hydrogen atom should appear as in Fig. 8. The electron revolves about the nucleus, which is assumed to be stationary, in discrete orbits corresponding to the stationary states

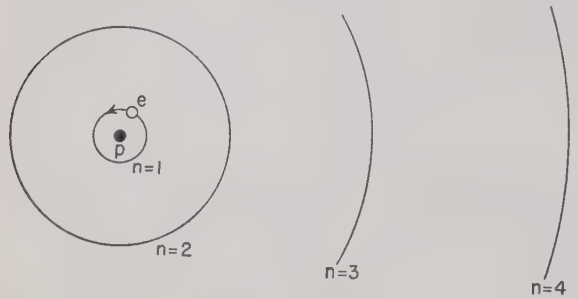


Fig. 8—The Bohr model of the hydrogen atom. The electron is in its lowest stationary state. The relative orbital radii are as shown but the proton and electron have been enlarged relatively about 10^5 times.

of the atom. Bohr further assumed that these stationary states are such that the momentum of the electron mv is *quantized* (restricted) to certain integral multiple values of $h/2\pi r$.

The first condition is fulfilled if the force required to accelerate the electron in its orbit (mv^2/r) is balanced by the electrostatic (coulomb) forces between the two particles. Therefore,

$$\frac{mv^2}{r} = \frac{e^2}{r^2}. \tag{7}$$

The second condition may be stated as

$$mvr = \frac{nh}{2\pi} \tag{8}$$

where

$$n = 1, 2, 3, 4, \cdots.$$

Using these relations we can calculate the total energy of an electron in one of its orbits. This calculation is sketched as follows: The total energy of the electron is equal to its kinetic plus its potential energy, and elementary electrostatic theory shows that the potential energy of a charge e distant r from an equal charge of opposite sign is $-(e^2/r)$. Then

$$E = \tfrac{1}{2}mv^2 - \frac{e^2}{r}. \tag{9}$$

Substituting from (7) and (8), we find that

$$E = -\frac{2\pi^2 me^4}{n^2 h^2}. \tag{10}$$

The fact that the energy is negative follows from the fact that energy of the electron far removed from the atom (that is, at infinity) is taken as zero. Since the force between the particles is attractive, work is done by the system in bringing the electron toward the atom; and hence the energy of the atom will be negative. The energy will have a different value for each value of n which corresponds to a different electron orbit in the atom.

By (4), the frequency of the radiation emitted when an electron falls from orbit n_2 to n_1 is

$$\nu = \frac{E_{n_1} - E_{n_2}}{h} = \frac{2\pi^2 me^4}{h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]. \tag{11}$$

Equation (11) is exactly the form of the empirical relation for the Balmer series (1) if $n_1 = 2$ and if the Rydberg constant is taken as

$$R = \frac{2\pi^2 me^4}{ch^3}. \tag{12}$$

All values on the right-hand side of (12) are known, and R can be calculated. When R is calculated, the result is in excellent agreement with the experimental value. This agreement furnishes a remarkable confirmation of the Bohr theory.

The mechanism of the emission of spectral lines requires that initially the electron be in one of the upper energy states $(n = 2, 3, 4, \cdots)$. This state has a greater energy content than the ground state $(n = 1)$, and the electron will return to the ground state either in one step or in a series of transitions through intermediate stationary states. The energy liberated by each transition to a lower state appears as a photon of the appropriate energy and frequency.

Excitation and Ionization

Normally, the electron in the hydrogen atom is in the ground state. In order to raise this electron to higher orbits farther away from the nucleus, energy must be supplied to the system. This energy is normally supplied by means of collisions with other atoms or electrons. The removal of an electron from an atom is known as *ionization*, and the resulting atom is said to be ionized. We can easily calculate how much energy must be supplied to the H-atom to completely remove the electron from it.

Illustrative Example

To calculate the ionization energy of hydrogen, n_2, corresponding to the final state, is set equal to infinity in (11). Then

$$E = \frac{2\pi^2 me^4}{h^3}\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$$

$$= \frac{2\pi^2 me^4}{h^3}.$$

When the values of the constants are substituted,

$$E = 2.2 \times 10^{-13} \text{ erg}$$

$$= 13.5 \text{ ev.}$$

The energy required to remove the electron from the H-atom can be measured and is found to check very closely with this calculated value. The potential necessary to ionize any atom is known as the *ionization potential.*

The experiment to determine the value of the ionization potential in hydrogen consists of shooting a beam of electrons of known energy (voltage) into a tube containing hydrogen gas at low pressure. If the bombarding electrons have high enough energy they will cause the characteristic spectrum lines to be emitted from the gas, and these lines can be observed with a spectroscope. As the energy of the beam is dropped below 13.5 ev, ionization ceases, but inelastic collisions between the incident electrons and hydrogen atoms occur and give rise to excitations of different series, and, indeed, of different lines in these series. This potential necessary to cause emission of a characteristic line in, say, the Lyman series is called the *excitation potential* for these lines.

Atomic spectra can be excited in many ways other than in a gaseous discharge tube. Commonly used methods are the carbon arc, the electric spark, and the hot flame.

In the heavier elements, it can be shown that the emission of ultraviolet, visible, and some infrared radiation is the result of transitions involving only the outermost electrons. Such radiations are completely independent of the inner electrons.

As the energy that is inelastically imparted to an atom increases, it is obvious from the relation $E = h\nu = hc/\lambda$ that the wavelength of the most energetic radiation which can be emitted decreases. Thus, if E is very small, the wavelengths that can be excited may be in the far infrared. Then, as E increases, the wavelength decreases, passing through the visible region and down into the ultraviolet. Through the use of special techniques, atomic spectral lines as short as 100 Angstroms can be observed. Radiations much shorter than 100 Å are usually known as X rays and are characteristically produced in special electron tubes.

APPENDIX[1]

Fundamentals of Waves and Radiation

Properties of Waves

It is easy to visualize the transmission of energy by moving particles that can give up their kinetic energy upon collision with stationary targets. Less obvious, but equally important, is the transfer of energy by wave motions. All of the sun's energy reaching the earth is transmitted by electromagnetic waves; all audible sounds are transmitted by sound waves; and some of the greatest radiation hazards arise from short-wavelength electromagnetic waves.

The general concept of wave transmission is simple. Some type of transmitter vibrates and sets up stresses in the surrounding medium. The medium, in attempting to return to its original unstressed state, propagates the stresses that travel out from the source and act on some suitable receiver. Thus, when human vocal cords vibrate, a succession of pressure impulses is applied to the surrounding air. Pressure above and below normal exist and are transmitted by each volume of air to the volume immediately beyond. The pressure variations travel out from the source and, upon striking an eardrum, set up vibrations that one interprets as sound.

It should be noticed that the medium transmitting the waves does not move from source to receiver. A disturbance of the medium does travel, but the medium itself does not move appreciably. A stone thrown into a quiet pool will produce a series of waves that spread out in concentric circles from the source of the disturbance. A cork floating on the surface bobs up and down as the waves pass, but it does not move outward from the source as it would if there were an actual outward flow of water.

The most obvious property of a wave is its *frequency*, or number of vibrations per second, denoted by the letter ν. Frequency is really to be associated with the source of the disturbance, since the wave is merely the transmission of the disturbance. The frequency of the wave is, of course, the same as the frequency of the source. It is sometimes more convenient to use the time of one vibration T, known as the *period*, instead of the frequency. The two are related by

$$\nu = \frac{1}{T}. \tag{13}$$

Since a wave is propagated, there will be a velocity associated with the wave motion. The velocity is a property of the medium and not of the source, for once the disturbance has been transmitted to the medium it is independent of the behavior of source. In a given medium a wave motion will be propagated only with its characteristic velocity. This behavior is in sharp con-

[1] Arranged from portions of "Nuclear Radiation Physics," by R. E. Lapp and H. L. Andrews, Prentice-Hall, Inc., New York, N. Y., 1948.

trast to the motion of particles, whose velocities depend upon their energies.

If a source is vibrating steadily with a frequency ν and the velocity of wave propagation is c, at the end of 1 second there will be ν waves spread over a distance c in space. One wave will occupy a distance λ, where

$$\lambda = \frac{c}{\nu} \cdot \tag{14}$$

The distance λ, or *wavelength*, is the shortest distance between consecutive similar points on the wave train. Thus it is the distance between consecutive crests or peaks, or consecutive troughs (see Fig. 9). Spectroscopists frequently specify a wave by the *wave number* τ or the number of waves occupying one centimeter,

$$\tau = \frac{1}{\lambda} \cdot \tag{15}$$

Fig. 9 illustrates another characteristic of wave motion; its amplitude, A. If $00'$ represents the undisturbed position of the medium, the amplitude is the distance from this undisturbed position to a point of maximum displacement. As might be expected, the amplitude is closely related to the energy carried by the wave.

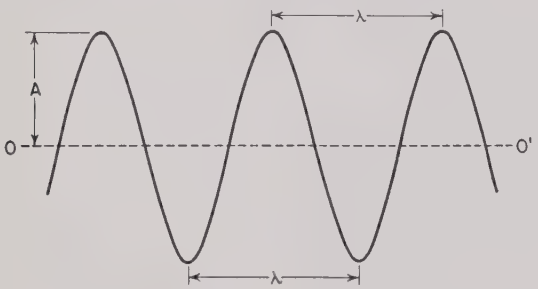


Fig. 9—The fundamental relation existing in a wave motion. The figure shows the spatial conditions at a given instant.

If a source starts a wave motion in the surrounding medium, the waves will, in general, spread out uniformly in all directions. Consider again the case of a stone thrown into a quiet pool. If, at some later time, an instantaneous photograph is taken of the water, a series of concentric rings of maximum upward displacement (crests) will be observed, and in between these there will be a series of concentric rings of maximum downward displacement (troughs). Each circle drawn through points having the same displacement (or phase) is known as a wave front. The wave fronts move out from the source as the wave proceeds. At every point the direction of propagation is perpendicular to the wave front. Lines drawn in the direction of propagation are called rays. Fig. 10 shows the situation for the water waves described above. U, U', U'' are *double-prime* wave fronts of maximum upward phase; D, D', D'' are wave fronts of maximum downward phase; and R, R', R'' are rays.

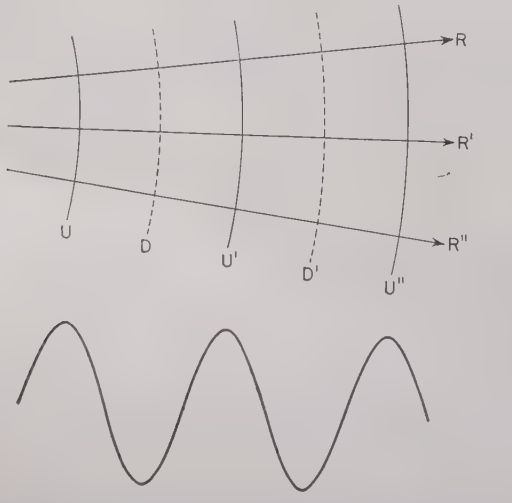


Fig. 10—The relations between wavefronts and rays. The lower curve is a vertical cross section through the ray R'.

There is sometimes a confusion in terminology between waves and rays. Thus it has become customary to speak of radio waves and X rays. Actually, each is a wave motion, having both wave fronts and rays. It should also be noted that in the older literature the terms "alpha ray" and "beta ray" are commonly used. These are now known to be high-speed charged particles and so should be called, respectively, alpha particles and beta particles, rather than rays.

All wave motions exhibit certain phenomena when they move from one medium to another having different characteristics. One of these phenomena is *reflection*. When a ray strikes a reflecting surface, one component of its velocity is reversed in direction, and the ray is propagated in a new direction such that the angle of reflection (the angle between the reflected ray and a perpendicular to the surface) is equal to the angle of incidence (the angle between the original ray and the perpendicular). All waves exhibit reflection, but not all reflections arise from wave motions. For example, a tennis ball will bounce off a smooth wall with an angle of reflection equal to the angle of incidence.

Wave motions are *refracted* when they enter a new medium having a different velocity of propagation. For example, a ray entering a new medium where it has a lower velocity will be bent so that it makes a smaller angle with the perpendicular to the surface. Refraction is also exhibited by motions other than waves.

Diffraction and *interference*, however, are phenomena that are unique to wave motions. If two waves of equal wavelength, but opposite phase, are superimposed, a crest will fall on a trough; the combined additive effect will be a complete cancellation, and the two waves are said to *interfere destructively*. If the two superimposed waves are in phase, a crest will fall on a crest, an increased intensity will result, and the two waves show *constructive interference*. It is extremely difficult to conceive of particles showing constructive or destructive interference.

Interference is most easily demonstrated through the

diffraction or bending of waves as they pass through small openings. Consider a wave front AB of visible light, striking a diaphragm with two small openings or slits S_1 and S_2 (Fig. 11). Only a small portion of the wave front will enter each of the slits. Because of diffraction, these small portions of the wave front will spread out in hemispherical waves after passing through the slits. If a point C on the screen is so chosen that the distance S_1C is exactly one wavelength greater than S_2C, there will be constructive interference, and light will be seen at C. If, again, S_1D is one-half wavelength greater than S_2D, there will be destructive interference and no light will be seen at D. Thus a series of light and dark bands will appear on the screen.

This experiment, which demonstrates the wave nature of light, can be used to measure the wavelength. As can be seen from Fig. 11, the angle of diffraction will be appreciable only if the distance between the slits is approximately equal to the wavelength. This fact accounts for the difficulty physicists have in identifying X and gamma rays, whose wavelengths are 10^{-8} cm or less.

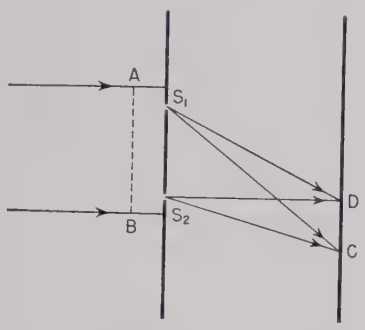


Fig. 11—Interference of light. The light passing through S_1 and S_2 will be diffracted and will interfere destructively at D and constructively at C. The interference pattern will be repeated on the other side of the center line of the slits.

The wave properties described are very general and hold for any type of wave motion. Waves may be classified in many ways, the most common being based upon the direction in which displacement occurs in the wave. In a sound wave, the small to-and-fro motions of the air molecules are along the direction of propagation. This condition represents a *longitudinal* wave. If the end of a rope is given a sideways displacement, a *transverse* wave is propagated down the rope at right angles to the displacement. Visible light, X rays, and gamma rays are all examples of transverse waves, and make up part of the *electromagnetic spectrum*.

The Quantum Nature of Electromagnetic Waves

All the components of the electromagnetic spectrum have wave properties; they show interference, and can be refracted and diffracted. Such phenomena as the radiation from hot bodies (black-body radiation), the production of X rays, and the photoelectric effect require the quantum concept for a complete explanation. The photoelectric effect will serve as an example.

A photoelectric cell consists of a metallic surface (cathode) sealed into an evacuated glass bulb together with a positively charged collecting electrode, called an anode (Fig. 12). When light falls on the cathode surface, electrons may be ejected from the metal. These electrons will be attracted to the anode and an electric current will flow.

According to wave theory, the vibrations of the incoming electromagnetic waves set the electrons of the metallic atoms into vibration with increasing amplitude until they have acquired sufficient energy to break loose from the metal. It would seem, then, that a weak light would require more time to liberate an electron than a strong light, and there might be a light wave so weak that the electrons could never attain the energy necessary for escape. Such a prediction is at complete variance with the facts.

The color or frequency of the light plays an important role in producing photoelectrons. Thus the most intense beam of red light (low frequency) will not yield a single photoelectron from most metals, whereas the feeblest blue light (high frequency) will instantly produce a few. This circumstance suggested to Einstein, who reasoned from earlier work by Planck, that radiation is not a smooth, continuous flow of energy, as pictured by the wave theory, but is, rather, a series of discontinuous packages of energy. The energy in each package, which is known as a *photon* or *quantum*, increases with the frequency of the light, being given by

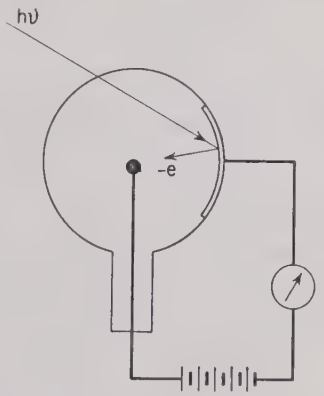


Fig. 12—The photoelectric effect. An incoming photon $h\nu$ ejects an electron $-e$ from the cathode surface.

$$E = h\nu \tag{16}$$

where $h = 6.6 \times 10^{-27}$ erg-sec is Planck's constant and E is the energy of the quantum in ergs.

A photon is apparently indivisible, and either exists or disappears completely upon giving up its energy in some process such as the photoelectric effect. The energy relations in the photoelectric effect are given by the Einstein equation

$$h\nu = \phi + \tfrac{1}{2}mv^2. \tag{17}$$

The left-hand side of this equation is the energy available in the incoming photon. The quantity ϕ (known as the work function) is the energy required to remove

an electron from the surface of the metal. If the photon has an energy greater than ϕ, the electron will be ejected with a kinetic energy given by the last term in (17).

This result immediately explains the photoelectric behavior of light of different frequency. Red light has a low frequency, and the associated quantum energy, by (13), may be less than ϕ, and hence no amount will liberate an electron. Blue light has a higher frequency, and the associated quantum energy may be larger than ϕ, and hence sufficient to produce a photoelectric current. A series of experiments by Millikan verified the prediction of (17), and there can be no doubt that here radiation behaves like a discontinuous series of energy packages.

Furthermore, a reverse application of (17) quantitatively explains the production of X rays. Here the incident electron has a kinetic energy $1/2mv^2$ and can produce no radiation of frequency greater than that given by

$$\tfrac{1}{2}mv^2 = hv. \tag{18}$$

In this case the work function ϕ is so small compared with $1/2mv^2$ that one can neglect it. This relation has also been completely verified by experiment.

In 1901, Planck made the first application of quantum ideas to the problem of the radiation emitted by hot bodies. Physicists had developed the idealized concept of a black body, which is capable of emitting and absorbing radiations of all frequencies. The ideal radiator is approached by an enclosed furnace with a small opening through which one may view the interior radiations. The radiation observed through the opening will depend only upon the temperature of the furnace and does not have "color" characteristic of an emitting surface. Careful measurements had been made of the amounts of energy radiated at various frequencies. Fig. 13 shows the experimental results. For each temperature there is a corresponding frequency at which maximum energy is radiated. The frequency of this maximum increases with temperature, according to Wien's displacement law, which is usually written in terms of wavelength rather than frequency:

$$\lambda_m T = \text{constant} \tag{19}$$

where

λ_m = wavelength of maximum energy emission
T = absolute temperature
constant = 0.29 cm degrees.

This is a very useful relation, since by its use the temperature of inaccessible hot bodies may be determined. For example, the maximum energy of the sun is emitted at a wavelength of about 4.5×10^{-5} cm (green light), which corresponds to a temperature of about 6000°C.

Many attempts had been made to explain the shape of the observed emission curves, assuming radiation to be a continuous wave moton, but all these efforts yielded curves that were too high at the short wavelengths. When Planck introduced the idea of radiation quanta, the difficulties disappeared and the predicted curves agreed in all details with those experimentally observed. This is another example of the dual nature of radiation, and it should be noted that the original theories failed at the high frequencies where the quantum characteristics become more pronounced.

Before leaving the discussion of black-body radiation, one should note that the curves of the Fig. 13 represent the total energy radiated at each wavelength. The size of each emitted quantum increases steadily as the wavelength decreases, in accordance with (13), but the energy emitted is, of course, equal to the product of the number of quanta and the energy of a single quantum. At high frequencies the number of emitted photons is so low that the emission curves decrease as shown.

The quantum energy as calculated from (16) will be expressed in ergs. A new energy unit, the *electron volt* (ev) has been found useful in studying nuclear radiations. Obvious extensions are the kiloelectron volt (kev) and million electron volt (Mev). It is customary to speak of a "50-kilovolt X ray" or a "2-Mev gamma ray." These terms are specifications of the quantum energy, and do not give the total energy of all the photons in a beam. In contrast to these energies which are commonplace in studies of nuclear radiation physics, the quantum energy associated with a 100-Mc radio wave will be 4.1×10^{-7} ev.

It is evident from the foregoing that we must ascribe a duality of characteristics to radiant energy. When propagation is being considered, the wave properties predominate and serve to explain the phenomena of interference and diffraction. When energy exchanges with matter are involved, the quantum characteristics are most important. This latter property is particularly apparent at the shorter wavelengths.

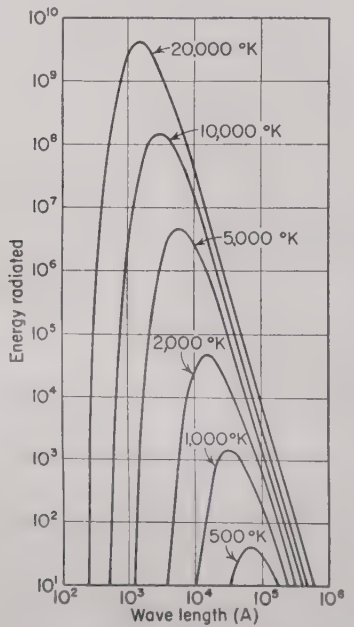


Fig. 13—The wavelength dependence of the energy emitted by a perfect black body.

Theory of Frequency-Modulation Noise*

F. L. H. M. STUMPERS†

Summary—**The energy spectrum of frequency-modulation noise is computed for different ratios of signal to noise. Numerical values are given for some simple filter amplitude characteristics. The theory is based on the Fourier concept of noise and treated in three steps: no signal, signal without modulation, and modulated signal. The result is given in the form of a series, and it is shown that this development is convergent. The suppression of the modulation by noise is also discussed.**

I. Introduction

SINCE THE PAPER by Armstrong[1] drew attention to the possibilities of frequency modulation with regard to the reduction of noise, a considerable amount of work has been published in this field. So far as is known, however, the theoretical treatment of noise and signal has been confined to the case in which the noise energy is small compared to the signal energy. In this paper we will try to give a rigorous treatment valid for all signal-to-noise energy ratios. The theory is developed by methods which Fränz[2,3] and Rice[4] applied to similar problems and which are based on the Fourier spectrum of the noise. An interesting idea of Mann[5] has been used for the counting of the number of zeros.

Usually the instantaneous frequency of a frequency-modulated signal is defined as the derivative of the phase with respect to the time. (For this definition, see van der Pol.[6]) In this section an alternative definition is given, which is more suitable for our further computations. It will be shown that, for a normal frequency-modulated signal, it gives the same result as the usual definition. Using this starting point, we further deduce a mathematical expression for the energy spectrum, and give a first example of its application. We shall confine ourselves to signals consisting of high-frequency components in such a way that all important components lie within a relatively narrow band $\omega_0 \pm \Delta\omega$ where $\Delta\omega \ll \omega_0$.

For a sinusoidal signal $\cos \omega_0 t$, the angular frequency is equal to the number of zeros in a time interval of π seconds. Now we choose a time interval τ, large compared to π/ω_0 but small compared to $\pi/2\Delta\omega$:

$$\omega_0 \tau \gg \pi, \qquad 2\Delta\omega\tau \ll \pi. \tag{1}$$

The *instantaneous frequency* is defined at the time t as the ratio of the number of zeros between $t-\tau/2$ and $t+\tau/2$ to τ/π, or as the mean density of the zeros averaged over the time interval τ/π.

As an example, let us take the signal

$$\cos \{\omega_0 t + f(t)\}.$$

If this function has consecutive zeros at $t=t_1$ and $t=t_2$, then

$$\omega_0(t_1 - t_2) + f(t_1) - f(t_2) = \pi.$$

If we assume that $f'(t)$ changes slowly compared to $\cos \omega_0 t$, then we can replace $f(t_1) - f(t_2)$ by $(t_1 - t_2) f'(t_1)$, and thus obtain:

$$t_1 - t_2 = \pi / \{\omega_0 + f'(t_1)\}.$$

τ being defined in such a way that $f'(t)$ is practically constant during a time interval τ, the number of zeros within the time τ is $\tau\{\omega_0 + f'(t)\}/\pi$. The definition of the instantaneous frequency above thus gives the result:

$$\omega_i(t) = \omega_0 + f'(t). \tag{2}$$

This, as we have stated, is the same result as is obtained on the basis of the usual definition.

The counting of the number of zeros of a function $v(t)$ within an interval τ is best done with the help of δ-functions such as are used in the operational calculus. The δ-function is defined by:

$$\delta(x) = 0, \qquad x \neq 0,$$
$$\delta(x) = \infty, \qquad x = 0,$$
$$\int_{-\infty}^{+\infty} \delta(x)dx = 1.$$

It is plausible to consider the integral

$$\int_{t_0-\tau/2}^{t_0+\tau/2} \delta\{v(t)\} v'(t)dt.$$

If $v(t)$ has a simple zero in a certain interval, the absolute value of the integral over that interval is 1. In the subsequent interval, which also is assumed to contain only one simple zero, the sign of the result will be different. The reason is that, when we introduce $v(t)$ as a new variable in the integral, this variable of integration runs from a negative to a positive value in one interval, and in the opposite direction in the next.

Therefore we modify our procedure so as to count only those zeros passed through with a positive slope.

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The instantaneous frequency is then

$$\omega_i(t_0) = \frac{2\pi}{\tau}\int_{t_0-\tau/2}^{t_0+\tau/2}\delta(v)v'U(v')dt \qquad (3)$$

in which

$$U(x) = 0, \qquad x < 0,$$
$$U(x) = \tfrac{1}{2}, \qquad x = 0,$$
$$U(x) = 1, \qquad x > 0.$$

The result is a function of τ, but as long as τ is subjected to the inequalities (1), the variation with τ will be unimportant.

The use of δ functions in the integrals can be avoided when we use the Stieltjes integral. In this way (3) is written:

$$\omega_i(t_0) = \frac{2\pi}{\tau}\int_{t_0-\tau/2}^{t_0+\tau/2}U(v')dU(v). \qquad (3a)$$

In a frequency-modulation receiver, a device is used which, when a signal is applied to it, gives an output voltage proportional to the instantaneous frequency of that signal. This device is the frequency detector, or discriminator. In general, the instantaneous frequency will not be constant. We can make a registration of it during a certain time. A Fourier analysis of that registration will give a spectrum of components at different frequencies. It is in this spectrum of the detected signal that we are now interested.

When we choose a time interval of 2π seconds for the Fourier analysis we have the formulas:

$$\omega_i(t) = \sum f_m e^{imt}$$
$$f_m = (2\pi)^{-1}\int_0^{2\pi}\omega_i(t)e^{-imt}dt. \qquad (4)$$

Let us consider a zero t_0 of $v(t)$, where the function has a positive slope whereas t_0 is not too near to 0 or 2π. Its contribution to the integral (4) is then

$$\tau^{-1}\int_{t_0-\tau/2}^{t_0+\tau/2}e^{-imt}dt = 2(m\tau)^{-1}\sin(m\tau/2)e^{-imt_0}.$$

In this result, $2(m\tau)^{-1}\sin(m\tau/2)$ approximates 1 for small values of τ. We have chosen $\tau \ll \pi/2\Delta\omega$. Therefore, for all frequencies smaller than $\Delta\omega$ we may replace $2(m\tau)^{-1}\sin(m\tau/2)$ by 1. Accordingly, for these frequencies we may use

$$f_m = \int_0^{2\pi}\delta(v)v'U(v')e^{-imt}dt. \qquad (5)$$

Using the Laplace transform, we get

$$\delta(v) = (2\pi)^{-1}\int_{-\infty-ic}^{+\infty-ic}e^{iuv}du, \quad c > 0 \qquad (6a)$$

$$v'U(v') = -(2\pi)^{-1}\int_{-\infty-ic}^{\infty-ic}e^{iuv'}u^{-2}du, \quad c > 0. \qquad (6b)$$

The path of integration can also be taken from $-\infty$ to ∞ along the real axis with a small indentation below the origin.

$$f_m = -(4\pi^2)^{-1}\int_{-\pi}^{\pi}e^{-imt}dt\int_{-\infty}^{+\infty}du_1\int_{-\infty}^{+\infty}du_2u_2^{-2}e^{iu_1v+iu_2v'}$$

This formula gives the spectral composition of the instantaneous frequency. If the frequency detector gives a potential difference of 1 volt over a resistance of 1 ohm for a frequency deviation of 1 radian per second, the same formula applies to the output of the frequency detector. We shall calculate the distribution of the energy, dissipated in that resistance, over the spectrum. The result is the energy spectrum of the output. In practical cases there will be a proportionality factor, which is omitted in our calculations. The energy corresponding to a certain frequency m is given by $2f_mf_m^*$ if $m \neq 0$, or by $f_0f_0^*$ for the dc term. f_m^* is the complex conjugate of f_m. Hence,

$$2f_mf_m^* = (8\pi^4)^{-1}\int_{-\pi}^{\pi}e^{-imt_1}dt_1\int_{-\pi}^{\pi}e^{imt_2}dt_2$$
$$\int\int\int\int_{-\infty}^{+\infty}u_2^{-2}u_4^{-2}e^{iu_1v(t_1)+iu_2v'(t_1)+iu_3v(t_2)+iu_4v'(t_2)}$$
$$du_1du_2du_3du_4. \qquad (7)$$

As a first example, we shall apply this formula to a frequency-modulated signal $\cos(\omega_0t+m_0\sin pt)$. $(m_0=\Delta\omega/p)$. In this case,

$$e^{iu_1v(t_1)+iu_2v'(t_1)} = \exp\{iu_1\cos(\omega_0t + m_0\sin pt_1) - iu_2(\omega_0 + \Delta\omega\cos pt_1)\sin(\omega_0t_1 + m_0\sin pt_1)\}.$$

We develop this form into a series:

$$\sum_{k=0}^{\infty}\frac{2^{-k}}{k!}\{(iu_1 - u_{2a})e^{i\omega_0t_1+i\,m_0\sin pt_1} + (iu_1 + u_{2a})e^{-i\omega_0t_1-im_0\sin pt_1}\}^k$$

in which $u_{2a}=u_2(\omega_0+\Delta\omega\cos pt_1)$.

Since, in (7), we have to integrate the last result with e^{-imt_1}, where $m \ll \omega_0$ we are only interested in those terms in the binomial development that have no $i\,\omega_0t_1$ in the exponent. Thus the series reduces to

$$\sum_{k=0}^{\infty}\frac{2^{-2k}}{k!k!}(u_1^2 + u_{2a}^2)^k.$$

This is the well-known development of the Bessel function of order zero. A relation between Bessel functions gives

$$J_0\{u_1^2 + u_2^2(\omega_0 + \Delta\omega\cos pt)^2\}^{1/2} = \sum_{-\infty}^{\infty}(-1)^mJ_{2m}(u_1)J_{2m}\{u_2(\omega_0 + \Delta\omega\cos pt_1)\}.$$

Now we perform the integration with respect to u_1 and u_2:

$$\int_{-\infty}^{+\infty} J_{2m}(u_1)du_1 = 2$$

$$\int_{-\infty}^{\infty} u_2^{-2}J_{2m}\{u_2(\omega_0 + \Delta\omega \cos pt_1)\}du_2$$

$$= (\omega_0 + \Delta\omega \cos pt_1)\{(2m-1)^{-1} - (2m+1)^{-1}\}.$$

(The last integral reduces to an easier type by one partial integration.) For $m=0$ the singularity at the origin is avoided by the small indentation. The result of the integration with respect to u_1 and u_2 is, therefore,

$$(\omega_0 + \Delta\omega \cos pt_1)\{-4 + 4(-1 + \tfrac{1}{3}) + 4(\tfrac{1}{3} - \tfrac{1}{5}) + \cdots\}$$

$$= -2\pi(\omega_0 + \Delta\omega \cos pt_1).$$

In the same way, the integrations with respect to u_3 and u_4 yield

$$-2\pi(\omega_0 + \Delta\omega \cos pt_2),$$

and, in total,

$$2f_m f_m^* = (2\pi^2)^{-1}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} e^{-imt_1}e^{+imt_2}(\omega_0 + \Delta\omega \cos pt_1)$$

$$(\omega_0 + \Delta\omega \cos pt_2)dt_1dt_2.$$

Only for $m=0$ and $m=p$ do we get a result different from zero. The dc energy is

$$f_0 f_0^* = \omega_0^2.$$

For the frequency p, the energy is

$$2f_p f_p^* = \frac{\Delta\omega^2}{2}.$$

These results are in complete agreement with the customary definition of instantaneous frequency. We have chosen this simple problem because the way in which it is solved will again be used in the more complicated problems further on. Its aim is also to give the reader confidence in the following computations, where the result is less obvious.

II. Frequency-Modulation Noise Without Signal

In this section we will first recall some properties of a noise spectrum, and then apply (7) to a noise band. As is usual in noise problems, an averaging procedure will be necessary.

By means of a filter we select a certain band of frequencies from a normal noise source, and apply these components as an input signal to an ideal frequency detector. As a first example, we shall take a filter with a rectangular amplitude-versus-frequency characteristic. This filter is not realizable but, as the phase characteristic is not important for these computations, it can be approximated. Later on we shall consider a filter with a gaussian amplitude-versus-frequency characteristic.

If we register the noise from a normal noise source during a time interval $(-T, T)$, we can make a Fourier analysis:

$$v(t) = \sum a_n \cos n\pi t/T - b_n \sin n\pi t/T. \tag{8}$$

When such a Fourier analysis is made a great many times consecutively, each time over an interval of the same length, the Fourier components will show a gaussian probability distribution:

$$W(a_n)da_n = (\pi C)^{-1/2}e^{-a_n^2/C}da_n \tag{9a}$$

$$W(b_n)db_n = (\pi C)^{-1/2}e^{-b_n^2/C}db_n. \tag{9b}$$

The value of a_n in one particular Fourier analysis is independent of the value of the other coefficients. This subject is treated extensively by Fränz and Rice.

The number of noise lines in a band of $2\Delta\omega$ radians will be $2\Delta\omega T/\pi$. As it makes the formulas simpler, we shall choose $T=2\pi$. If we made another choice, the formulas (4) to (7) which are also based on a time interval 2π, would need an appropriate modification. The choice of 2π is, however, quite arbitrary, and, whenever we find it advisable to increase the number of lines in a part of the spectrum, we shall do so.

The average energy per component is

$$\overline{\tfrac{1}{2}(a_n^2 + b_n^2)} = (2\pi C)^{-1}\iint_{-\infty}^{+\infty}(a_n^2 + b_n^2)e^{-(a_n^2+b_n^2)/C}da_n db_n$$

$$= C/2.$$

The effective voltage corresponding to a noise band extending from $\omega_0 - \Delta\omega$ to $\omega_0 + \Delta\omega$ is

$$v_{n0} = (\Delta\omega C)^{1/2}.$$

As the number of lines in the band increases proportionally to the length of the considered time interval, the average amplitude has to be reduced so as to keep the average power constant.

When we introduce the $v(t)$ of (8) in (7), the result will be a function of the $4\Delta\omega$ variables a_n and b_n. As is usual in noise computations, the average of the result of (7) over all a's and b's is used to obtain the effective energy spectrum after detection. Thus,

$$\overline{2f_m f_m^*} = 2\int\int\int\cdots\int da_{N_1}\cdots db_{N_2}$$

$$W(a_{N_1}\cdots b_{N_2})f_m f_m(a_{N_1}\cdots b_{N_2})$$

$$N_1 = \omega_0 - \Delta\omega; \qquad N_2 = \omega_0 + \Delta\omega.$$

The integration does not lead to great difficulties (see Appendix I). After introduction of a new variable $s = n - \omega_0$, we obtain

$$\overline{f_0 f_0^*} = \omega_0^2 + \Delta\omega^2/3. \tag{13a}$$

The dc corresponding to the central frequency is usually suppressed by balanced detection. For the frequency m we obtain the energy in the form of an integral:

$$\overline{2f_m f_m^*}$$

$$= (2\pi)^{-1} \sum_{k=1}^{\infty} k^{-1} N^{-2k} \int_{-\pi}^{\pi} e^{-imv} \{ (\sum e^{isv})^{2k-1} (\sum s^2 e^{isv}) - (\sum e^{isv})^{2k-2} (\sum s e^{isv})^2 \} dv \quad (N = 2\Delta\omega). \tag{13b}$$

The summation over s has to be taken over all integers satisfying

$$-\Delta\omega \leqq s \leqq \Delta\omega.$$

To get the energy in a part of the spectrum, the results for all frequencies m in this part are totaled. Now we are free to increase the number of lines by enlargement of the intervals $2T$, which we have so far chosen $2T = 2\pi$. In this way a continuous energy distribution $E_0(u)$ will be approximated, and the sum

$$N^{-1} \sum e^{isv}$$

can be replaced by the integral

$$\int_{-1/2}^{1/2} e^{iuv} du.$$

Instead of s/N we have introduced the continuous variable u. In the same way,

$$N^{-1} \sum_{-\Delta\omega}^{\Delta\omega} s^2 e^{isv} = N^2 \int_{-1/2}^{1/2} u^2 e^{iuv} du.$$

Equation (13b) can now be written:

$$E_0(u) = \sum_{k=1}^{\infty} 4\,(k)^{-1} \Delta\omega^2 h_{2k}(u) \tag{14}$$

where $h_{2k}(u)$ is given by the relation

$$\int_{-\infty}^{+\infty} h_{2k}(u) e^{iuv} du = \left\{ \int_{-1/2}^{1/2} e^{iuv} du \right\}^{2k-1} \left\{ \int_{-1/2}^{1/2} u^2 e^{iuv} du \right\} - \left\{ \int_{-1/2}^{1/2} e^{iuv} du \right\}^{2k-2} \left\{ \int_{-1/2}^{1/2} u e^{iuv} \right\}^2. \tag{15}$$

The values of $h_{2k}(u)$ can be derived directly from this integral, but a shorter computation will be treated in the next section. There we shall also see that, for k large,

$$h_{2k}(u) \approx (12)^{-1} (5k - 3)^{-1/2} \pi^{-1/2} (15)^{1/2} e^{-15u^2/(5k-3)}. \tag{16}$$

The series in (14) is convergent, since for large k the general term behaves as $k^{-3/2}$. We have calculated the values for $E_0(u)$ as shown in Table I.

Let a filter with a symmetrical, but otherwise arbitrary, amplitude characteristic be used, the calculations being slightly modified. If now the characteristic is given by $f(\omega)e^{i\phi(\omega)}$, the input signal will be:

$$v(t) = \sum f(n) \{ a_n \cos(nt + \phi_n) - b_n \sin(nt + \phi_n) \} \tag{8a}$$

$f(\omega)$ be normalized in such a way that its maximum value is 1, and the bandwidth $2\Delta\omega$ of the filter be defined by

$$\int_{-\infty}^{+\infty} f^2(\omega) d\omega = 2\Delta\omega.$$

Then (13a) does not change.

As before, we introduce a new variable $u = \omega/2\Delta\omega$. The function $F(u)$ is so defined that $F(u) = f^2(\omega - \omega_0)$; then

$$\int_{-\infty}^{+\infty} F(u) du = 1. \tag{18}$$

Instead of (14), we obtain (see Appendix I)

$$E_0(u) = \sum_{k=1}^{\infty} 4(k)^{-1} (\Delta\omega)^2 \quad H_{2k}(u)$$

in which $H_{2k}(u)$ is now given by

$$\int_{-\infty}^{\infty} H_{2k}(u) e^{iuv} du = \left\{ \int F(u) e^{iuv} du \right\}^{2k-1} \left\{ \int F(u) u^2 e^{iuv} du \right\} - \left\{ \int F(u) e^{iuv} du \right\}^{2k-2} \left\{ \int u F(u) e^{iuv} du \right\}^2. \tag{15a}$$

When $F(u)$ is given, all further functions can be found successively by direct integration. Here, too, the operational calculus may furnish a shorter method of calculation, as is shown in the next section.

As an example, take a gaussian amplitude characteristic. The normalized squared amplitude characteristic is given by $F(u) = e^{-\pi u^2}$. Then, as is shown in the next section,

TABLE I

Noise energy and noise voltage as a function of the frequency ($u = \omega/2\Delta\omega$). Energy per unit bandwidth. No carrier wave present. Rectangular filter amplitude characteristic.

$2u = \omega/\Delta\omega$	$E_0(u)$	$v_0(u) = \{E_0(u)\}^{1/2}$
0	1.2241 $\Delta\omega^2$	1.1064 $\Delta\omega$
0.1	1.1274	1.0634
0.2	1.0381	1.0189
0.3	0.9557	0.9776
0.4	0.8799	0.9380
0.5	0.8101	0.9001
0.6	0.7462	0.8638
0.7	0.6881	0.8295
0.8	0.6354	0.7971
0.9	0.5877	0.7666
1.0	0.5445	0.7379

TABLE II

Noise energy and noise voltage (per unit bandwidth) as a function of the frequency ($u = \omega/2\Delta\omega$). No carrier wave. Gaussian amplitude characteristic.

$2u = \omega/\Delta\omega$	$E_0(u)$	$V_0(u) = \{E_0(u)\}^{1/2}$
0	1.17594 $\Delta\omega^2$	1.08441 $\Delta\omega$
0.2	1.15719	1.07573
0.4	1.10377	1.05061
0.6	1.02373	1.01180
0.8	0.92702	0.96282
1	0.82527	0.90844

$$H_{2k}(u) = 2^{-3/2}k^{-1/2}\pi^{-1}e^{-\pi u^2/2k}. \quad (18)$$

Table II shows the result. For filters with a nonsymmetrical amplitude characteristic, the computation leads to longer formulas, as shown in Appendix I. For this case we have not computed a numerical example.

III. Some Distribution Functions

In our computations of the energy spectra some functions occur regularly, and we shall treat them together in this section. At first let us consider the problem of finding the product distribution when the two functions f_1 and f_2 are given.

$$\int_{-\infty}^{+\infty} f_1(u)e^{iuv}du \int_{-\infty}^{+\infty} f_2(w)e^{iwv}dw = \int_{-\infty}^{+\infty} f_3(x)e^{ixv}dx. \quad (19)$$

This can be done directly by considering the product as a double integral and by the introduction of $x=u+w$ as a new variable in this integral. The Laplace transforms, when known, are of much help. Let $f_{p,1}(p)$ be the image of $f_1(u)$:

$$f_{p,1}(p) \doteq f_1(u),$$

which shorthand notation stands for:

$$f_{p,1}(p) = p\int_{-\infty}^{+\infty} f_1(u)e^{-pu}du.$$

Then, upon introducing $p=-iv$ in (19), we get at once

$$f_{p,3} = p^{-1}f_{p,1}f_{p,2}.$$

Thus, for the rectangular distribution, the product functions are found by

$$\left\{\int_{-1/2}^{1/2} e^{iuv}du\right\}^k = \int_{-\infty}^{+\infty} f_k(u)e^{iuv}du \quad (20a)$$

$$f_1(u) = 1, \qquad -\tfrac{1}{2} \leqq u \leqq \tfrac{1}{2}$$
$$= 0, \qquad -\infty < u < -\tfrac{1}{2}, \qquad \tfrac{1}{2} < u < \infty.$$
$$f_1(u) \doteq e^{p/2} - e^{-p/2} = 2\sinh p/2$$
$$f_k(u) \doteq p^{1-k}(e^{p/2} - e^{-p/2})^k = p^{1-k}(2\sinh p/2)^k.$$

Therefore,

$$f_k(u) = \sum_{r=0}^{k} r\binom{k}{r}\frac{(u+k/2-r)^{k-1}}{(k-1)!}U(u+k/2-r). \quad (20b)$$

For the definition of U, see (3).

A function is computed from its Laplace transform by means of the inversion integral (Bromwich); for instance,

$$f_k(u) = (2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty} p^{-1}f_{p,k}(p)e^{pu}dp.$$

For large k the integrand of this integral (here to be taken along the imaginary axis) has its maximum for $p=0$, and can be approximated by its development for small p.

$$p^{1-k}(e^{p/2} - e^{-p/2})^k \approx pe^{p^2k/24}.$$

The result is an approximation of $f_k(u)$ for large k:

$$f_k(u) \approx 6^{1/2}k^{-1/2}\pi^{-1/2}e^{-6u^2/k}. \quad (21)$$

We arrive at another type of distribution function by differentiation of (20a):

$$\left\{\int_{-1/2}^{1/2} e^{iuv}du\right\}^{k-1}\left\{\int_{-1/2}^{1/2} ue^{iuv}du\right\} = \int_{-\infty}^{+\infty} c_k(u)e^{iuv}du$$
$$c_k(u) = uf_k(u)/k.$$

A third type, of which we have already met examples, is

$$\left\{\int_{-1/2}^{1/2} e^{iuv}du\right\}^{k-1}\left\{\int_{-1/2}^{1/2} u^2e^{iuv}du\right\} = \int_{-\infty}^{+\infty} a_k(u)e^{iuv}du$$
$$a_1(u) = u^2, \qquad -\tfrac{1}{2} \leqq u \leqq \tfrac{1}{2}$$
$$a_1(u) = 0, \qquad -\infty < u < -\tfrac{1}{2}, \qquad \tfrac{1}{2} < u < \infty.$$
$$a_1(u) \doteq 4^{-1}p^{-2}(p^2 - 4p + 8)e^{p/2} - 4^{-1}p^{-2}(p^2 + 4p + 8)e^{-p/2}$$
$$a_k(u) \doteq 4^{-1}p^{-1-k}\{(p^2 - 4p + 8)e^{p/2} - (p^2 + 4p + 8)e^{-p/2}\}(e^{p/2} - e^{-p/2})^{k-1}.$$

This gives, for instance,

$$a_2(u) = \tfrac{1}{3}u^3 + \tfrac{1}{2}u^2 + \tfrac{1}{4}u + \tfrac{1}{12}, \qquad -1 \leqq u \leqq 0$$
$$= -\tfrac{1}{3}u^3 + \tfrac{1}{2}u^2 - \tfrac{1}{4}u + \tfrac{1}{12}, \qquad 0 \leqq u \leqq 1$$
$$= 0, \qquad -\infty < u < -1, \qquad 1 < u < \infty.$$

Approximation of the inversion integral leads to the result:

$$a_k(u) \doteq a_{p,k}(p) \approx \frac{p}{12}e^{(5k+4)p^2/120}$$
$$a_k(u) \approx (12)^{-1}(30)^{1/2}(5k+4)^{-1/2}\pi^{-1/2}e^{-30u^2/(5k+4)}, \qquad k \text{ large.} \quad (22)$$

Another function worth consideration is

$$\left\{\int_{-1/2}^{1/2} ue^{iuv}du\right\}^2 = \int_{-\infty}^{+\infty} b_2(u)e^{iuv}du$$
$$\left\{\int_{-1/2}^{1/2} e^{iuv}du\right\}^{k-2}\left\{\int_{-1/2}^{1/2} ue^{iuv}du\right\}^2 = \int_{-\infty}^{+\infty} b_k(u)e^{iuv}du \qquad k \geqq 2.$$
$$b_1(u) = 0.$$

The Laplace transform gives in this case:

$$b_k(u) \doteq 4^{-1}p^{-1-k}\{(2-p)e^{p/2} - (2+p)e^{-p/2}\}^2(e^{p/2} - e^{-p/2})^{k-2}.$$

Again the originals are easily found; for instance,

$$b_2(u) = \tfrac{1}{6}u^3 - \tfrac{1}{4}u - \tfrac{1}{12}, \qquad -1 \leqq u \leqq 0$$
$$= -\tfrac{1}{6}u^3 + \tfrac{1}{4}u - \tfrac{1}{12}, \qquad 0 \leqq u \leqq 1$$
$$= 0, \qquad -\infty < u < -1, \qquad 1 < u < \infty.$$

For large k we get the approximation:

$$b_k(u) \approx -(72)^{-1}(30)^{3/2}(\pi)^{-1/2}(5k-4)^{-3/2}\left\{1 - 60u^2/(5k-4)\right\}e^{-30u^2/(5k-4)}. \quad (23)$$

The function $h_k(u)$ which we have used in the second section is defined by $h_k(u) = a_k(u) - b_k(u)$. Therefore,

$$h_k(u) \doteq p^{-1-k}\left\{e^p - (p^2+2) + e^{-p}\right\}(e^{p/2} - e^{-p/2})^{(k-2)}.$$

For $h_2(u)$, the result is

$$h_2(u) = \tfrac{1}{6}(u+1)^3, \qquad -1 \leqq u \leqq 0$$
$$= \tfrac{1}{6}(1-u)^3, \qquad 0 \leqq u \leqq 1$$
$$= 0, \qquad -\infty < u < -1, \qquad 1 < u < \infty.$$

For large k,

$$h_k(u) \doteq h_{p,k}(p) \approx \tfrac{1}{12}pe^{(5k-6)p^2/120}$$
$$h_k(u) \approx (12)^{-1}(30)^{1/2}(5k-6)^{-1/2}\pi^{-1/2}e^{-30u^2/(5k-6)}. \quad (24)$$

The approximate formulas are already fairly good for low values of k; for instance,

$h_4(u) = 0.0667$ (exact) 0.0688 (approx.)
$h_6(u) = 0.0512$ (exact) 0.0526 (approx.)

By differentiating the equation (20a) twice, we arrive at a relation between the functions:

$$kh_k(u) = -k^2b_k(u) + u^2f_k(u).$$

The functions $f_k(u)$ in particular have been treated frequently since De Moivre.[7] All these functions have a place in the theory of averages. One may compare Maurer's[8] paper, where some asymptotic formulas are derived in a more precise way. So far the rectangular distribution has been our starting point, but the computation can be made for another type as well. The gaussian frequency distribution is attractive because it gives simple results. Moreover, we have already pointed out that a gaussian amplitude-versus-frequency characteristic may be better approximated by real conditions than a rectangular one.

Corresponding to the original distribution $F_1(u) = e^{-\pi u^2}$, we get, in the same way as before,

$$\left\{\int_{-\infty}^{+\infty} e^{-\pi u^2 + iuv}du\right\}^k = \int_{-\infty}^{+\infty} F_k(u)e^{iuv}du$$
$$F_k(u) = k^{-1/2}e^{-\pi u^2/k}. \quad (21a)$$

Analogous to $a_k(u)$, we get

$$A_k(u) = \tfrac{1}{2}k^{-5/2}e^{-\pi u^2/k}\left\{2u^2 + k(k-1)/\pi\right\} \quad (22a)$$
$$k = 1, 2, 3, \cdots,$$

and, instead of $b_k(u)$, we get

$$B_k(u) = \tfrac{1}{2}k^{-5/2}e^{-\pi u^2/k}(2u^2 - k/\pi) \quad (23a)$$
$$k = 2, 3, \cdots, \qquad B_1(u) = 0.$$

Here,

$$\int_{-\infty}^{+\infty} A_k(u)e^{iuv}du = \left\{\int_{-\infty}^{+\infty} F_1(u)e^{iuv}du\right\}^{k-1}\left\{\int_{-\infty}^{+\infty} u^2F_1(u)e^{iuv}du\right\}.$$

In the same way, $h_k(u)$ is replaced by

$$H_k(u) = \frac{1}{2\pi}k^{-1/2}e^{-\pi u^2/k} \quad (24a)$$
$$k = 2, 3, \cdots, \qquad H_1(u) = A_1(u).$$

All these functions have simple Laplace transforms, and are therefore easily found by this method.

IV. Frequency-Modulation Noise in the Presence of a Nonmodulated Carrier Wave

When an unmodulated carrier-wave is present, together with a rectangular noise spectrum symmetrically around it, the input signal is given by:

$$v(t) = \cos\omega_0 t + \sum_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega}(a_n\cos nt - b_n\sin nt) \quad (25)$$

This function is substituted in (7) and the average is taken in the same way as in (12). Some comments on the integration are given in Appendix II.

The dc energy is now

$$\overline{f_0f_0^*} = \omega_0^2 + \tfrac{1}{3}(\Delta\omega)^2e^{-1/NC} \quad (29)$$

$1/NC$ is the quotient of signal energy and noise energy at the input of the frequency detector ($N = 2\Delta\omega$). After introduction of the continuous variable u in the same way as in Section 2, the energy spectrum is given by:

$$E_1(u)/4(\Delta\omega)^2 = \sum_{r=1}^{\infty} r^{-1}e^{-2/NC}\,{}_1F_1{}^2(-r+1, 1, 1/NC)h_{2r}(u)$$
$$\sum_{r=1}^{\infty}\sum_{r=0}^{1/2(k-1)}\binom{k-r}{r}\frac{e^{-2/NC}}{(k-r)^2(k-2r)!(NC)^{k-2r}}$$
$${}_1F_1{}^2(-r+1, k-2r+1, 1/NC)\left\{kh_k(u) + (k-2r)^2b_k(u)\right\}. \quad (30)$$

In this expression ${}_1F_1$ is the confluent hypergeometric function, and $h_k(u)$ and $b_k(u)$ are the functions defined in Section 3. For the calculation we begin with the term for $k=1$ and add the terms for the higher values of k until they are sufficiently small. The convergence of the development is shown in Appendix II. The first term in the development of $E_1(u)$ is

$$4(\Delta\omega)^2NC(1 - e^{-1/NC})^2h_1(u)$$
$$= 4(\Delta\omega)^2NC(1 - e^{-1/NC})^2u^2, \qquad 0 \leqq u \leqq \tfrac{1}{2}.$$

[7] A. De Moivre, "Mensura sortis," 1711; "Miscellanea analytica," 1730.

[8] L. Maurer, "Ueber die Mittelwerthe der Funktionen einer reellen Variabelen," *Math. Ann.*, vol. 47, pp. 263–280; 1896.

As we are interested in the energy spectrum, only non-negative values of u are important. For small values of NC the first term gives a good approximation of the energy. Then the effective noise voltage is $2\Delta\omega u(NC)^{1/2}$. This gives the well-known triangular noise spectrum which is already given by the simplified analysis.

The second term in the development of $E_1(u)$ is

$$4\Delta\omega^2\left[e^{-2/NC}h_2(u) + N^2C^2\left\{1 - (1 + 1/NC)e^{-1/NC}\right\}^2 \cdot \left\{h_2(u) + 2b_2(u)\right\}\right].$$

The third term is

$$4\Delta\omega^2\left[2N^3C^3\left\{1 - (1 + 1/NC + 1/2N^2C^2)e^{-1/NC}\right\}^2 \cdot \left\{h_3(u) + 3b_3(u)\right\} + (2NC)^{-1}e^{-2/NC}\left\{3h_3(u) + b_3(u)\right\}\right].$$

For small values of NC the terms containing N^kC^k form an asymptotic expansion (asymptotic for $NC \to 0$). We were led to this development when trying to get a more precise estimate from the same starting point as the simplified analysis.[9] However, for the calculation of the output noise for larger ratios of input noise energy to signal energy, one has to take into account the full terms of the development (30). For very large values of NC it is seen that the terms of the development (14) are predominant, thus affirming the result of Section 2.

With the help of (30) we have calculated the energy spectrum by adding up the terms up to $k = 10$, or $2r = 10$, and making a graphical estimate for the remainder. In the following table the effective noise voltage $\{E_1(u)\}^{1/2}$ is given for $u = 0$, 0.1, 0.2, 0.3, 0.4, and 0.5 (corresponding to frequencies 0, $0.2\Delta\omega$, $0.4\Delta\omega$, $0.6\Delta\omega$, $0.8\Delta\omega$, and $\Delta\omega$). It is seen that for $NC = 0.1$ the deviation from the triangular spectrum is still very small. For this value of NC, the output noise still grows linearly with the input noise. (Strict linearity would give 0.3162 instead of 0.3208). There is already a marked deviation from the triangular spectrum for $NC = 0.2$. All results are shown graphically in Fig. 1.

TABLE III

Effective noise voltage as a function of frequency and input noise-to-signal energy ratio. Rectangular amplitude characteristic.

Input noise-to-signal ratio NC	$\{E_1(u)\}^{1/2}/\Delta\omega$					
	$u=0$	$u=0.1$	$u=0.2$	$u=0.3$	$u=0.4$	$u=0.5$
0.01	0	0.0200	0.0400	0.0600	0.0800	0.1000
0.1	0	0.06485	0.1294	0.1935	0.2574	0.3208
0.2	0.04032	0.1014	0.1900	0.2802	0.3702	0.4594
0.5	0.2763	0.2988	0.3679	0.4564	0.5658	0.6750
1	0.5275	0.5191	0.5500	0.5933	0.6739	0.7616
2	0.7071	0.6664	0.6500	0.6582	0.6882	0.7377
5	0.8836	0.8143	0.7593	0.7207	0.6991	0.6930
10	1.0260	0.9512	0.8868	0.8336	0.7920	0.7630

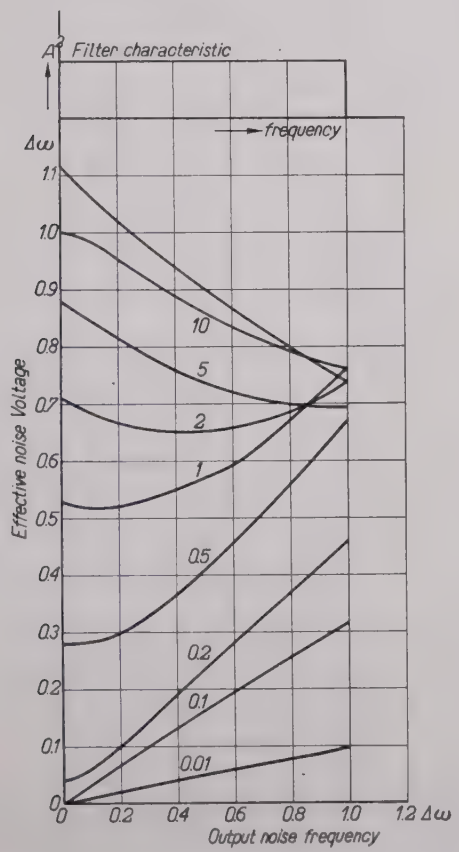

Fig. 1—Spectrum of effective noise voltage after detection. Parameter is input noise-to-signal energy ratio (NC). Rectangular amplitude-versus-frequency characteristic of the filter. Remark the triangular spectrum for $NC = 0.01$ and 0.1. The rms voltage of the noise in a small band of B cps is $(B/2\Delta\omega)^{1/2}$ times the value given by the curve.

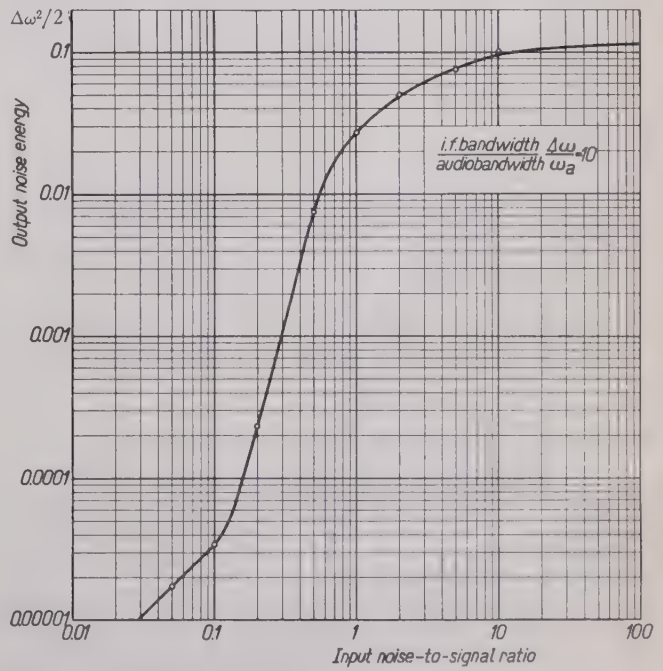

Fig. 2—Output noise energy as a function of input noise-to-signal energy ratio. If bandwidth is 10 times af bandwidth. Rectangular amplitude characteristic. Owing to the slow convergence of the series, the values for $NC = 5$ and 10 are less accurate.

[9] F. L. H. M. Stumpers, "Eenige onderzoekingen over trillingen et frequentiemodulatie," (in Dutch), diss. *Delft*, pp. 38–46; 1946.

For radio reception only the audible noise is important. The ratio of the audio-frequency bandwidth to the intermediate-frequency bandwidth can vary between 0.1 and 1. The noise energy is computed by integration of the noise energy $E_1(u)$ between appropriate boundaries. For a ratio of 0.1 one has to take into account the noise between 0 and $0.1\Delta\omega$ (or $0 \leqq u \leqq 0.05$). Figs. 2 and 3 show the energy of the output noise as a function of the ratio of input noise to signal. Typical is the strong increase of the noise above $NC = 0.1$ in the curves for $\Delta\omega/\omega_a = 5$, or 10, as compared to the curve for $\Delta\omega/\omega_a = 1$. This effect was found experimentally by Guy and Morris.[10] The influence of pre-emphasis can be calculated by multiplying the energy distribution after detection by $(1+R^2C^2\omega^2)^{-1}$. This we have done for an audio-frequency bandwidth of 15,000 cps and an RC time of 75.10^{-6} seconds. The result is shown in Figs. 4 and 5.

TABLE IV

Effective noise voltage (per unit frequency) as a function of frequency and input noise-to-signal ratio. Gaussian amplitude characteristic.

NC	$\{E_1(u)\}^{1/2}/\Delta\omega$					
	$u=0$	$u=0.1$	$u=0.2$	$u=0.3$	$u=0.4$	$u=0.5$
0.01	0	0.01969	0.03756	0.05209	0.06222	0.06752
0.1	0.004684	0.06369	0.1214	0.1686	0.2009	0.2200
0.2	0.06681	0.1129	0.1865	0.2513	0.2983	0.3253
0.5	0.3486	0.3706	0.4223	0.4809	0.5278	0.5554
1	0.5932	0.6054	0.6363	0.6724	0.7008	0.7137
2	0.7999	0.8050	0.8176	0.8311	0.8380	0.8334
5	0.9579	0.9577	0.9563	0.9513	0.9399	0.9203
10	1.0183	1.0160	1.0090	0.9963	0.9772	0.9578

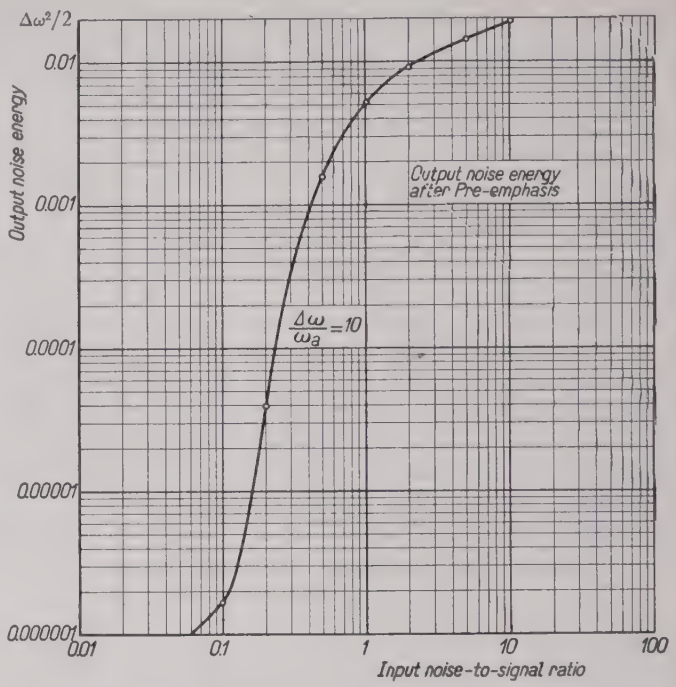

Fig. 4—Output noise energy as a function of input noise-to-signal ratio, when pre-emphasis is applied with an RC time constant of 75.10^{-6} seconds. Analogous to Fig. 2.

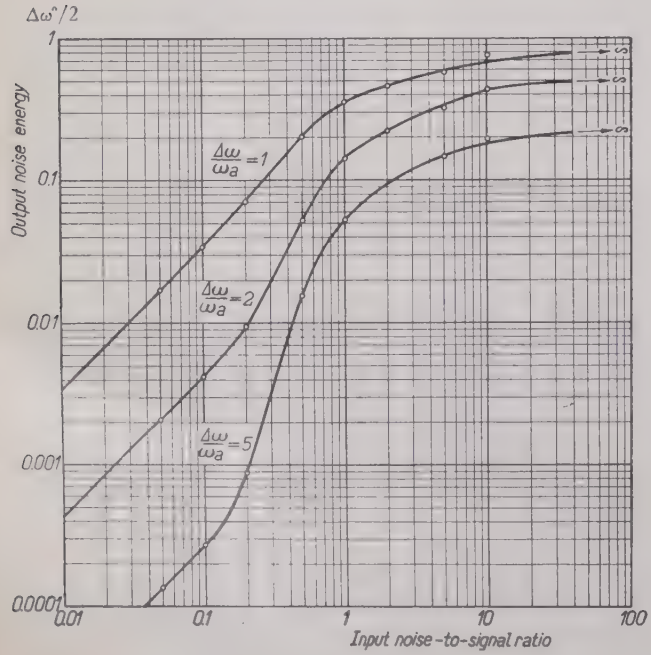

Fig. 3—Output noise energy as a function of input noise-to-signal energy ratio. If bandwidth is 5, 2, or 1 times af bandwidth. Rectangular filter characteristic.

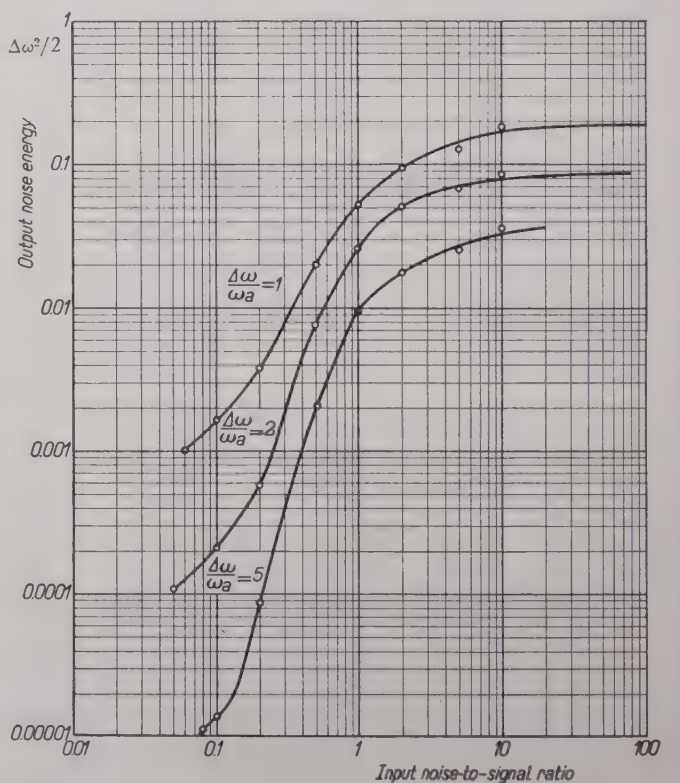

Fig. 5—Output noise energy as a function of input noise-to-signal energy ratio. Pre-emphasis applied. RC time constant, 75.10^{-6} seconds. Analogous to Fig. 3.

[10] R. F. Guy and R. M. Morris, "N.B.C. frequency modulation field test," *RCA Rev.*, vol. 5, pp. 190–225; October, 1940.

So far, the filter amplitude characteristic has been idealized to a rectangular form. As in Section 2, we shall consider now a gaussian amplitude characteristic, which provides a better approximation of actual conditions. All calculations are similar to those already given, and we have only to replace the functions $h_k(u)$ by $H_k(u)$, $a_k(u)$ by $A_k(u)$, $b_k(u)$ by $B_k(u)$, etc., in the final result. Compare (21a) to (24a). The effective noise voltage is given as a function of the frequency and the input noise-to-signal ratio in Table IV.

The results are shown in Fig. 6. Comparison with Fig. 1 makes it clear that the general behavior does not change, although there are minor deviations. In Figs. 7 and 8 the energy of the noise is drawn as a function of the input noise-to-signal ratio in the same way as in Figs. 2 and 3, but now for a gaussian amplitude characteristic.

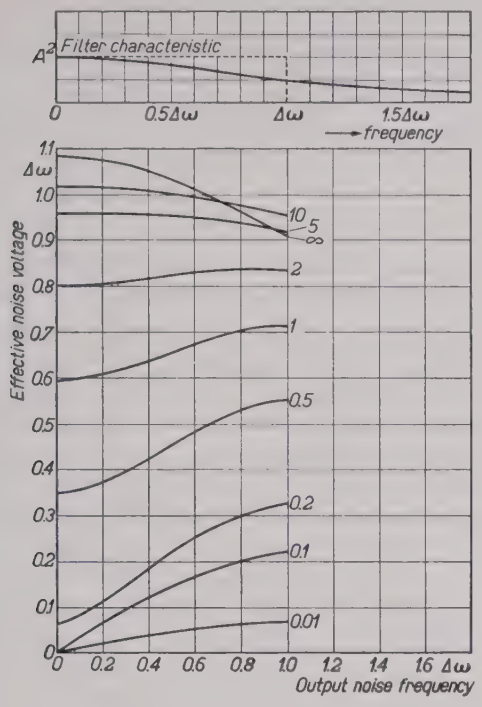

Fig. 6—Spectrum of effective noise voltage (per unit frequency bandwidth) after detection. Parameter is input noise-to-signal ratio (*NC*). Gaussian amplitude-versus-frequency characteristic of the filter. See also Fig. 1.

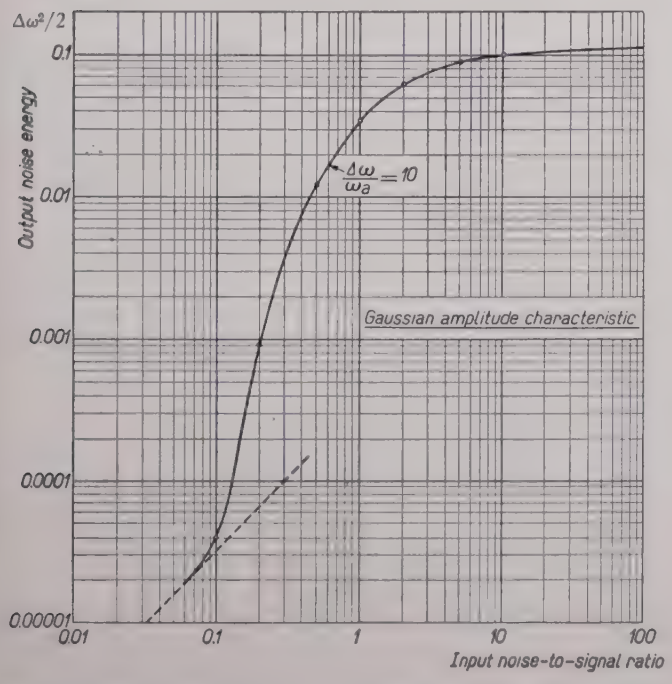

Fig. 7—Output noise energy of a receiver with an ideal frequency detector as a function of input-noise-to-signal energy ratio (*NC*). Gaussian amplitude characteristic of the filter. If bandwidth (energetically defined) 10 times af bandwidth.

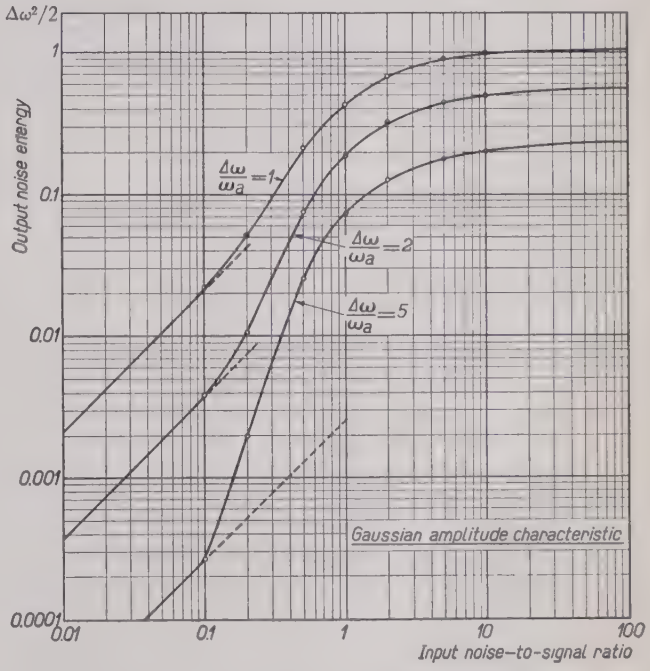

Fig. 8—Analogous to Fig. 7, but if bandwidth 5, 2, or 1 times the af bandwidth.

V. Noise in the Presence of a Frequency-Modulated Signal. Suppression of the Modulation by Noise

When a frequency-modulated signal is amplified in a receiver, there may be some distortion of the modulation due to insufficient bandwidth or to a nonlinear phase characteristic. In the following calculations we shall leave this effect out of account and assume that the signal passes the filter undistorted.

With a rectangular amplitude characteristic of the filter, the input-signal is given by

$$v(t) = \cos(\omega_0 t + m_1 \sin pt) + \sum_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} (a_n \cos nt - b_n \sin nt) \qquad (31)$$

$$m_1 = \Delta\omega_1 / p.$$

It is necessary to substitute this function $v(t)$ in (7) and to take the average, as in (12). Some remarks on the integration are given in Appendix III. Use is made of the following abbreviations:

$$\alpha = \frac{1}{(k-r)^2(k-2r)!}\binom{k-r}{r}\frac{e^{-2/NC}}{(NC)^{k-2r}}$$

$$J_m\{(k-2r)m_1\} = J_m$$

$${}_1F_1(-r+1,\ k-2r+1,\ 1/NC) = X$$

$${}_1F_1(-r,\ k-2r+1,\ 1/NC) = Y$$

$$u - mp/2\Delta\omega = u_m.$$

The noise energy is then given by

$$E_2(u)/4\Delta\omega^2 = \sum_{r=1}^{\infty} r^{-1}e^{2/NC}{}_1F_1(-r+1,1,1/NC)h_{2r}(u)$$
$$+\sum_{k=1}^{\infty}\sum_{r=0}^{1/2(k-1)}\sum_{m=-\infty}^{+\infty}[\alpha X^2 J_m{}^2\{kh_k(u_m)$$
$$+(k-2r)^2 b_k(u_m)\}$$
$$+2\alpha k^{-1}X\{kX-2(k-r)Y\}mp(2\Delta\omega)^{-1}J_m{}^2u_m f_k(u_m)$$
$$+\alpha(k-2r)^{-2}\{kX-2(k-r)Y\}^2m^2p^2(2\Delta\omega)^{-2}J_m{}^2 f_k(u_m)]$$
$$+\sum_{r=0}^{\infty}\tfrac{1}{2}(NC)^{-2}e^{-2/NC}{}_1F_1{}^2(-r+1,\ 2,\ 1/NC)$$
$$(\Delta\omega_1)^2(2\Delta\omega)^{-2}\{f_{2r}(u_{-1})+f_{2r}(u_{+1})\}. \qquad (33)$$

J_m is the Bessel function of order m and argument $(k-2r)m_1$. The functions $h_k(u)$, $b_k(u)$, $f_k(u)$ are discussed in Section III. We have not yet used $f_0(u)=\delta(u)$ (this is the same δ-function as used in Section I).

In calculating the spectrum from (33), one has to start with the terms of the lowest order. Here the last term of (33) gives the only term of order zero. It gives a result different from zero only if $u=p/2\Delta\omega$; that is, only for the frequency p. The energy for that frequency is

$$\tfrac{1}{2}(NC)^{-2}e^{-2/NC}\ {}_1F_1{}^2(1,\ 2,\ 1/NC)(\Delta\omega_1)^2$$
$$= \tfrac{1}{2}(\Delta\omega_1)^2(1-e^{-1/NC})^2.$$

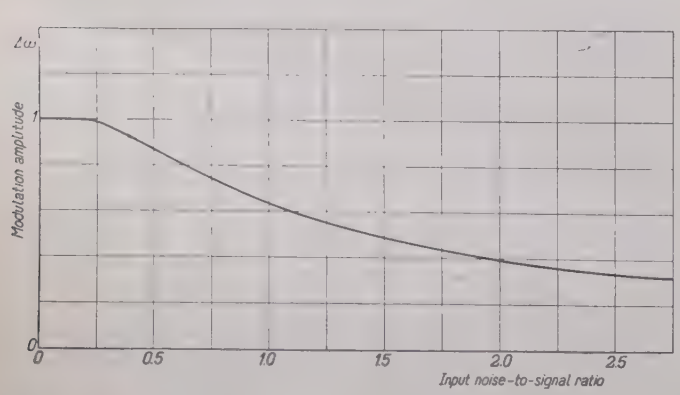

Fig. 9—Suppression of the modulation by noise. Ordinate: amplitude of the modulation. Abscissa: noise-to-signal energy ratio.

Whereas, in general, $E_2(u)$ gives the energy per unit bandwidth, here the energy is concentrated in a single line (this is the meaning of the δ-function). It is the energy of the modulation. If no noise is present, the amplitude of the modulation is $\Delta\omega_1$. In the presence of noise this amplitude is modified to $\Delta\omega_1(1-e^{-1/NC})$ where $1/NC$ is the ratio of signal energy to noise energy (if). (See Fig. 9). Thus (33) takes into account the suppression of the modulation by noise.

The first-order term in (33) gives:

$$4(\Delta\omega)^2NC(1-e^{-1/NC})u^2 + 4(\Delta\omega_1)^2(NC)^{-1}e^{-2/NC}.$$

For small values of NC, this is a fair approximation for the output noise. Then the sweep of the modulation has no effect on the noise energy. We see from this term, however, that, when the noise energy is not small compared to the signal energy, the sweep of the modulation affects the noise after detection. This effect was found experimentally by Guy and Morris,[10] and is fully described by (33). When, instead of the result for a rectangular filter, one wishes to know the result for another symmetrical filter, one has only to substitute the appropriate functions for $h_k(u)$, $b_k(u)$, $f_k(u)$. For a gaussian amplitude characteristic, these functions have been discussed in Section III. As the amount of work involved in numerical calculations of the noise by means of (33) is considerable, a numerical example is omitted.

All of the above calculations refer to the noise energies inherent in the system of frequency modulation. They will give an increasingly better approximation of the practical results as the frequency detector more nearly approaches the ideal.

Appendix I[11]

Starting from (12), we integrate first with respect to a_n. This integral has the form

$$(\pi C)^{-1/2}\int_{-\infty}^{+\infty}\exp\{-a_n{}^2/C + ia_n(u_1\cos nt_1 - nu_2\sin nt_1$$
$$+u_3\cos nt_2 - nu_4\sin nt_2)\}da_n$$
$$\exp\{-(C/4)(u_1\cos nt_1 - nu_2\sin nt_1$$
$$+u_3\cos nt_2 - nu_4\sin nt_2)^2\}.$$

In the same way, the integration over b_n gives

$$\exp\{-C/4(u_1\sin nt_1 + nu_2\cos nt_1$$
$$+u_3\sin nt_2 + nu_4\cos nt_2)^2\}.$$

Multiplying all probability integrals, we get

$$\exp[-(C/4)\sum\{u_1{}^2 + n^2u_2{}^2 + u_3{}^2 + n^2u_4{}^2$$
$$+2(u_1u_3 + n^2u_2u_4)\cos nv + 2n(u_1u_4 - u_2u_3)\sin nv\}] \qquad (16)$$

[11] In the Appendixes a more specified outline of the calculations is given, but for space considerations much ordinary algebra has been left to the reader.

where the summation has to be taken over all n satisfying

$$\omega_0 - \Delta\omega \leqq n \leqq \omega_0 + \Delta\omega$$

and

$$v = t_1 - t_2.$$

Let us introduce new variables $\beta_2 = u_2\omega_0$, $\beta_4 = u_4\omega_0$, $s = n - \omega_0$, and make a series development of that part of the exponential form which contains cos $(\omega_0 v + sv)$ and sin $(\omega_0 v + sv)$:

$$\sum_{k=0}^{\infty} (-C/4)^k (1/k!) \Big[\sum_{s=-\Delta\omega}^{\Delta\omega} \{ 2\alpha_s \cos(\omega_0 v + sv) + 2\gamma_s \sin(\omega_0 v + sv) \} \Big]^k.$$

Here

$$\alpha_s = u_1 u_3 + \beta_2\beta_4(1 + s/\omega_0)^2;$$
$$\gamma_s = (u_1\beta_4 - \beta_2 u_3)(1 + s/\omega_0).$$

This form can also be written:

$$\sum_{k=0}^{\infty} (-C/4)^k (1/k!) \Big[\sum_{s=-\Delta\omega}^{\Delta\omega} \{ (\alpha_s - i\gamma_s) e^{i\omega_0 v + isv} + (\alpha_s + i\gamma_s) e^{-i\omega_0 v - isv} \} \Big]^k.$$

As in Section I, we are only interested in such values of m in (7) and (12) which are small compared to ω_0. Therefore, as in the example treated in Section I, we use the binomial formula and retain only those terms which contain no $\omega_0 v$. The result is

$$\sum (C/4)^{2k} (k!k!)^{-1} \Big\{ \sum (\alpha_s - i\gamma_s) e^{isv} \Big\}^k \cdot \Big\{ \sum (\alpha_s + i\gamma_s) e^{-isv} \Big\}^k. \quad (17a)$$

Now we develop the integrand with respect to s/ω_0 and stop at $(s/\omega_0)^2$. The result can be integrated straightforwardly. The following types of integrals occur $(\gamma = NC/4,\ N = 2\Delta\omega)$:

$$\int\int_{-\infty}^{+\infty} e^{-\gamma(u_1^2+u_2^2)} (u_1^2 + u_2^2)^k u_2^{-2} du_1 du_2 = 0, \text{ if } k \neq 0$$
$$= -2\pi, \text{ if } k = 0$$

$$\int\int_{-\infty}^{+\infty} e^{-\gamma(u_1^2+u_2^2)} (u_1^2 + u_2^2)^{k-1} u_1^2 u_2^{-2} du_1 du_2 = -(k-1)!\pi\gamma^{-k}, \quad k \geqq 1$$

$$\int\int_{-\infty}^{+\infty} e^{-\gamma(u_1^2+u_2^2)} (u_1^2 + u_2^2)^{k-1} du_1 du_2 = (k-1)!\pi\gamma^{-k}, \quad k \geqq 1$$

$$\int\int_{-\infty}^{+\infty} e^{-\gamma(u_1^2+u_2^2)} (u_1^2 + u_2^2)^{k-1} u_1^4 u_2^{-2} du_1 du_2 = -3/2(k-1)!\pi\gamma^{-k}, \quad k \geqq 1.$$

This leads directly to (13a) and (13b). The introduction of (8a) (symmetrical amplitude characteristic) modifies (16) into

$$\exp\left[-C/4 \sum f^2(n) \{ u_1^2 + nu_2^2 + u_3^2 + n^2u_4^2 + 2(u_1u_3 + n^2u_2u_4)\cos nv + 2n(u_1u_4 - u_2u_3)\sin nv \} \right]. \quad (16a)$$

The phase characteristic does not influence the calculations. On account of the symmetry in the characteristic, we only have to introduce an extra factor $f^2(s)$ in both sums of (17a). After changing to the new variable u, this leads directly to (15a). In case the amplitude characteristic is not symmetrical, the change in (17a) is greater. Instead of (17a), we now get:

$$\sum (C/4)^{2k} (k!k!)^{-1} \Big\{ \sum f^2(s)(\alpha_s - i\gamma_s) e^{isv} \Big\}^k \cdot \Big\{ \sum f^2(s)(\alpha_s + i\gamma_s) e^{-isv} \Big\}^k.$$

After introduction of the new variable u, the analogue of form (15a) is, then,

$$\int_{-\infty}^{+\infty} H_{2k}(u) e^{iuv} du = \frac{1}{4} \Big\{ \int F(u) e^{iuv} du \int u^2 F(-u) e^{iuv} du + \int F(-u) e^{iuv} du \int u^2 F(u) e^{iuv} du - 2 \int uF(u) e^{iuv} du \int uF(-u) e^{iuv} du \Big\} \cdot \Big\{ \int F(u) e^{iuv} du \int F(-u) e^{iuv} du \Big\}^{k-1}.$$

In this expression all integrals are from $-\infty$ to ∞.

Appendix II

Equation (25) is substituted in (7) and the average is taken as in (12). The integration over a_n and b_n goes exactly in the same way as in Appendix I. As in (16), the result is a function of $t_2 - t_1 = v$. When we introduce new variables t_1 and v, instead of t_1 and t_2, the integration over t_1 gives the result:

$$1/(4\pi^3) \int dv e^{-inv} J_0 \{ u_1^2 + u_2^2\omega_0^2 + u_3^2 + u_4^2\omega_0^2 + 2(u_1u_3 + u_2u_4\omega_0^2)\cos nv + 2(u_1u_4 - u_2u_3)\omega_0 \sin nv \}^{1/2}. \quad (26)$$

The Bessel function can also be written

$$\sum_{-\infty}^{+\infty} (-1)^q J_q(u_1^2 + u_2^2\omega_0^2)^{1/2} J_q(u_3^2 + u_4^2\omega_0^2)^{1/2} e^{iq(nv+\phi)} \quad (27)$$

where ϕ is defined by

$$\cos\phi = \frac{u_1u_3 + u_2u_4\omega_0^2}{(u_1^2 + u_2^2\omega_0^2)^{1/2}(u_3^2 + u_4^2\omega_0^2)^{1/2}}$$

and

$$\sin\phi = \frac{(u_2u_3 - u_1u_4)\omega_0}{(u_1^2 + u_2^2\omega_0^2)^{1/2}(u_3^2 + u_4^2\omega_0^2)^{1/2}}.$$

Introduce new variables as in Appendix I, and expand into a series that part of the exponent which contains cos $(\omega_0 v + sv)$ and sin $(\omega_0 v + sv)$. This gives the same re-

sult as in (17). Binomial development of the terms of this sum results in the double sum:

$$\sum_{k=1}^{\infty}\sum_{r=0}^{k}(C/4)^k\{(k-r)!r!\}^{-1}\Big\{\sum(\alpha_s-i\gamma_s)e^{isv}\Big\}^{k-r}\cdot\Big\{\sum(\alpha_s+i\gamma_s)e^{-isv}\Big\}^{r}\cdot e^{i(k-2r)\omega_0 v}. \quad (28)$$

Now we have to integrate the product of (27) and (28) and

$$(4\pi^3)^{-1}e^{-imv}\exp\left\{-(C/4)\sum(u_1^2+\beta_{2a}^2+u_3^2+\beta_{4a}^2)\right\}$$

in which

$$\beta_{2a}=\beta_2(1+s/\omega_0);\qquad \beta_{4a}=\beta_4(1+s/\omega_0).$$

As m is small compared to ω_0, we have to choose $q+k-2r=0$. $q=2r-k$. Again we expand into a series with respect to s/ω_0, and stop at $(s/\omega_0)^2$. We introduce the continuous variable u and we use the functions introduced in Section III. The result is

$$\sum_{k=1}^{\infty}\sum_{r=0}^{k}\left\{2u^2f_k(u)-8(k-r)rb_k(u)\right\}\{(k-r)!r!\}^{-1}\gamma^k\cdot\int\int_0^{\infty}dxdyJ_{k-2r}(x)J_{k-2r}(y)x^{k-1}y^{k-1}e^{-\gamma(x^2+y^2)}.$$

Here $\gamma=NC/4$. These integrals are of the type called by Watson[12] "Weber's first exponential integral." Their computation leads to equation (30).

To show the convergence of the development, we return to a single sum. The part of the formula containing $b_k(u)$ is modified into

$$\sum_{2}^{\infty}-8b_k(u)(k!)^{-1}(2\gamma)^k(2\pi)^{-1}\cdot\int_0^{2\pi}d\psi\int_0^{\infty}dx\int_0^{\infty}dyJ_0\{(x^2+y^2-2xy\cos\psi)^{1/2}\}\cdot(\cos\psi)^{k-2}e^{-\gamma(x^2+y^2)}x^{k-1}y^{k-1}.$$

This series is even convergent when J_0 is replaced by 1 when the rest of the integrand is positive, and by -1 when the rest is negative. For k large, the general term of the series behaves as b_k, or is smaller, and converges to zero at least with $k^{-3/2}$. The part of the formula containing $f_k(u)$ is still faster convergent.

[12] G. N. Watson, "A Treatise on the Theory of Besselfunctions," second edition, p. 393; Cambridge, 1944.

Appendix III

After the introduction of (31) in (7), the averaging has to be done as in (12). The first steps in the computation are the averaging over all a_n, b_n, and the removal of all terms containing $\cos\omega_0 t$. Further, we introduce new variables as in Appendix II. If, now,

$$G(\beta_2,\beta_4,\psi)=u_1^2+\beta_2^2+u_3^2+\beta_4^2+2(u_1^2+\beta_2^2)^{1/2}(u_3^2+\beta_4^2)^{1/2}\cos\psi,$$

the result of these first steps can be written in the form:

$$\overline{2f_mf_m^*}=(8\pi^4)^{-1}\omega_0^2\int dt_1\int dve^{-inv}\cdot\int\int\int\int\exp-(C/4G)\sum(\beta_{2a},\beta_{4a},\psi_a)\cdot J_0[\{G(\beta_{2b},\beta_{4b},\psi_b)\}^{1/2}]\beta_2^{-2}\beta_4^{-2}du_1d\beta_2du_3d\beta_4. \quad (32)$$

In this formula we have used the following abbreviations:

$$\beta_{2a}=\beta_2(1+s/\omega_0);$$

$$\beta_{2b}=\beta_2\left(1+\frac{\Delta\omega_1}{\omega_0}\cos pt_1\right);$$

$$\beta_{4a}=\beta_4(1+s/\omega_0);$$

$$\beta_{4b}=\beta_4\left(1+\frac{\Delta\omega_1}{\omega_0}\cos p(t_1-v)\right);$$

$$\psi_a=\omega_0v+sv-\phi_a;$$

$$\psi_b=\omega_0v-m_1\sin p(t-v)+m_1\sin pt-\phi_b;$$

$$m_1=\Delta\omega_1/p$$

$$\cos\phi(\beta_2,\beta_4)=\frac{u_1u_3+\beta_2\beta_4}{(u_1^2+\beta_2^2)^{1/2}(u_3^2+\beta_4^2)^{1/2}};$$

$$\sin\phi(\beta_2,\beta_4)=\frac{u_1\beta_4-u_3\beta_2}{(u_1^2+\beta_2^2)^{1/2}(u_3^2+\beta_4^2)^{1/2}};$$

$$\phi_a=\phi(\beta_{2a},\beta_{4a});\qquad \phi_b=\phi(\beta_{2b},\beta_{4b}).$$

As before, we take the terms in the exponent containing ω_0v and expand into a series. We also use the series of (27) for J_0, and take the terms together in such a way that ω_0v disappears from the result $(q=2r-k)$.

This time we are interested in terms up to $(s/\omega_0)^2$, $(s\Delta\omega_1/\omega_0^2)$, and $(\Delta\omega_1^2/\omega_0^2)$.

The integrals are of the same type as in Appendix II. The result of the integration is given in (33).

A Note on a New Ionospheric-Meteorological Correlation*

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Summary—**Past investigations in the field of ionospheric-meteorological correlation are reviewed, with special emphasis on ionospheric characteristics correlating with barometric pressure.**

An attempt to verify past correlations involving pressure has been made with negative results. However, a new correlation is presented between the time-of-occurrence of the maximum F_2 critical frequency during a single day and the average daily barometric pressure.

Introduction

IN GENERAL, ionospheric characteristics have been correlated with two types of meteorological phenomena—the variation of barometric pressure at the surface of the earth, and the occurrence of thunderstorms. Very little work has been done in this field by American investigators. It would seem desirable to attack the great mass of ionospheric data which has been gathered during the past ten years with a view toward verifying past correlations and discovering new ones. What can be done along this line is indicated by the new correlation noted below. This was uncovered in a definitely limited study. It would seem quite possible to discover other correlations, if available data are subjected to a thorough, systematic study.

Correlation with Barometric Pressure

The first experimental evidence of an ionospheric-meteorological correlation was reported by Ranzi from Italy. He found that there was an increase of E-layer ionization after sunset when barometric depressions occurred at the place of observation or north of it.[1] In one of the few American investigations, Colwell reported on some data which indicated a lack of E-layer reflection in high-pressure regions.[2] From Australia, Martyn reported a very close correlation between E-layer ionization density at night and the barometric pressure at ground level 12 to 36 hours later.[3] Many interesting correlations were pointed out by Martyn and Pulley, one of which was between the occurrence of nocturnal E-layer ionization and barometric pressure. Another showed direct correlation between E-layer critical frequency and barometric pressure. The published curves showing these and other correlations cover a period of only eleven days, but the authors state that they found similar correlation for several months.[4] The only negative results that have been reported were obtained by Best, Farmer, and Ratcliffe in England. They reported that E-layer ionization did not correlate with barometric pressure.[5]

Martyn and Pulley also presented several correlations with F_2-layer characteristics. They found direct correlation between maximum F_2 critical frequency and pressure. In addition, they found inverse correlation between pressure and both the minimum height of the F_2-layer and the time-of-occurrence of the minimum height. They were the first to attempt even a qualitative explanation of their results. They suggest that, since pressure changes on the surface of the earth cause a variation in the ozone content of the atmosphere, this variation of ozone content causes a variation in temperature. Thus, over a barometric depression, the ozone content of the atmosphere is high, the temperature is, therefore, high, and ionization densities are low.[4] Bajpai and Pant, working in India, found fair inverse correlation between the time-of-occurrence of the minimum height of the F_2 layer and pressure, but poor direct correlation between F_2 critical frequency and pressure.[6]

In an investigation using Australian data, Bannon, Higgs, Martyn, and Munro found a correlation between F_2-layer characteristics and certain meteorological conditions. Rather than attempting to compare data on a day-by-day basis, they instead divided days into two types, "frontal" and "nonfrontal." "Frontal" days are those in the transition zone between two successive anticyclones. In this region, there is a sudden discontinuity in the characteristics of the air. In an analysis extending over a period of 530 days, it was found that the average value of the square of F_2 critical frequency was higher on "frontal" days than on "nonfrontal" days. They think convection or diffusion times of pressure effects would be so long as to be prohibitive in explaining correlations obtained. Rather, they suggest large-scale movements of air in the high atmosphere apparently paralleling movements at lower levels. They conclude that a great, and probably the major part, of the day-to-day fluctuation of F_2-layer ionization density is

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[1] I. Ranzi, "Causes of ionization in the upper atmosphere," *Nature* (London), vol. 130, p. 545; October 8, 1932.

[2] R. C. Colwell, "Cyclones, anticyclones, and the Kennelly-Heaviside layer," PROC. I.R.E., vol. 21, pp. 721–725; May, 1933.

[3] D. F. Martyn, "Atmospheric pressure and the ionization of the Kennelly-Heaviside layer," *Nature* (London), vol. 133, pp. 294–295; February 24, 1934.

[4] D. F. Martyn and O. O. Pulley, "The temperature and constituents of the upper atmosphere," *Proc. Roy. Soc. A*, vol. 154, pp. 455–486; April, 1936.

[5] J. E. Best, F. T. Farmer, and J. A. Ratcliffe, "Studies of region E of the ionosphere," *Proc. Roy. Soc. A*, vol. 164, pp. 96–116; January 7, 1938.

[6] R. R. Bajpai and B. D. Pant, "Further studies of F-region at Allahabad," *Indian Jour. Phys.*, vol. 13, pp. 57–71; February, 1939.

associated with meteorological changes at the ground.[7]

Most recently, descriptive reports from Russia indicate a correlation has been noted there. Kessenikh and Bulatov compare data taken at Tomsk and Moscow. These stations differ only thirty minutes in latitude but have a widely different climate. It was found that, in winter, when Tomsk mean monthly atmospheric pressure was higher than that of Moscow, so was F_2 critical frequency. In summer, when the pressure equalized, so did F_2 critical frequency.[8] It should be noted, however, that from geomagnetic considerations, higher winter F_2 critical frequencies would be expected in Tomsk than in Moscow.

Correlation with Thunderstorms

Some eight years in advance of any experimental data, C. T. R. Wilson predicted qualitatively that the electrostatic fields of thunderclouds might affect the ionization of the Kennelly-Heaviside layer.[9] In a mathematical analysis based upon ionization by the radiation field of lightning flashes, Bailey and Martyn arrived at the same conclusion.[10] However, an error in their work was detected by Healey and, upon subsequent correction, the analysis predicted no correlation.[11] In a later paper, Bailey suggested a more accurate method of approach.[12] Accordingly, the work was reviewed by Healey and another analysis was carried out using the modified approach. From this it was concluded that, during the day, less than 1 per cent of the lightning flashes will cause a notable increase in the ionization density of the E layer, but that at night this proportion is considerably greater; and that if there always exists in the ionosphere a constant electric field of the order of 0.5 volt per meter, the increase will often be very marked.[13]

The first experimental verification of a correlation between thunderclouds and E-layer ionization was made by Ratcliffe and White in England. A statistical study of the results appeared to indicate that a correlation did exist.[14] On the basis of an investigation carried out in India, Bahr and Syam reported a correlation coefficient of 0.5 between thunderstorms and an abnormal increase in E-region ionization.[15] Best, Farmer, and Ratcliffe again rejected the correlation. Further, they re-examined data used by Ratcliffe and White and decided that, when analyzed under stricter standards, correlation did not exist.[5]

Investigation by the Author

Since attempts at correlation with thunderstorms necessitate the establishment of a somewhat arbitrary standard as to just what constitutes a thunderstorm, it was decided to let pressure represent meteorological phenomena. It should be noted that there is a unilateral relationship between thunderstorms and barometric depressions; i.e., thunderstorms are always accompanied by barometric depressions,[16] but barometric depressions frequently occur without thunderstorms. Consequently, a correlation of ionospheric characteristics with thunderstorms should of necessity also indicate a correlation with pressure. This suggests the possibility that the variation of pressure is associated with the fundamental agency affecting the ionosphere, and that the occurrence of thunderstorms is merely an indirect measure of occasional decreases in pressure. Unfortunately, the truth of this conjecture could not be investigated at this locality because of the almost complete absence of thunderstorms.

Continuous pressure information was available from a recording barograph in the Physics Department of Stanford University, and hourly ionospheric information was obtainable from the Electrical Engineering Department as a result of research sponsored by the Bureau of Standards. Daily means of pressure were used in all correlation studies, although examination showed little difference between noon values and daily means, pressure being a relatively slowly varying function of time.

At first, it was thought that a purely statistical study would be most satisfactory, but this approach was discarded after further consideration since most elementary statistical methods are not suitable for studying two sets of data between which there might possibly be a lag. Instead, during a two-week period of violent pressure variation, ionospheric data and corresponding daily means of pressure were plotted simultaneously. If there appeared to be any indication of correlation, the

[7] J. Bannon, A. J. Higgs, D. F. Martyn, and G. H. Munro, "The association of meteorological changes with variation of ionization of the F_2 region of the ionosphere," *Proc. Roy. Soc. A*, vol. 174, pp. 298–309; February 21, 1940.

[8] V. N. Kessenikh and H. D. Bulatov, "The continental effect in the geographic distribution of electron concentration in the F_2 layer," *Compt. Rend. Acad. Sci.* (U.R.S.S.), vol. 45, pp. 234–237; November 30, 1944. Abstract: *Wireless Eng.*, vol. 22, p. 394; August 1945.

[9] C. T. R. Wilson, "The electric field of a thundercloud and some of its effects," *Proc. Phys. Soc.* (London), vol. 37, pp. 32D–37D; November 28, 1924.

[10] V. A. Bailey and D. F. Martyn, "The influence of electric waves on the ionosphere," *Phil. Mag.*, vol. 18, pp. 369–386; August, 1934.

[11] R. H. Healey, "The influence of the radiation field from an electrical storm on the ionization density of the ionosphere," *Phil. Mag.*, vol. 21, pp. 187–198; January, 1936.

[12] V. A. Bailey, "The motion of electrons in a gas in the presence of variable electric fields and a constant magnetic field," *Phil. Mag.*, vol. 23, pp. 774–791; April (Sup.), 1937.

[13] R. H. Healey, "The effect of a thunderstorm on the upper atmosphere," *A.W.A. Tech. Rev.*, vol. 3, pp. 215–227; April, 1938.

[14] J. A. Ratcliffe and E. L. C. White, "Some automatic records of wireless waves reflected from the ionosphere," *Proc. Phys. Soc.* (London), vol. 46, pp. 107–115; January, 1934.

[15] J. N. Bahr and P. Syam, "Effect of thunderstorms and magnetic storms on the ionization of the Kennelly-Heaviside layer," *Phil. Mag.*, vol. 23, pp. 513–528; April, 1937.

[16] S. Petterssen, "Weather Analysis and Forecasting," p. 85, McGraw-Hill Book Co., New York, N. Y., 1940.

period was extended. Correlation was sought between pressure at the ground and the following ionospheric characteristics: (1) maximum E-layer critical frequency, (2) occurrence of sporadic-E layer, (3) maximum sporadic-E critical frequency, (4) minimum F_2-layer height, (5) time of minimum F_2-layer height, and (6) maximum F_2-layer critical frequency. All attempts at correlation were unsuccessful.

Upon further examination, the data yielded an unexpected correlation. It was found that on days when the daily mean pressure at the ground was high the maximum F_2 critical frequency occurred in the morning; whereas on days of low pressure the maximum critical frequency occurred in the late afternoon. This tendency prevailed quite markedly during the winter of 1945–1946, in particular during the months of December and February. Simultaneous plots of mean pressure and the peak ionization time of the F_2 layer for this period are given in Fig. 1. Both plots are of winter-time conditions. Correlation was not found during the summer for two reasons: Firstly, pressure variations are much less pronounced during the summer months than during the winter months; consequently, any possible correlative variations in the ionosphere are masked by other effects. Secondly, during the summer months the F_2 critical frequency has a very broad peak during any one day, which makes determination of the hour of maximum critical frequency difficult and rather meaningless.

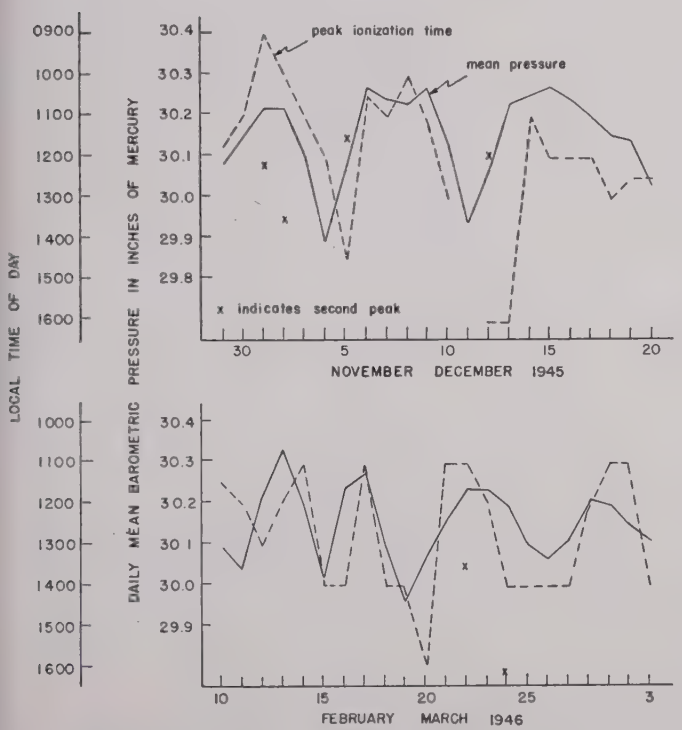

Fig. 1—Simultaneous plots of daily mean barometric pressure at the ground and peak-ionization time of the F_2 layer.

During the winter months there is usually a definite peak of ionization, although quite often there are secondary peaks during morning increases and afternoon decreases of ionization density. Less often there are two equal peaks of ionization, one in the morning and one in the afternoon. In such a case there is no purely objective method of determining which peak corresponds to the pressure variation. During the forty-four days covered in Fig. 1, there were six such double peaks. The alternative points are marked. Values giving best correlation were used for drawing the curve, but it can be seen that correlation would not be destroyed by using the less desirable points. This double-peak complication indicates that a modified method of analysis would be desirable.

Data were also examined for months other than those covered in Fig. 1. Data for January and February, 1945, showed interesting correlative tendencies, but ionospheric data was not complete, preventing comparison of data for more than a few days at a time. In November, 1945, there was a preponderance of secondary and multiple peaks, making data difficult to interpret. In January, 1946, ionospheric data were not complete for the first fifteen days, whereas pressure variation and shift in peak ionization time were indecisive during the last fifteen days. During the four months of winter in 1946–1947, correlation was not as good as that obtained during the previous winter.

It is difficult to attempt an explanation for the correlation found, since ionization and recombination processes in the F_2 layer are not well understood. However, if a decrease in air pressure at the ground may be associated with a corresponding decrease in atmospheric density in the F_2 layer and a consequent increase in the mean time necessary for recombination, it might be expected that the peak of ionization would occur later in the afternoon than normal. This checks with the experimental data.

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A Phase-Shift Oscillator with Wide-Range Tuning*

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Summary—**A new audio oscillator of the phase-shift type is discussed. The oscillator can cover continuously the entire audio-frequency range, and tuning is achieved by varying but one element of the feedback network. The theory of the new principle and a description of the oscillator are given.**

Introduction

ADEQUATE FREQUENCY stability, the most important demand of modern audio oscillators, has been achieved by the utilization of the *RC* principle.[1–5] *RC* oscillators have a low harmonic content and a simple electrical and mechanical structure, largely because of the direct manner of their oscillation and the use of thermistor amplitude control.

Regeneration networks of *RC* oscillators contain only resistive and capacitive elements. The circuits generally used cover a frequency range of 1 to 10, obtained by the simultaneous varying of two circuit elements. Tuning of a wider frequency range makes band-switching necessary. This weakness of *RC* oscillators explains why heterodyne oscillators still are utilized.

The oscillator described in this paper is of the phase-shift type, but its feedback network is of a more complex structure than that of *RC* oscillators. Besides *RC* elements it contains self inductances, and it employs also mutual inductances.

A phase-shift oscillator is an amplifier in combination with a feedback network. Steady-state sinusoidal oscillation fulfills the equation

$$A = \frac{1}{G}. \tag{1}$$

Both A (the feedback factor) and G (the gain of the amplifier) are vector quantities. Equality of absolute values in the above equation indicates constancy of the amplitude, and equality of phases ensures stability of the generated frequency.

According to (1), fundamentally it is possible to build an oscillator so long as $G>1$, and for this reason the oscillator discussed in this paper was developed for a single-tube amplifier. As the phase-shift of a single-tube amplifier with resistive load is 180°, an oscillator can be obtained with the help of a feedback network which gives a further phase shift of 180° at the generated frequency. The feedback path should have minimum attenuation, so as not to require unnecessary gain from the amplifier.

Such a network can be obtained by means of a reactive filter. If there is no energy reflected from the input terminals and there is no loss in the filter itself, energy passes without attenuation.

The All-Pass Lattice Section

The characteristic impedance and the voltage ratio of a symmetrical filter section can be expressed[6] by the two pairs of impedances of its lattice-type equivalent (Fig. 1):

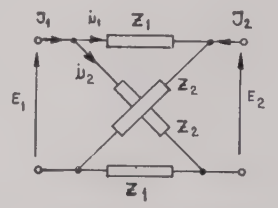

Fig. 1—Circuit of the lattice.

$$Z = \sqrt{Z_1 Z_2} \tag{2}$$

$$\frac{E_1}{E_2} = \frac{\sqrt{\frac{Z_2}{Z_1}} + 1}{\sqrt{\frac{Z_2}{Z_1}} - 1}. \tag{3}$$

With a resistive characteristic impedance R, the relation of Z_1 and Z_2 of the lattice is given by (2) to

$$Z_2 = \frac{R^2}{Z_1}, \tag{4}$$

and the voltage ratio becomes, with (3),

$$\frac{E_1}{E_2} = \frac{\sqrt{\frac{R^2/Z_1}{Z_1}} + 1}{\sqrt{\frac{R^2/Z_1}{Z_1}} - 1} = \frac{R + Z_1}{R - Z_1}.$$

Fig. 2 shows the vector diagram of the above circuit if Z_1 is a pure reactance. In that case $R+Z_1$ and $R-Z_1$ are conjugate, so that

$$\left|\frac{E_1}{E_2}\right| = \frac{|R + Z_1|}{|R - Z_1|} = 1,$$

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[1] H. H. Scott, "A new type of selective circuit and some applications," PROC. I.R.E., vol. 26, pp. 226–235; February, 1938.

[2] F. E. Terman, R. R. Buss, W. R. Hewlett, and F. C. Cahill, "Some applications of negative feedback with particular reference to laboratory equipment," PROC. I.R.E., vol. 27, pp. 653–655; October, 1939.

[3] E. L. Ginston and L. M. Hollingsworth, "Phase shift oscillators," PROC. I.R.E., vol. 29, pp. 43–49; February, 1941.

[4] G. Willoner and F. Tihelka, "Tongenerator ohne Schwingunskreis," *Arch. für Tech. Messen.*, No. 117, T44; March, 1941.

[5] G. Willoner and F. Tihelka, "Über Phasenschiebergeneratoren," *Hochfrequenzt. und Elektroakustik*, vol. 61, pp. 48–51; February, 1943.

[6] See Appendix.

the lattice network has zero attenuation at all frequencies, and thus it is an all-pass section. The angle of the voltage ratio represents the phase shift of the network (Fig. 2):

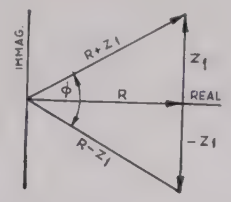


Fig. 2—Vector diagram showing phase-shift of an all-pass lattice section.

$$\phi = \text{arc}\,\frac{E_1}{E_2} = 2\ \text{arctg}\,\frac{|Z_1|}{R}.$$

If Z_1 takes the form of an inductance, then $Z_1 = j\omega L$, and

$$\phi = 2\ \text{arctg}\,\frac{\omega L}{R}. \tag{5}$$

The impedance Z_2 of the lattice is, according to (4),

$$Z_2 = \frac{R^2}{j\omega L} = \frac{1}{j\omega\dfrac{L}{R^2}}.$$

Hence, Z_2 must take the form of a capacitance of the value

$$C = \frac{L}{R^2}. \tag{6}$$

Fig. 3 shows the resulting all-pass lattice section. Its characteristic impedance is, from (2) or (6),

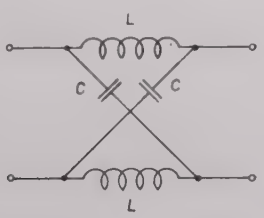


Fig. 3—The all-pass lattice section.

$$R = \sqrt{\frac{L}{C}} \tag{7}$$

and its phase shift is

$$\phi = 2\ \text{arctg}\,\omega\sqrt{LC}.$$

The above discussion is based on the lattice which is the general type of four-terminal network. For use in phase-shift oscillators, a three-terminal equivalent of the lattice is more convenient. This form has the advantage of a common input and output terminal, and can be easily applied as a feedback network.

Fig. 1 shows the lattice, and Fig. 4 its three-terminal equivalent. The equivalence is evident in view of the symmetry theory of Bartlett which was generalized by Brune, and which states[7] that a two-terminal network

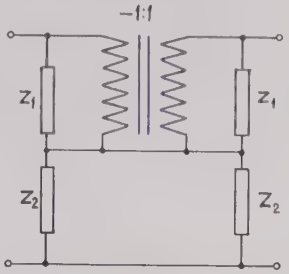


Fig. 4—Three-terminal equivalent of the lattice.

of the lattice impedance Z_2 is obtained if a four-terminal network is divided through its axis of symmetry, and if the ideal transformers on the half network with the ratio $-1:1$, connecting the two halves, are short-circuited. The lattice impedance Z_1 is obtained in a similar way, but in this case the imagined transformers with the ratio 1:1 are short-circuited.

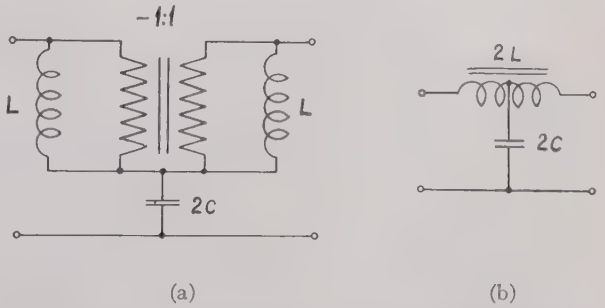


Fig. 5—(a) Three-terminal all-pass section. (b) Simplified circuit.

With the lattice impedances of Fig. 3, the three-terminal all-pass section changes into that of Fig. 5(a), and, as the transformer is an ideal one, the circuit is equivalent to that of Fig. 5(b).

The All-Pass Filter

The feedback path of the oscillator discussed is built up of all-pass sections as seen in Fig. 5(b). To obtain continuous tuning, the filter sections are shunted by a high-resistance potential divider from which the feedback voltage can be obtained by means of a sliding contact (Fig. 6).

[7] O. Brune, "Note on Bartlett's bisection theorem," *Phil. Mag.*, Ser. 7, vol. 14, p. 806; November, 1932. A proof of the equivalence of the networks of Figs. 1 and 4 is given in the Appendix.

A vector diagram of this feedback device is shown in Fig. 7. The frequency corresponding to each slide position is the frequency that gives 180° phase shift from filter input to the tapped point:[8]

$$\phi_1 + \phi_2 + \cdots + \phi_{n-1} + \phi = 180°.$$

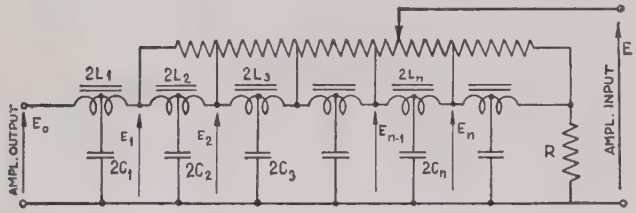


Fig. 6—Shunted all-pass filter as feedback path of the oscillator.

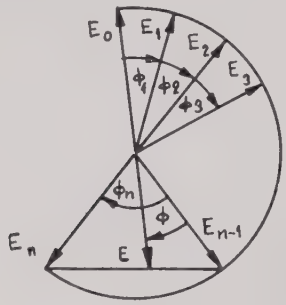
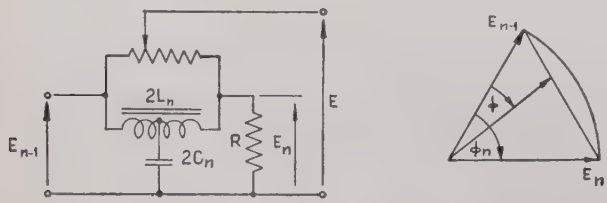


Fig. 7—Vector diagram to Fig. 6.

Feedback voltage E has a fluctuation according to the position of the slide (Figs. 8(a) and (b)). To keep



(a) (b)

Fig. 8—(a) All-pass filter section. (b) Its vector diagram at tuning.

fluctuations low, the maximum phase shift of a filter section, in its own frequency range, should not exceed 90°. In this case, according to (5),

$$2 \operatorname{arctg} \frac{\omega_{n-1} L_n}{R} = 90°$$

[8] Using an all-pass filter with many filter sections, oscillation would be possible at frequencies for which the phase shift through the feedback path is $180+n(360)$ degrees, where n is an integer. Oscillation at the higher frequencies is generally automatically avoided in consequence of the smaller gain of the amplifier at these frequencies.

and

$$\frac{\omega_{n-1} L_n}{R} = 1$$

or

$$L_n = \frac{R}{\omega_{n-1}} \tag{8}$$

where

L_n = the half self-inductance of the nth section
R = the characteristic impedance of the filter
$\omega_{n-1} = 2\pi f_{n-1}$, where f_{n-1} is the highest frequency obtained at the nth section, i.e., the lowest frequency at the $n-1$ section.

The half-capacitance of the nth section, according to (6), is

$$C_n = \frac{L_n}{R^2}. \tag{9}$$

Design of the Oscillator

An oscillator designed on the above principles for the frequency range of 100 to 10,000 cps is briefly described, and the calculation of the filter is shown. Fig. 9 shows the circuit diagram, where a power pentode is used as the amplifier. The matching transformer between the tube and the filter is built for high self-inductance

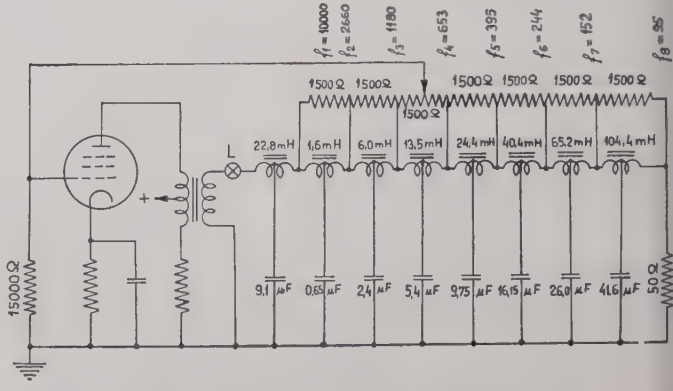


Fig. 9—Circuit diagram of a phase-shift-oscillator for a frequency range of 100 to 10,000 cps.

and minimum leakage, so as not to cause unnecessary phase shift. The magnetic circuit of the transformer contains no air gap, and dc magnetization is compensated by a secondary winding. (The transformer can be omitted if the following filter has a high characteristic-impedance.)

The characteristic impedance of the filter was chosen $R = 50$ ohms. In designing the filter elements, it may be noted that the first section is unbridged, and so its maximum phase shift can be over 90°. In the case of the filter discussed, ϕ_1 was chosen to be 172° at the upper end of the frequency range. According to (5),

$$2 \operatorname{arctg} \frac{\omega_1 L_1}{R} = 172°,$$

and

$$\frac{\omega_1 L_1}{R} = 14.3.$$

With the angular velocity of $\omega_1 = 2\pi \times 10{,}000 = 62{,}800$ radians per second, and the characteristic impedance $R = 50$ ohms, the half self-inductance of the first filter section becomes $L_1 = 11.4$ mh. According to (6), the half-capacitance of the section is $C_1 = L_1/R^2 = 4.55$ μf.

The second filter section is designed for a maximum phase shift of 90°. According to (8),

$$L_2 = \frac{R}{\omega_1}.$$

As $R = C_2 50$ ohms and $\omega_1 = 62{,}800$ rad/sec^{-1}, $L_2 = 0.8$ mh, and $C^2 = L_2/R^2 = 0.32$ μf.

The angular velocity ω_2 can be calculated according to (5):

$$2 \operatorname{arctg} \frac{\omega_2 L_1}{R} + 2 \operatorname{arctg} \frac{\omega_2 L_2}{R} = 180°,$$

or

$$\operatorname{arctg} \frac{\omega_2}{4390} + \operatorname{arctg} \frac{\omega_2}{62{,}500} = 90°,$$

which gives $\omega_2 = 16{,}700$ rad/sec^{-1} and $f_2 = 2660$ cps.

The elements of the third section are $L_3 = R/\omega_2 = 3.0$ mh and $C_3 = L_3/R^2 = 1.2$ μf ω_3 is given by

$$\operatorname{arctg} \frac{\omega_3}{4390} + \operatorname{arctg} \frac{\omega_3}{62{,}500} + \operatorname{arctg} \frac{\omega_3}{16{,}700} = 90°$$

and gives $\omega_3 = 7400$ rad/sec^{-1} and $f_3 = 1180$ cps.

Calculation of the remaining filter section is similar. The described procedure may be continued until the lowest desired frequency is reached. The values of the filter elements $2L$ and $2C$ are shown in Fig. 9.

To have linear filter reactances and a close coupling within the filter sections, iron-powder ring cores were used with double windings.

Limitation of the amplitude, necessary in all types of oscillators, can be achieved by various methods. In the circuit shown in Fig. 9 a nonlinear element is used. An incandescent lamp L, with a resistance close to the characteristic impedance of the filter, is in series with the filter input circuit and keeps the amplitude at a constant value.

Conclusion

The most usual all-pass filter is the transmission line. Transmission lines are often used as resonant circuits in the ultra-high-frequency region, but they cannot be applied in the audio-frequency range on account of their dimensions, which are determined by the wavelength. To obtain smaller dimensions in the audio-frequency range, the distributed inductances and capacitances of the transmission line must be concentrated into filter sections. The results of this concentration are the tee- and pi-type filters. As the square of the cutoff frequency is inversely proportional to the product LC of a filter section, the all-pass character of a transmission line with infinitesimal sections changes into a low-pass character at the tee and pi filters.

An all-pass filter in the audio range is obtained in the lattice network. Its three-terminal equivalent, discussed in this paper, is convenient for designing phase-shift oscillators.

Appendix

Matrix of a Four-Terminal Network

A linear four-terminal network can be characterized by the following equations (Fig. 10):

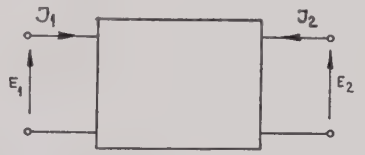


Fig. 10—Schematic diagram of a four-terminal network.

$$E_1 = Z_{11} J_1 + Z_{12} J_2 \tag{10}$$

$$E_2 = Z_{21} J_1 + Z_{22} J_2. \tag{11}$$

In these equations, $Z_{11} = E_1/J_1$ (for $J_2 = 0$) and $Z_{22} = E_2/J_2$ (for $J_1 = 0$) are the open-circuit input and output impedances, $Z_{12} = E_1/J_2$ (for $J_1 = 0$) and $Z_{21} = E_2/J_1$ (for $J_2 = 0$) are the transfer impedances of the network. Equation (12) is the impedance matrix of the four-terminal network:

$$Z = \left\| \begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix} \right\|. \tag{12}$$

All networks containing only passive elements have

$$Z_{12} = Z_{21}, \tag{13}$$

and the symmetrical ones have

$$Z_{11} = Z_{22}. \tag{14}$$

Matrix of the Lattice

The input impedance of the unterminated lattice is, from Fig. 1,

$$Z_{11} = Z_{22} = \frac{Z_1 + Z_2}{2}. \tag{15}$$

The open-circuit output voltage is

$$E_2 = i_1 Z_2 - i_2 Z_1,$$

and, as $i_1 = i_2 = J_1/2$, the transfer impedance

$$Z_{12} = Z_{21} = \frac{E_2}{J_1} = \frac{Z_2 - Z_1}{2}. \tag{16}$$

Thus the impedance matrix of the lattice is, from equations (15) and (16),

$$\mathcal{Z} = \left\| \begin{matrix} \frac{Z_1 + Z_2}{2} & \frac{Z_2 - Z_1}{2} \\ \frac{Z_2 - Z_1}{2} & \frac{Z_1 + Z_2}{2} \end{matrix} \right\|. \tag{17}$$

Characteristic Impedance of the Lattice

A symmetrical four-terminal network terminated in its characteristic impedance presents also at its input terminals the characteristic impedance, so $E_1 = ZJ_1$ and $E_2 = -ZJ_2$. These values introduced into (10) and (11) give, taking (13) and (14) into consideration,

$$ZJ_1 = Z_{11}J_1 + Z_{12}J_2$$

and

$$-ZJ_2 = Z_{12}J_1 + Z_{11}J_2.$$

J_1/J_2 expressed of both equations gives

$$\frac{Z_{12}}{Z - Z_{11}} = -\frac{Z + Z_{11}}{Z_{12}}$$

or

$$Z_{12}^2 = Z_{11}^2 - Z^2$$

and

$$Z^2 = Z_{11}^2 - Z_{12}^2 = (Z_{11} - Z_{12})(Z_{11} + Z_{12}).$$

With (15) and (16),

$$Z^2 = Z_1 Z_2,$$

and the characteristic impedance of the lattice

$$Z = \sqrt{Z_1 Z_2}. \tag{18}$$

Voltage Ratio of the Lattice

If current J_1 of the four-terminal network (Fig. 10) is constant, i.e., the generator has infinite impedance, then the generated no-load voltage in the output circuit is given by

$$J_1 Z_{12} = -J_2(Z_{11} + Z)$$

and the voltage ratio

$$\frac{E_1}{E_2} = -\frac{J_1}{J_2} = \frac{Z_{11} + Z}{Z_{12}}.$$

From (15), (16), and (18),

$$\frac{E_1}{E_2} = -\frac{\frac{Z_1 + Z_2}{2} + \sqrt{Z_1 Z_2}}{\frac{Z_2 - Z_1}{2}}$$

$$= \frac{(\sqrt{Z_1} + \sqrt{Z_2})^2}{(\sqrt{Z_2} + \sqrt{Z_1})(\sqrt{Z_2} - \sqrt{Z_1})}$$

$$= \frac{\sqrt{\frac{Z_2}{Z_1}} + 1}{\sqrt{\frac{Z_2}{Z_1}} - 1}. \tag{19}$$

Three-Terminal Lattice Equivalent

Matrices can be summed up by adding the corresponding elements. According to this rule, matrix (17) can be split into

$$\mathcal{Z} = \left\| \begin{matrix} \frac{Z_2 + Z_1}{2} & \frac{Z_2 - Z_1}{2} \\ \frac{Z_2 - Z_1}{2} & \frac{Z_2 + Z_1}{2} \end{matrix} \right\|$$

$$= \left\| \begin{matrix} \frac{Z_2}{2} & \frac{Z_2}{2} \\ \frac{Z_2}{2} & \frac{Z_2}{2} \end{matrix} \right\| + \left\| \begin{matrix} \frac{Z_1}{2} & -\frac{Z_1}{2} \\ -\frac{Z_1}{2} & \frac{Z_1}{2} \end{matrix} \right\| = \mathcal{Z}' + \mathcal{Z}''.$$

The elements of matrix $\mathcal{Z}'$ are $Z_{11}' = Z_{12}' = Z_{21}' = Z_{22}' = Z_2/2$; thus its equivalent network is that of Fig. 11(a). Replacement of matrix $\mathcal{Z}''$ with the elements $Z_{11}'' = Z_{22}'' = Z_1/2$; and $Z_{12}'' = Z_{21}'' = -(Z_1/2)$ can be seen on Fig. 11(b).

According to the network theory, addition of matrices corresponds to the series connection of networks, and this can be obtained by connecting the input and output terminals separately in series. Fig. 12 shows the series connection of the networks shown in Figs. 11(a) and (b). The result is identical with the circuit of Fig. 4, which is a three-terminal equivalent of the lattice.

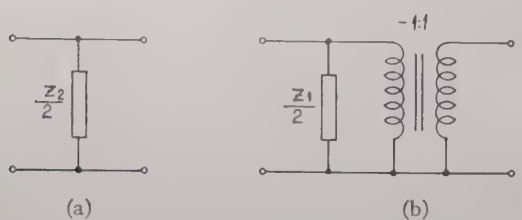


Fig. 11—(a) Circuits for matrices $\mathcal{Z}'$. (b) $\mathcal{Z}''$.

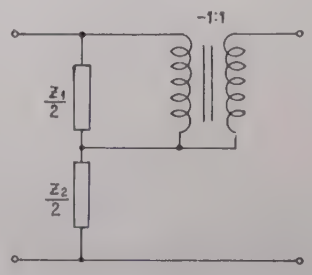


Fig. 12—Three-terminal equivalent of the all-pass lattice section.

An Experimental Investigation of the Radiation Patterns of Electromagnetic Horn Antennas*

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Summary—**A systematized set of radiation patterns of rectangular electromagnetic horn antennas have been measured and are shown as a function of electric and magnetic plane flare angles and radial length of the horn. A total of two hundred and fifty patterns are included for representative flare angles from zero to 50°, and radial lengths from zero to fifty wavelengths. This set of patterns reveals many characteristic properties which may serve as a guide in the design of electromagnetic horn antennas.**

Introduction

CONSIDERABLE WORK of a theoretical nature on radiation-pattern characteristics of rectangular electromagnetic horn antennas has appeared in the literature. Of particular interest are the patterns calculated by Chu and Barrow.[1,2] The theoretical patterns which they felt were most accurate were calculated on the basis of Huygens' principle, considering each individual element of the assumed Hertzian vector potential at the mouth of the horn to be radiating a spherical wave, the composite radiation pattern being calculated by a vector integration of the individual elements. Since it was assumed that the radiating elements were the electric and magnetic fields which would exist at the horn mouth if the horn were infinitely long, the pattern calculated on this basis for short horns becomes inaccurate.[3] At the present time there are no accurate theoretical radiation patterns for short horns. For information on horns less than six wavelengths long it is necessary to revert to experimental methods.

Some experimental work on horn patterns has been published,[2,4,5] but due to the limited radio-frequency spectrum then available it was physically impractical to measure more than the minimum necessary to establish the validity of the theoretical work. With the currently available oscillators developed during the war, it is possible to obtain several watts output at frequencies up to 24,000 Mc, thus increasing many times the range of electrical dimensions which can be conveniently realized physically for antenna measurements. A horn having an electrical length of fifty wavelengths has a physical length of two feet at 24,000 Mc. Such a horn as this can easily be supported and rotated to obtain its radiation pattern.

Most of the published work on rectangular horn antennas deals only with sectoral horns,[1,2,4] or horns which are flared in one principal plane alone. There is little mention of the pyramidal type of horn which is flared in both planes, other than to say that the pattern in any one plane is approximately dependent upon the flare angle in that plane only, independent of the flare angle in the other plane. Generally, a horn is designed to have a specific radiation pattern in each of the two principal planes. Hence, a knowledge of the pattern in the two principal planes and the effect of a flare angle in one plane upon the pattern in the other plane would be quite valuable.

It is the purpose of this paper to present a series of experimentally determined patterns over the most useful range of horn parameters for use in the design of electromagnetic horn antennas requiring a radiation pattern having specific characteristics. The patterns shown represent relative electric field intensity at a large, fixed distance from the antenna for any direction in the plane of measurement. Since the patterns are only relative and give no indication of gain of the antenna, all patterns have been drawn to the same maximum amplitude for a more pleasing presentation.

Horn Pattern Measurement

The radiation pattern measurements described in this paper were prompted by a need for data for use in the design of transmitting horn antennas to illuminate model aircraft antenna systems. The principal requirement of such an antenna is that it must produce an essentially plane wave over the area in which the model aircraft is to be measured. This requirement can be realized by proper separation of transmitting and receiving antennas and by using a transmitting horn whose directional properties will permit the radiated energy to illuminate the model directly, without part of the energy being first reflected from the ground before arriving at the model.

The measuring techniques used were similar to those described by Sinclair, Jordan, and Vaughn.[6] Due to

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† The Antenna Laboratory, Ohio State University, Columbus, Ohio.

[1] W. L. Barrow and L. J. Chu, "Theory of the electromagnetic horn," PROC. I.R.E., vol. 27, pp. 51–64; January, 1939.

[2] L. J. Chu and W. L. Barrow, "Electromagnetic horn design," *Trans. A.I.E.E. (Elec. Eng.*, July, 1939), vol. 58, pp. 333–338; July, 1939.

[3] A more rigorous approach in which radiation from currents flowing on the outer surface of the horn can be included is given by S. A. Schelkunoff, "On diffraction and radiation of electromagnetic waves," *Phys. Rev.*, vol. 56, p. 308; August 15, 1939.

[4] W. L. Barrow and F. D. Lewis, "The sectoral electromagnetic horn," PROC. I.R.E., vol. 27, pp. 41–50; January, 1939.

[5] W. L. Barrow and Carl Shulman, "Multiunit electromagnetic horns," PROC. I.R.E., vol. 28, pp. 130–136; March, 1940.

[6] George Sinclair, E. C. Jordan, and Eric W. Vaughn, "Measurements of aircraft antenna patterns using models," PROC. I.R.E., vol. 35, pp. 1451–1462; December, 1947.

mechanical difficulties inherent in the measurement of a transmitting antenna, the horn patterns were measured using the horn as a receiving antenna. By applying the reciprocity theorem, it can be seen that the pattern of an antenna when transmitting is the same as when receiving.

The most useful radiation patterns of a rectangular horn antenna excited in its dominant mode are those in the two principal planes of the horn, usually designated by the terms E plane and H plane because of the fact that these planes are parallel to the lines of electric and magnetic field intensities, respectively. Because of previous reference in the literature to flare angles in the E and H planes by the symbols θ and ϕ, respectively, and to radial length of a horn by the symbol R, these symbols will be adhered to. The three parameters, θ, ϕ, and R, and the two principal planes of measurement are shown in Fig. 1.

In an effort to determine the degree of approximation involved in assuming that the principal-plane patterns of a pyramidal horn antenna are dependent only upon the radial length of the horn and the flare angle in the plane of measurement, independent of the flare angle in the other principal plane, a series of patterns were measured at 24,000 Mc on a set of horns whose flare angles in the E plane were held constant, while the H plane flare

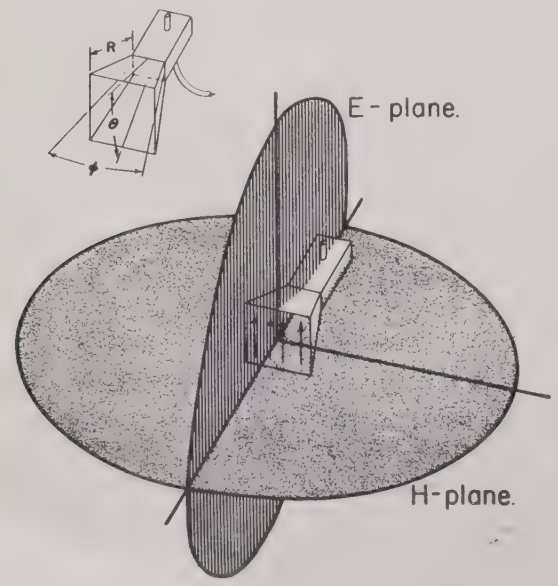

Fig. 1—Co-ordinate system showing the two principal planes of a rectangular horn and the parameters ϕ, θ, and R.

angles varied from 10° to 50° in 10° increments. The angle of flare in the E plane was 40°. This set of patterns is shown for various horn lengths R in Fig. 2. A comparison of the patterns in any vertical column in Fig. 2 shows that the E-plane patterns, for a horn of fixed flare angle in that plane, are essentially independent of the flare angle in the H plane, assuming that the radial length of the horn is constant. This same phenomenon has been observed for H-plane patterns for a horn of fixed flare angle in the H plane, but only the patterns for the E plane are presented because of the fact that the H-plane patterns are much less sensitive to changes in parameters, as will be shown later.

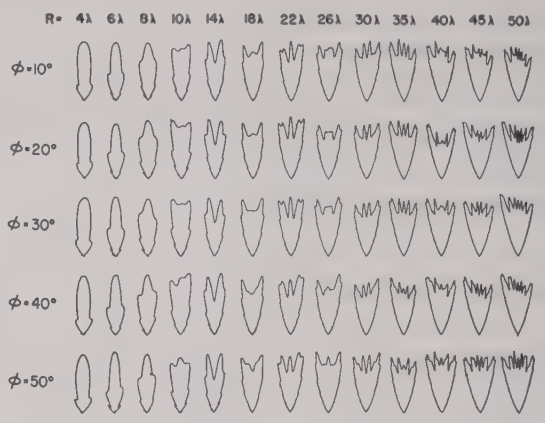

Fig. 2—Comparison of E-plane patterns for a horn having a constant flare angle in the E plane but a variable H-plane flare angle.

For horns which are only a few wavelengths long, the radiating elements are not confined to the mouth aperture. Considerable energy is diffracted around the mouth, causing currents to flow on the outside walls of the horn which, in turn, contribute to the radiation pattern of the horn. In the extreme case of a horn of zero length (an open waveguide) the energy radiated in back

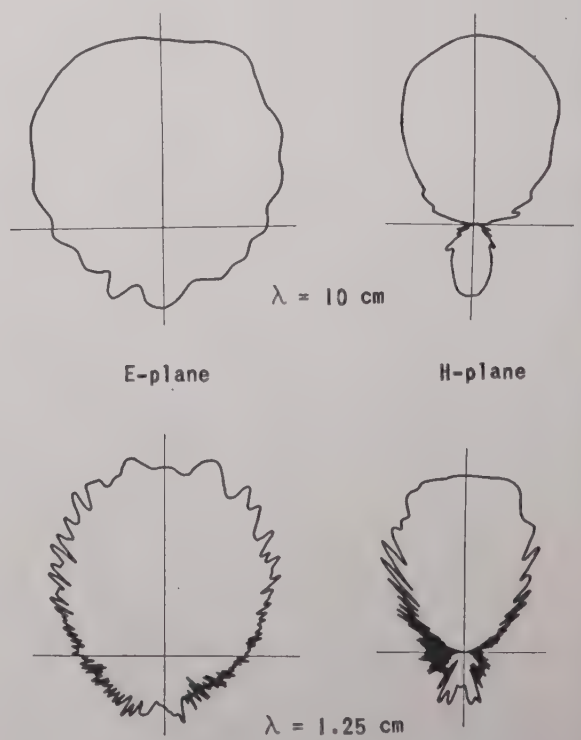

Fig. 3—Comparison of principal plane patterns of electrically equivalent open waveguide at a wavelength of 10 and 1.25 cm.

of the waveguide is comparable to that which is radiated forward. In Fig. 3 are shown the principal plane patterns for electrically equivalent open waveguides at wavelengths of 10 and 1.25 cm, the patterns being measured under similar conditions. Energy reflection from surrounding objects is quite objectionable at 1.25 cm, but at 10 cm becomes almost negligible. For this reason the patterns of those horns whose lengths were less than or equal to four wavelengths were measured at 10 cm.

Utilizing the fact that the radiation pattern in one principal plane of a pyramidal horn antenna is independent of the flare angle in the other principal plane, the effect of variation of the three horn parameters θ, ϕ, and R can be seen by plotting the patterns for the flare angles θ and ϕ independently against the horn length R. This has been done for several values of θ and ϕ up to 50°, and the results are shown in Figs. 4 and 5. The patterns shown are taken from the original patterns as recorded on a Speedomax polar recorder and have not been retouched.

Pattern Bandwidth

Pattern bandwidth, or the width of the frequency spectrum over which a given antenna can operate without an appreciable change in its radiation pattern, is quite frequently one of the primary considerations in

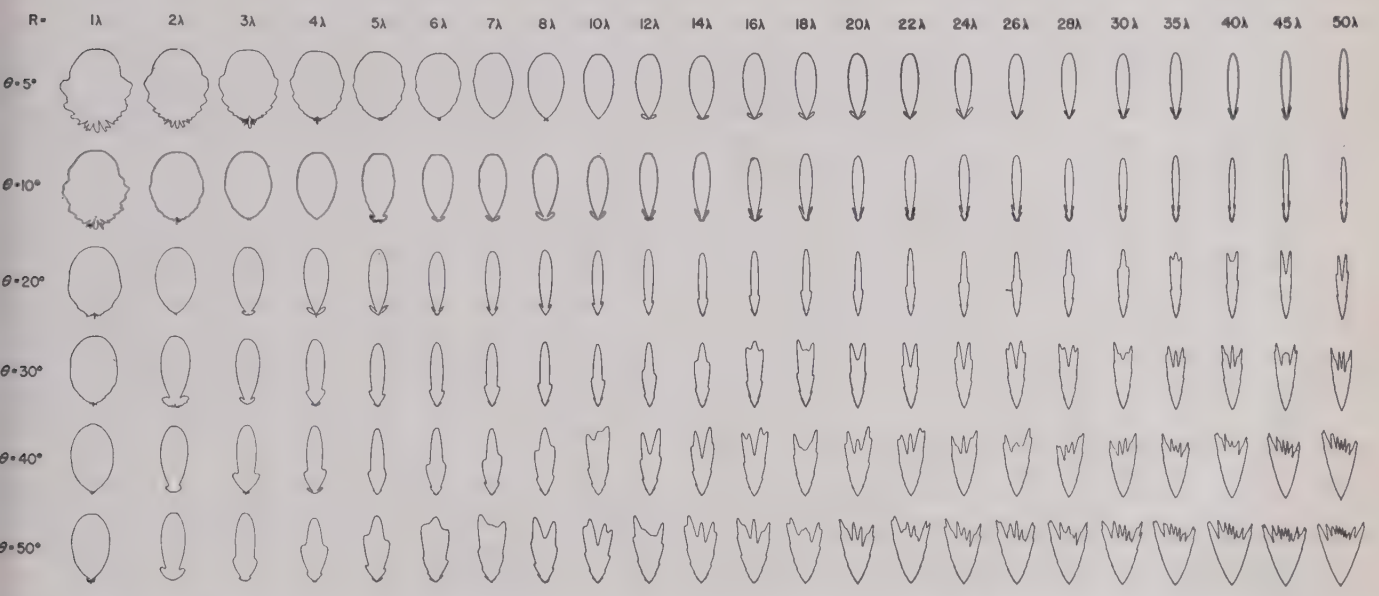

Fig. 4—*E*-plane patterns of a rectangular horn antenna as a function of radial horn length R and *E*-plane flare angle θ.

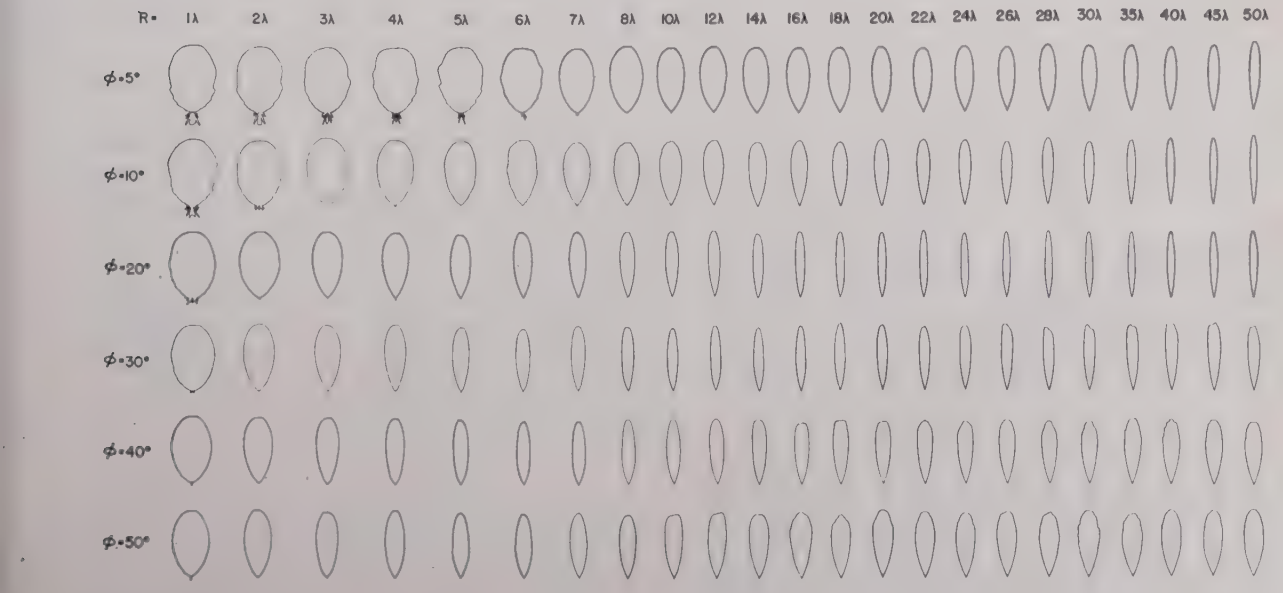

Fig. 5—*H*-plane patterns of a rectangular horn antenna as a function of radial horn length R and *H*-plane flare angle ϕ.

the design of an electromagnetic horn. When operating a given horn at two different frequencies, the dimensions of the throat aperture, mouth aperture, and horn length are transformed in the ratio of the two frequencies, but the flare angles are preserved. There are no electrical limitations on the dimensions of mouth aperture for a particular horn used over a frequency band; there are, however, both high- and low-frequency limits which must be placed upon the throat-aperture dimensions for dominant-mode (TE_{01}) excitation. The lowest usable frequency is limited by the cutoff frequency of the particular waveguide in question, this frequency being

$$\nu_{\text{cutoff}} = \frac{c}{2b}$$

where b is the width of the waveguide in the H plane and c is the velocity of light. The highest usable frequency is limited by the frequency at which waves other than the dominant-mode waves can exist in the waveguide; this limitation is determined by how the waveguide is excited. If the waveguide is symmetrically excited, the TE_{02} mode cannot exist; however, the TE_{03} mode can exist at three times the cutoff frequency of the dominant mode. Hence, the usable frequency limits for a given horn and waveguide combination which is symmetrically excited are

$$\nu_{\text{low}} = \frac{c}{2b}$$

$$\nu_{\text{high}} = 3\nu_{\text{low}} = \frac{3c}{2b}.$$

For long horns the horn lengths will be proportional to the ratio of frequencies used (to a very close approximation); for shorter horns the error introduced by this approximation becomes greater than for long horns, due to the fact that the horn length is measured from the mouth aperture to the throat aperture rather than to the virtual intersection of the conducting walls. However, the approximation is close enough for most applications.

When choosing a horn to operate over a band of frequencies for a specific application, the throat dimension should be chosen such that all frequencies in the specified band fall within the frequency limits given above. The variation in the two principal plane patterns for this range of frequencies can be found from Figs. 4 and 5, for the particular flare angles and radial length, to lie between the longest and shortest electrical lengths as determined from the low- and high-frequency limits, respectively.

Conclusions

There are two important characteristics of the principal-plane patterns of a pyramidal horn excited in its dominant mode which can be observed from the patterns in Figs. 4 and 5. They are the secondary-lobe structure in the E plane and the total absence of secondary lobes in the H plane. From the theory of arrays carrying equal currents, it is to be expected that a uniform field distribution (which is equivalent to a uniform current distribution), such as that which exists in the E plane, will produce side lobes in the plane of the uniform field. An approximate explanation of the absence of secondary lobes in the H plane is the following. In the H plane a sinusoidal field distribution exists. It has been shown by Stone[7] that the ideal array without secondary lobes has its currents distributed according to the binomial coefficients. The binomial distribution reaches a maximum at the center of the array and goes to zero at the array extremities. The sinusoidal distribution of fields found at the aperture of a horn has the same general characteristics as the binomial distribution; hence, it is to be expected that its radiation pattern should be essentially free of secondary lobes.

Several interesting trends can be observed by close examination of the E-plane patterns. For any particular flare angle ($\theta = 30°$, for example), a side lobe can be traced from its origin at $R = 2\lambda$ through a series of lengths during which the magnitude of the side lobe is growing larger relative to the main lobe until, at $R = 16\lambda$, the major lobe and the first side lobes are of equal amplitude. Beyond the length $R = 16\lambda$ the side lobes become of greater amplitude than the main lobe. It is this growth of secondary lobe structure that is responsible for the divided characteristic of the beam which occurs at large horn lengths. This same evolutionary process can be observed by tracing any other side lobe through its various stages of development. As the horn becomes of very large aperture, the pattern has a large number of side lobes of comparable magnitude, as, for example, the long horns at $\theta = 50°$.

A characteristic trend which is most noticeable in the evolution of a waveguide as it grows into a horn is the rapid decrease in beam width with the first few wavelengths of a horn's development. An illustration is given in Fig. 6, showing patterns measured at a wavelength of

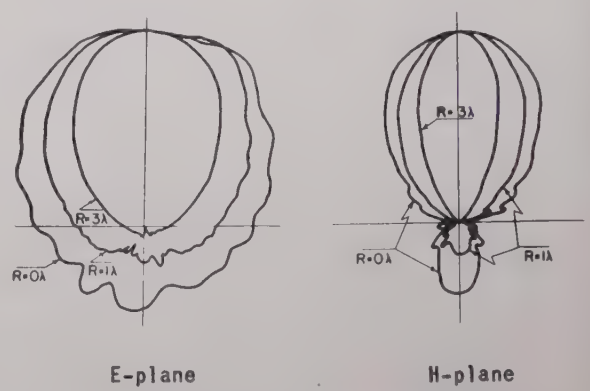

Fig. 6—Superposition of principal plane patterns for short horns having constant flare angles.

[7] John Stone Stone, U. S. Patent Nos. 1,643,323 and 1,715,433.

10 cm for three lengths of a horn whose flare angles are $\theta = 10°$ and $\phi = 30°$. The pattern showing the largest beam angle (the outer pattern) is for a horn of zero length; for lengths $R = 1\lambda$ and $R = 3\lambda$ the patterns become considerably narrower, and radiation in back of the horn becomes negligible.

ACKNOWLEDGMENT

The author wishes to express appreciation to those members of the Antenna Laboratory staff whose assistance made possible this investigation, and in particular to Robert A. Fouty for his invaluable assistance, and George Sinclair for his many helpful suggestions.

Fields in Nonmetallic Waveguides*

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Summary—**An infinite plate of dielectric material is taken as the prototype of nonmetallic waveguides. The Green's function is found for the case in which the electric field is parallel to the surfaces of the plate. The solution is set up as a Fourier integral, which is then replaced by a complex contour integral. There are but a finite number of real poles, the residues at which correspond to the propagating modes in the metallic guide. An integral around a branch cut gives a wave radiating into space, which is the analog of the attenuated modes of the metallic guide.**

The modal field distributions are discussed for the plate and for circular rods. The surface fields are not small, but they are attenuated transversely at rates of 28 db per radius and higher; a system using a dielectric guide would be at least fairly well shielded. The nonmetallic guide should be useful wherever a low-cost flexible conductor is needed and imperfect shielding can be tolerated.

INTRODUCTION

IF VISIBLE LIGHT be injected at one end of a glass rod, it is transmitted to the other end with very little loss, in spite of rather violent convolutions of the rod. We explain the effect in terms of ray optics and total internal reflection. It occurs quite naturally to an investigator working in the centimeter region that something similar might be done with microwaves, using plastic rods or strips. These might be mechanically flexible, and they could be more easily and cheaply fabricated than metallic guides, especially for extremely short wavelengths. It was this idea, brought to the attention of the author by H. Kallmann in the MIT Radiation Laboratory, which started the present investigation. The subject of dielectric guides turned out to be an old one; it was studied both theoretically and experimentally in the first two decades of this century.[1–3] This paper contains some new results, and some old material restated in modern language.

It is now a familiar concept that, when the walls of a waveguide are perfectly conducting, any field distribution can be described in terms of two sets of modes; those of one group, finite in number, are propagated along the axis of the guide without attenuation, while the infinite number in the other group are reactively attenuated without change in phase. That is, the propagation factors of the first group are pure reals; those of the second are pure imaginaries. The objects of this paper are to determine to what extent the mode description is applicable to nonmetallic guides, and to discuss their engineering value.[4,5]

In a circular dielectric rod there are modes of the E_{0n} and H_{0n} types. These were studied in preference to those resembling the H_{11} field (which is the lowest mode in a circular metallic guide), because in the dielectric any field having an angular dependence is complicated by longitudinal components of both electric and magnetic fields.[6] However, a dielectric plate shows the same characteristics as a rod; since here we find a simpler geometry and more familiar functions, the detailed discussion will be confined to this example.

FORMULATION

We propose to find the field in and about a dielectric plate due to a unit filament of current. The result is the Green's function, in terms of which the field due to any actual distribution of current having the same symmetry can be expressed.

Consider the slab of dielectric material shown in Fig. 1. It has an index of refraction n (dielectric constant n^2); it extends from $x = -a$ to $x = +a$ and to $\pm\infty$ in the y and z directions. The filament of current is at $x = b$, $z = 0$, and it extends to $y = \pm\infty$. We then have only a y component of the electric field, and we assume that

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[1] D. Hondors and P. Debye, "Elektomagnetische Wellen an dielektrischen Drahten," *Ann. der Phys.*, vol. 32, pp. 465–476; 1910.

[2] H. Zahn, "Uber den Nachweis elektromagnetischer Wellen an dielektrischen Drahten," *Ann. der Phys.*, vol. 49, pp. 907–933; 1916.

[3] O. Schriever, "Elektromagnetische Wellen an dielektrischen Drahten," *Ann. der Phys.*, vol. 63, pp. 645–673; 1920.

[4] Some of the results have already been given limited circulation in R. M. Whitmer, "Waveguides without metal walls," MIT Rad. Lab. Report No. 726, May 10, 1945.

[5] Some conclusions similar to those found here have been stated by G. Roe, "Normal modes in the theory of waveguides," *Phys. Rev.*, vol. 69, p. 255 (A); March, 1946.

[6] J. A. Stratton, "Electromagnetic Theory," McGraw-Hill Book Company, New York, N. Y., sec. 9.15, 1941.

$\partial/\partial y$ of any quantity vanishes identically. If we call the electric field ψ, the wave equation which it must satisfy is

$$(\nabla^2 + k^2n^2)\psi(x, z) = -2\pi\delta(x - b)\delta(z) \tag{1}$$

where k is 2π times the free-space wave number, n is a constant greater than unity for $|x| < a$, $n = 1$ for $|x| > a$, and δ is the Dirac delta function.[7] A time-dependence factor of $e^{-i\omega t}$ has been omitted; with this sign, convention e^{ikz} represents a wave moving in the positive z direction. All statements can be put into conventional engineering form by replacing i by $-j$; e^{-jkz} is a wave moving in the positive z direction.

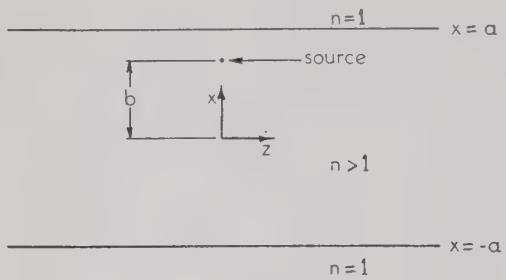

Fig. 1—A slab, of index of refraction $n > 1$, extends between $x = \pm a$, and to $\pm\infty$ in the y and z directions. A filament of current at $x = b$, $z = 0$ produces an electric field parallel to the surfaces of the slab.

The Fourier integral

$$\psi(x, z) = \int_{-\infty}^{\infty} v(x, h) \exp [ihz]dh \tag{2}$$

is a solution of (1), provided that

$$\left(\frac{d^2}{dx^2} + k^2n^2 - h^2\right)v(x, h) = -\delta(x - b) \tag{3}$$

independently of h, as may be shown by direct substitution.

The necessary continuities of the electric and magnetic fields require that ψ be continuous everywhere, and that $\partial\psi/\partial x$ be continuous at $x = \pm a$. These conditions on ψ and its derivative must apply to each Fourier component of ψ. Hence,

$$v(\pm a - 0) = v(\pm a + 0)$$

and

$$\frac{d}{dx}v(\pm a - 0) = \frac{d}{dx}v(\pm a + 0). \tag{4}$$

[7] The function δ, when its argument is time, is called the "unit impulse" in engineering analysis. The unit impulse is, in turn, the derivative of the familiar step-function. $\delta(u)$ has the property that $\int f(u)\delta(u)du = \begin{cases} f(0) \text{ if the integration interval contains } u = 0. \\ \text{zero if the interval does not contain } u = 0. \end{cases}$

Also,

$$v(b - 0) = v(b + 0). \tag{5}$$

Integrating (3) over a small interval of x at b,

$$\int_{b-0}^{b+0}\left(\frac{d^2}{dx^2} + k^2n^2 - h^2\right)vdx = -\int_{b-0}^{b+0}\delta(x - b)dx = -1$$

$$\frac{dv}{dx}\Big]_{b-0}^{b+0} = -1$$

or

$$v'(b + 0) - v'(b - 0) = -1. \tag{6}$$

Let

$$h^2 - k^2 = (\lambda/a)^2; \quad h^2 - k^2n^2 = (\mu/a)^2. \tag{7}$$

Then (3) may be written as

$$\left(\frac{d^2}{dx^2} - (\lambda/a)^2\right)v = 0 \quad \text{for} \quad |x| > a, \tag{8a}$$

and

$$\left(\frac{d^2}{dx^2} - (\mu/a)^2\right)v = -\delta(x - b) \quad \text{for} \quad |x| < a. \tag{8b}$$

The solutions of (8) are

$$x < -a, \qquad v_1 = A_1e^{-\lambda x/a}$$

for

$$\begin{aligned} -a < x < b, &\qquad v_2 = A_2e^{\mu x/a} + A_3e^{-\mu x/a} \\ b < x < a, &\qquad v_3 = A_4e^{\mu x/a} + A_5e^{-\mu x/a} \\ a < x, &\qquad v_4 = A_6e^{\lambda x/a}. \end{aligned} \tag{9}$$

Note that λ is given by a square root; its sign must be so chosen that v_1 and v_4 each represents an outgoing wave.

The conditions stated in (4)–(6) on v and v' give us six equations (inhomogeneous because (6) is) from which to find the six coefficients of (9). For our purposes, it is sufficient to calculate only A_6. For $x > a$, we find that

$$\psi(x, z) = a\int_{-\infty}^{\infty} \frac{(\mu - \lambda)\exp[\mu(1 + b/a)] + (\mu + \lambda)\exp[-\mu(1 - b/a)]}{(\mu - \lambda)^2\exp[2\mu] - (\mu + \lambda)^2\exp[-2\mu]} \exp[\lambda(x/a - 1) + ihz]dh. \tag{10}$$

Singularities

Equation (10) is to be integrated as a contour integral in the complex h plane. The residues at the poles of the integrand will give rise to modal-type waves. Let us first consider those poles; they are given by the roots of the denominator:

$$(\mu - \lambda)^2e^{2\mu} - (\mu + \lambda)^2e^{-2\mu} = 0$$

which, with $\mu = iw$ and $ka\sqrt{n^2 - 1} = d$, may be rearranged to take the form

$$\tan 2w = \frac{2w\sqrt{d^2 - w^2}}{2w^2 - d^2}. \qquad (11)$$

This will be recognized as the transcendental equation which gives the quantum-mechanically allowed energy levels for a one-dimensional rectangular potential "well," with the index of refraction of the slab corresponding to the depth of the well. There are a number of analogies between the two problems.

This equation could have been obtained more easily, simply by assuming waves like those in (9) as solutions of the wave equation ((1) with the right-hand side zero), and applying the boundary conditions. The Green's-function method was chosen in order to be certain that we have all possible types of fields which could arise from the assumed excitation. To be more specific, the branch-cut wave to be discussed later would not have appeared at all if we had started with the exponential waves of (9).

Equation (11) has a finite number of roots for $0<w<d$. These give real values of h between k and kn, in which region λ is real and negative. The residue of the integrand of (10) at a pole h_m gives a wave whose z dependence is $e^{ih_m z}$; clearly, these poles correspond to the real propagation factors of the metallic guide, and one looks for the analogs of the imaginary factors. There are no roots of (11) which give imaginary h's, but one might expect to find complex h's which would give "leaky" modes; that is, waves which would propagate with attenuation in the z direction while losing power sidewise. The author has been guilty of saying that such modes exist.[4] That they cannot exist may easily be shown; outside the slab the space dependence of such a mode would be of the form of

$$\exp\,[(ip - q)x + (ir - s)z]$$

with p, q, r, and s each positive for x, $z>0$, in order to satisfy the radiation condition. Substitution into the wave equation shows that this function is not a solution unless

$$pq + rs = 0,$$

which cannot be true for the type of wave suggested above. Hence, there are no attenuated, leaky modes.

One may ask what does happen to the propagation factor as a mode is "squeezed out," as by reducing ka. It may be shown from (11) that as ka is made smaller the propagation factor h_m of the highest mode[8] moves down the real h axis to the point $h=k$, which is a branch point of the integrand. If, now, ka is made still smaller, the root does acquire an imaginary part, but the signs are such that it represents an incoming wave, and it is not on the Riemann surface over which we integrate.[9]

Branch Cut

The integrand of (10) has branch points at $h=\pm k$, the roots of λ. The integrand is even in μ, and so does not have branch points at its roots. We employ the radiation condition to determine the paths of integration around $\pm k$; if λ be real, it must be negative; if imaginary, it must be positive imaginary. This tells us that the path of integration goes above the branch point at $-k$ and below the one at $+k$, as in Fig. 2, so that when the contour is closed in the upper half-plane there must be a branch cut upward from $+k$, and conversely. For $|h| \gg k$, $\lambda = -ha$ in the first and fourth quadrants, and $\lambda = +ha$ in the second and third. The integrand of (10) then vanishes along the infinite semicircle in the upper half-plane for $z>0$, in the lower half-plane for $z<0$. The contour goes above the poles on the negative real axis, below those on the positive side.

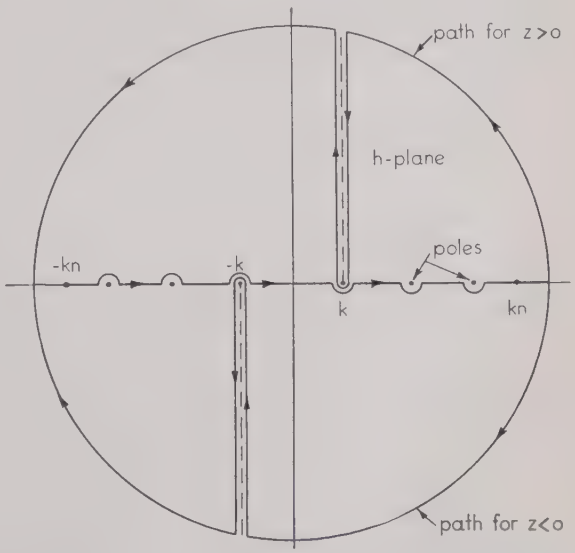

Fig. 2—The path of integration in the complex h plane is shown by the arrows. There are poles along the real axis between $\pm k$ and $\pm kn$. There are branch points at $\pm k$.

If the integral around the branch cut could be performed rigorously, it is probable that it would be found to represent a radiated field, and hence loss of power. Roe[10] has found this to be true for a special case of an analogous problem. This "branch-cut wave" is attenuated in the z direction because the integrand contains e^{ihz}, and along the branch cut the imaginary part of h is positive. It is apparent that this wave corresponds, for the dielectric slab, to the attenuated modes of the metallic guide.

The branch-cut integration has been performed using approximations valid when $kz \gg 1$, $z \gg x-a$. That is, the point of observation is a long way from the source and

[8] For the modes, $k<h_m<kn$. That is, the guide wavelength is shorter than the free-space wavelength, because the field travels partly in a dielectric. In accordance with standard waveguide terminology we speak of the mode nearest cutoff as the highest; it has the longest guide wavelength and the smallest value of h.

[9] The author is indebted to Glenn Roe for pointing out what happens to the root h_m after it has become equal to k.

[10] G. Roe, unpublished Ph.D. thesis, University of Minnesota, March, 1947.

close to the guide surface. The result is a wave having the form of

$$\psi \text{ (branch cut)} \backsim \frac{Ck(x-a)+D}{(kz)^{3/2}} e^{ikz}$$

where C and D are real constants.

It must be borne in mind that this is *not* a modal-type wave. It arises, not from a pole at a single value of h, but from an integration over a continuous range of h. Even if the integral could be performed rigorously, this wave would not have the form of a function of the transverse variable times a single exponential containing the axial variable.

Distribution of the Propagating-Mode Fields[11]

In the dielectric slab there is always at least one propagating mode, no matter how small ka may be, nor by how little the index of refraction exceeds unity. The fields are sinusoidal functions of x inside the guide; outside, they decay exponentially.

In a dielectric rod the electric field for an H_0 mode has only a tangential component. If the index be independent of the radius r, it depends upon r like

$$J_1(kr\sqrt{n^2-1-(h/k)^2})$$

inside the surface, and outside like

$$H_1^{(1)}(ihr).$$

Here J_1 is the Bessel function of the first order and $H_1^{(1)}$ is the Hankel function of the first order and first kind. The quantity h is a root of a transcendental equation corresponding to (11).

The dielectric rod with

$$n^2 = 1 + V + 3/(2kr)^2,$$

with V an arbitrary constant, corresponds to a cylindrical quantum-mechanical problem with a "square" potential well of depth V. It was considered here in order to learn whether a single mode could be effectively focused by making n large near the center; this particular dependence of n upon r was chosen simply because the wave functions for it are known. Inside the rod the field behaves like

$$r^{-1/2} \sin\left[kr\sqrt{V-(h/k)^2}\right];$$

outside, it again goes like

$$H_1^{(1)}(ihr)$$

where this h is the root of still another transcendental equation. Although increasing the index of refraction toward the center of the rod decreases the surface field a little, it also decreases the rate at which the field falls off with distance from the rod.

TABLE I

Case and mode	Cutoff ka	Next mode, cutoff ka	Surface field / Maximum internal field, per cent	Transverse attenuation, db per radius or half-thickness
Circular rod, constant n				
H_{11}-like mode	Zero (guide supports principal mode)	$\frac{3.831}{\sqrt{n^2-1}}$		28.2
E_{01}	$\frac{2.4048}{\sqrt{n^2-1}}$	$\frac{5.5201}{\sqrt{n^2-1}}$		37.5
H_{01}	$\frac{2.4048}{\sqrt{n^2-1}}$	$\frac{5.5201}{\sqrt{n^2-1}}$	44	39.0
Circular rod, $n \backsim 1/r$				
H_{01}	$\frac{2.029}{\sqrt{n_s^2-1}}$	$\frac{4.89}{\sqrt{n_s^2-1}}$	39	35.3
Flat plate, constant n				
H_0	Zero (guide supports principal mode)	$\frac{\pi/2}{\sqrt{n^2-1}}$	59	13.8

Cutoff Conditions and Comparisons of Field Intensities, Lowest Modes

Here n is the index of refraction of the dielectric, and n_s its value at the surface for the case $n \backsim 1/r$ (see text); $ka = 2\pi a/\lambda$ where a is the radius or half-thickness, λ is the free-space wavelength.

Although the cutoff values of ka for the E_{on} and H_{on} modes are the same, there is no degeneracy for intermediate values of ka.

The surface-field ratios and the transverse attenuations are calculated for the lowest mode of each type with the next mode just at cutoff, in order to obtain the most favorable condition.

[11] The differential equations and the transcendental equations whose roots give the propagation factors are discussed in some detail in footnote reference 4. They also appear in great detail and for more general cases in Roe's thesis; see footnote reference 10.

Mode cutoff conditions and data on transverse decay are given in Table I. The decay data were calculated with the parameters such that the second mode (of its type) was just at cutoff; this gives the most favorable condition (weakest external field) with only the lowest mode propagating. For hr large, $H_1^{(1)}(ihr)$ behaves like $(r/a)^{-1/2}\exp\ [-(ha)r/a]$, where a is the radius of the rod. This asymptotically exponential attenuation has been expressed in db per radius in Table I.

The H_{11}-like mode in the rod and the H_1 mode of the dielectric plate have zero cutoff wave number. Because of this characteristic they have been described in Table I as principal modes, but they have longitudinal field components and their phase velocities depend upon frequency.

Engineering Applications

Although the external modal field is not in itself radiated, it will induce currents in objects near the guide surface, and hence indirectly cause both reflections and radiation. By reciprocity, power could be received from another system.

If the guide were designed to support a few or several modes, their resultant field could be concentrated near the center. For a given dielectric material and a given frequency this would require a guide mechanically larger than for a single mode, and over any appreciable distance dispersion would still push the field to the surface. In the present state of the art it is difficult to couple in and out of any kind of guide which propagates more than one mode, but we may learn how in the future.

For comparison with the visible light analog, where we know the external field to be very weak, a glass rod 1 cm in diameter, with an index of 1.4 and yellow light, will support 17,000 H_{on} modes.

The dielectric slab might be made useful by bounding it with metal plates parallel to the $x-z$ plane and extending them so far in the x direction that the field at their edges is weak. This is just an involved way of describing a parallel-strip transmission line with a bar of dielectric material to concentrate the field near the center.

The dielectric guide will be useful for transmitting power over short distances where mechanical flexibility and low cost are important, and imperfect shielding can be tolerated.

Acknowledgment

The author is indebted to C. W. Horton of the University of Texas for permission to use his data on the H_{1n}-like and the E_{on} modes to obtain their cutoff values and their transverse attenuations.

The Relationship Between the Emission Constant and the Apparent Work Function for Various Oxide-Coated Cathodes*

HAROLD JACOBS†, GEORGE HEES†, AND WALTER P. CROSSLEY†

Summary—**Measurements were made of the emission of oxide-coated cathodes on six chemically different metal wires over a period of 500 hours. An empirical emission equation was employed which was found to be as accurate as the conventional Dushman equation and to be simpler to use. It was found that the logarithm of the emission constant A' varied as the sum of the apparent work function times a constant, and the constant B.**

Introduction

IN THE HISTORY of oxide-coated cathodes, there has been considerable discussion on the meaning of the constants of the various emission equations. A considerable difference in opinion exists at the present time as to how these constants vary. According to Blewett,[1] Espe[2] concludes that "activation is due to an increase in the A factor while the work function remains constant." Heinze and Wagener[3] claim that activation follows from a decrease in the work function alone. Others, Detels[4] and Huxford,[5] show that the work function and A factor both decrease in such a way that $\log A$ is a more or less linear function of the work function. DeBoer[6] theorizes (using a concept of thermal ionization of surface atoms) that the A factor should change and the work function remain constant. Actually, some of the difficulty in attaining unified results and opinions

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† Advanced Development Laboratory, Sylvania Electric Products Inc., Kew Gardens, L. I., N. Y.

[1] J. P. Blewett, "Properties of oxide coated cathodes," *Jour. Appl. Phys.*, vol. 10, pp. 831–848; December, 1939.

[2] W. Espe, "Thermionic constants of oxide coated cathodes," *Zeit. fur. tech. Phys.*, vol. 10, p. 489; 1929.

[3] W. Heinze and S. Wagener, "Variation of emission constant of oxide cathodes during activation," *Zeit. fur. Phys.*, vol. 110, p. 1[illegible]4; 1938.

[4] F. Detels, "Formation processes in oxide cathodes," *Zeit. fur. Hochfrequenz*, vol. 30, pp. 10–52; 1927.

[5] W. S. Huxford, "Photoelectric emission from oxide coated cathodes," *Phys. Rev.*, vol. 38, p. 379; 1931.

[6] J. H. DeBoer, "Electron Emission and Absorption Phenomena," Cambridge University Press, 1935, p. 34.

may reside in studying the constants under different experimental conditions, where, due to different pressures, temperatures, and general conditions, the actual experiments may have differed.

In the work to be reported an attempt was made to observe the changes of the constants of an empirical emission equation

$$I = A' \exp\left(\frac{-e\chi}{kT}\right) \tag{1}$$

over a 500-hour period of life, after the oxide-coated cathodes had been broken down and aged rather completely. Using this equation, we have set up the problem of describing the electron emission from oxide-coated cathodes as a function of life, and to determine whether or not any relationship exists which may be indicated experimentally between the A' factor and the work function. It was decided to use this equation for two reasons: firstly, it is in the simplest form for practical use; secondly, this equation can perhaps have future possibilities, since it can be shown to have some relation to the modern theory of solids.[7]

Description of Experimental Tubes and Tube Processing

To study thermionic electron emission, special emission-test diodes were constructed as sketched in Fig. 1. These tubes contained three grade-A nickel cylinders to be used as anodes in a standard soft-glass "lock-in"-type

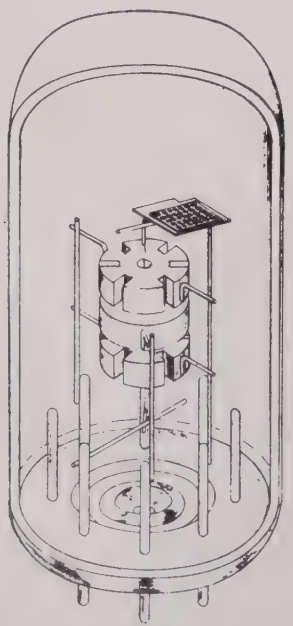

Fig. 1—Experimental diode.

vacuum-tube bulb. The purpose of the slotted end cylinders was to act as guard rings and help provide a uniform electrostatic field at that portion of the filament being studied. The use of guard rings also minimized "end effect"—cooling of the ends of the filament by thermal conduction. The center anode was isolated electrically and used as the collector for the emission studies. To reduce the probability of evolution of gas, no mica or lava spacers are used as is customary in commercial tubes, but all spacing was accomplished by means of nickel supports. The outside diameter of the anode cylinders was 0.500 inch; the inside diameter, 0.125 inch. The center anode was 0.375 inch long, and the guard rings 0.125 inch long. A 0.050-inch peep hole was provided in the center anode so that the brightness temperature of that portion of the filament being studied could be measured directly with an optical pyrometer. A spectral-emissivity factor of 0.6[8] was used as a correction for the emissivity of the oxide coating, and resulted in a correction of 33°C at a color temperature of 900°C brightness.

The filaments consisted of 0.005-inch diameter wire, coated with a standard triple-carbonate suspension to an approximate outside diameter of 0.0065 inch. The coating density was held within 1.0 ± 0.3 grams per cubic centimeter.

The diodes are exhausted, activated, aged, and life-tested under identical conditions to eliminate, to the greatest extent possible, the effects of these factors on the ultimate thermionic emission. Table I gives exhaust, activation, and aging schedules used, and Table II gives the life-test conditions.

Methods of Measurement

Data to plot the emission curves were obtained in the filament-temperature ranges of 850 to 1100°K by raising the plate voltage, and recording plate current and voltage until temperature-limited current was drawn. The highest filament temperature was, in general, that which gave a temperature-limited current of 12.0 ± 2 ma at a plate voltage of 200.0 volts dc. It has been observed that the filaments can be heated markedly by the cathode current being drawn, and because of this it was sometimes necessary to run emission curves at initial currents less than 12.0 ma.

The filament temperature in the region below 1050°K was determined from the filament volt-amperes by extrapolation on a temperature calibration curve. Filament temperature was measured as a function of filament volt-amperes at several points in the visible temperature region (900, 860, 820, and 790°C brightness) and extrapolated to room temperature at 0 volt-amperes input.

The logarithm of the plate current was plotted against the plate voltage. The slopes of the space-charge region and the temperature-saturation region were extrapolated. The point of intersection of these projected lines was taken as the point of zero effective field on electrons leaving the filament. The current at this point

[7] A. L. Reimann, "Thermionic Emission," John Wiley and Sons, Inc., New York, N. Y., 1934; pp. 227–229.

[8] J. P. Blewett, "Properties of oxide coated cathodes," *Jour. Appl. Phys.*, vol. 10, pp. 668–679; October, 1939.

was used as the total thermionic emission I from the filament independent of field effects.

The log of the total emission in amperes per cm² was plotted against the reciprocal of the absolute temperature (see Fig. 2) to evaluate the emission constant A' and the apparent work function χ of the empirical emission equation (1).

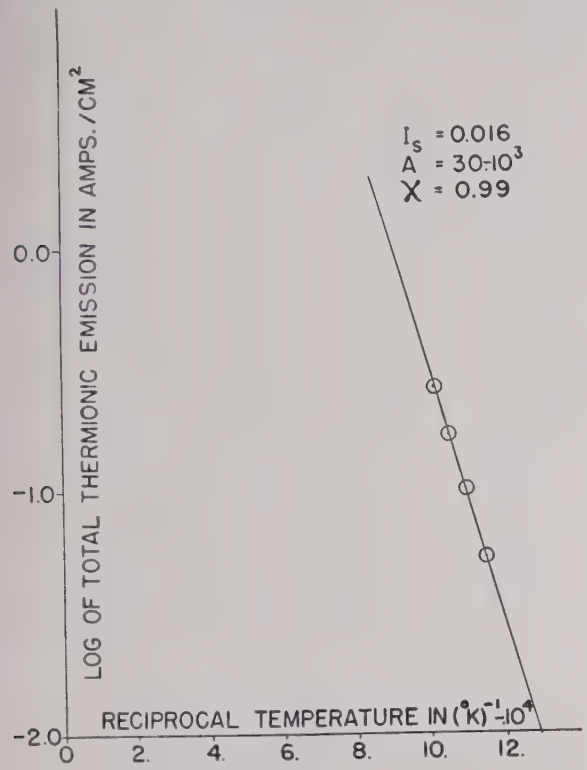

Fig. 2—Plot of emission equation to evaluate A' and χ: $I = A'\epsilon^{-e\chi/kT}$.

It is of interest to note at this point how the Dushman equation can serve just as well to describe the experimental determination of emission as a function of temperature. Using the same data as obtained in Fig. 2 and plotting $\log I_0/T^2$ versus $1/T - 0.10^{-4}$, we find that a fairly good straight line can still be approximated (see Fig. 3). However, the Dushman equation is more complicated in reading, since to get the emission one must multiply by a T^2 factor at any given temperature.

Data

To define the thermionic-emission properties of a given filamentary material, the emission constant A', the work function χ, and the total emission at 800°K I_s were determined as a function of life. The temperature ranges used in the measurements of the above values are also reported. To present this large mass of data in a comprehensive form, a statistical breakdown was employed.

The standard deviation has been selected as a measure of dispersion.

Results are given in Table III.

A particularly interesting feature of the data can now be presented concerning the relationship of A' and χ as a function of life.

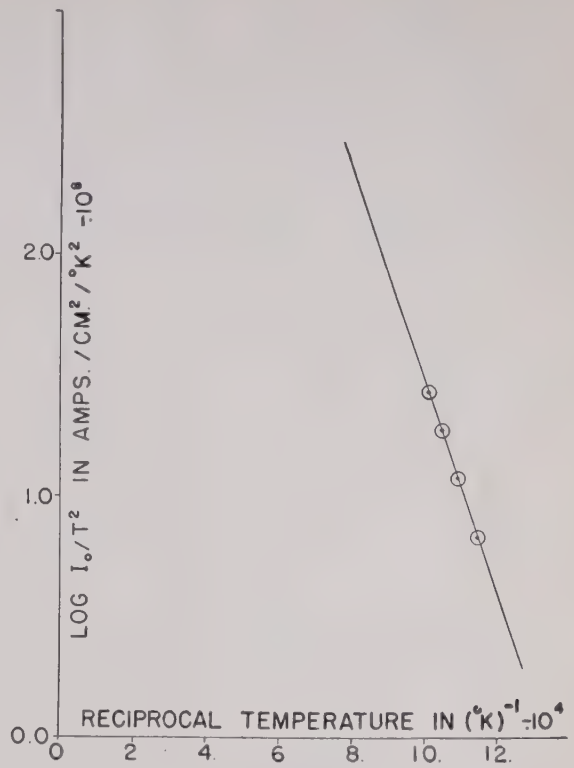

Fig. 3—Plot of Richardson's equation: $I = AT^2\epsilon^{-e\phi/kT}$.

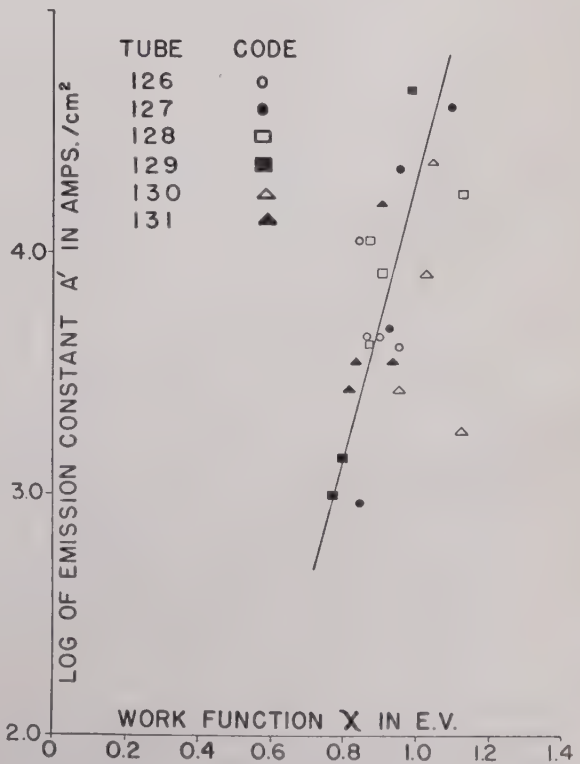

Fig. 4—Variation of A' with χ for pure nickel filamentary alloy.

The measurements of A' and χ were made for each tube at approximately 24, 100, 250, and 500 hours of life. If the tubes were placed in groups such that all of the oxide-coated filaments coated in one type of filament wire were grouped, a straight-line relationship could be found between the log A' and χ. For instance, in Fig. 4 the A' and χ values for six tubes containing a pure nickel filament, coated with the oxide, were studied over life. Each particular reading for this type of material was plotted. The log A' and χ can be seen to be related to each other. Similarly, Fig. 5 represents the data taken for A' and χ of each particular tube at each measurement on life, where the oxide is coated on a nickel wire with boron impurity. Fig. 6 illustrates the data for the commercial Tensite filament coated with the oxide. In the case of all wire samples, as the points of A' and χ for each tube are plotted over various periods of life, a straight-line relation is formed between the log A' and χ. It is also of interest to note that the slopes of these lines are somewhat different, depending on the nature of the core material. It would be of interest to test other core materials and to determine how much the slope will actually be different for different samples. To the authors' knowledge, work of this type has never been published.

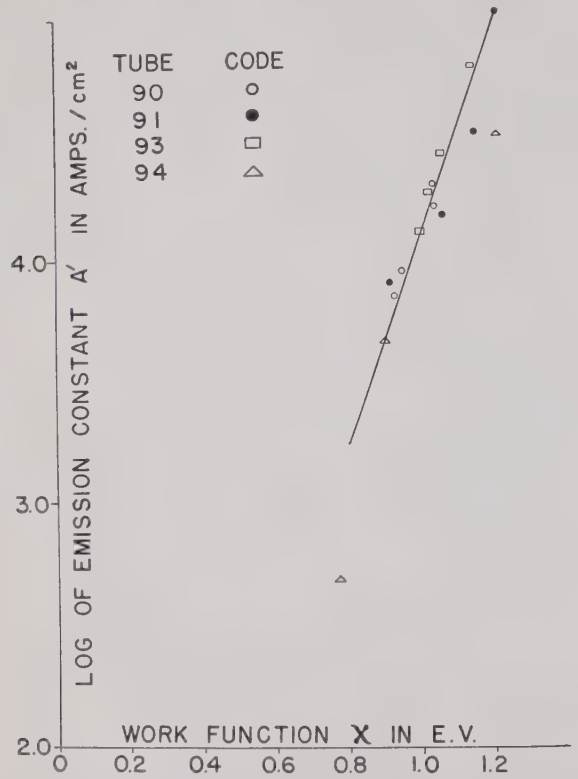

Fig. 5—Variation of log A' with χ for boron-nickel filamentary alloy.

One questionable point is whether this change in A' is a real change in the constant, or whether a change in A' is due to an exponential temperature factor inherent in A'. Suppose, for instance, that in our temperature range of measurement A' changes exponentially with temperature. This could give a result which would mask, to some extent, the nature of log A' varying with χ.

In order to investigate the effect of temperature on A' and the temperature range of the emission measurements, the following device was utilized.

According to Reimann,[9] from photoelectric evidence, a variation in the work function with temperature does not exist.

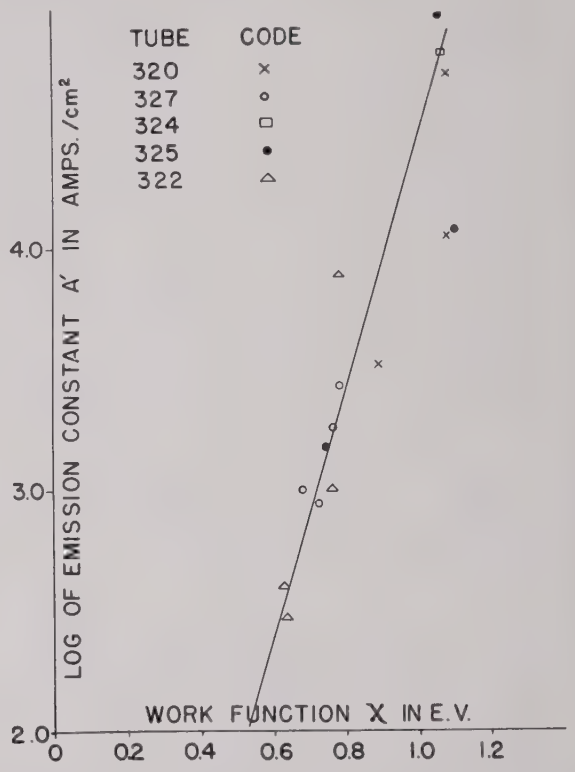

Fig. 6—Variation of A' with χ for Tensite filamentary alloy.

Over the small range in temperatures studied it would not be expected that the work function would shift with temperature. For the case of oxide-coated pure nickel wire, an effective work function of 0.93 electron volts was assumed. From the data in Table III, this is not far from the average value determined.

Then, for each tube, A' was calculated for each I_0 at each particular temperature. This method of determining A' is somewhat independent of the previous method, since here we are assuming a constant work function and calculating A' by a measurement of I_0 as a function of T.

The resulting A' values, as determined from (1), are tabulated in Table IV.

An inspection of these results shows that, if we assume χ to be constant over the range of temperatures in which I_0 was measured, then log A' has relatively small changes in value with temperature, and certainly changes much less than the observed variations in Figs. 4–9.

[9] A. L. B. Reimann, "Thermionic Emission," John Wiley and Sons, Inc., New York, N. Y., 1934, pp. 267–269.

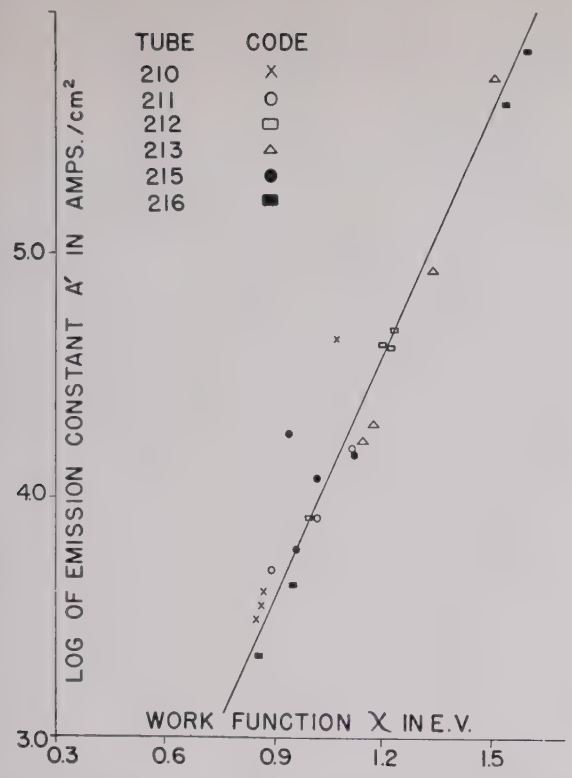

Fig. 7—Variation of A' with χ for vanadium-nickel filamentary alloy.

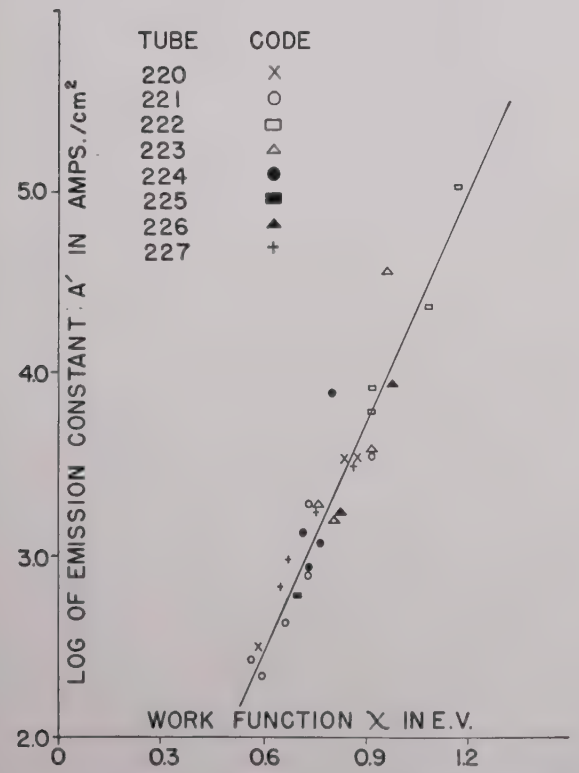

Fig. 8—Variation of A' with χ for cobalt-nickel filamentary alloy.

It might be inferred from this that variations in log A' with χ do not represent a temperature effect but are inherently due to physical and chemical changes in the cathode for small temperature ranges.

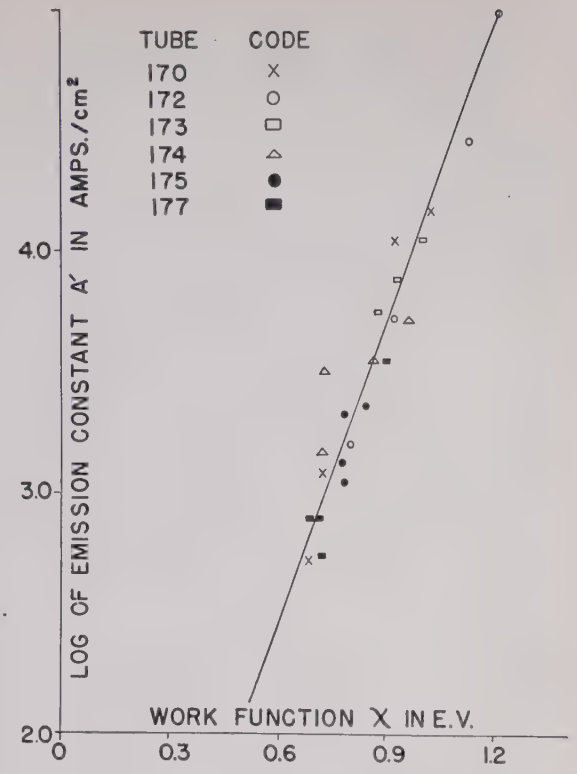

Fig. 9—Variation of A' with χ for platinum-nickel filamentary alloy.

Interpretation

From the data above, it appears that log A' varies linearly with the apparent work function χ, over various periods of life, for the case of oxide-coated cathodes on nickel. Why this should occur is by no means clear, since there is not enough fundamental work yet available upon which to build a general theory. In addition, the results indicate that emission is very much a function of the impurities in the core metal.

TABLE I

1. Bake at 400°C for 30 minutes to drive off occluded gases on glass.
2. Degas elements at red heat not to exceed 850°C by rf heating to thoroughly degas metal parts. (This required about 15 minutes of heating.)
3. Set filament temperature at 900±25°C for 2 minutes (keeping anodes at a red heat) to reduce the alkaline earth carbonates to the oxides.
4. Following this, the processing was established as below:
   (a) Set T_F=1000°C brightness; E_p=10 v for 1 minute
   (b) Set T_F= 900°C brightness; E_p= 50 v for 5 minutes
   (c) Set T_F= 900°C brightness; E_p=150 v for 10 minutes
5. Flash getter and tip off.

TABLE II

Life Test Conditions

T_F=775°C brightness
E_p=40.0 V rms, 60-cycle ac
Emission was measured at 24, 100, 250, and 500 hours of life.

Acknowledgment

Acknowledgment is made for the suggestions offered by W. B. Nottingham, of The Massachusetts Institute of Technology; W. F. Schockley, of the Bell Telephone Laboratories; and George Kuczynski, of Sylvania Electric Products Inc.

TABLE III

Mean Value of Emission Constants versus Life

Alloy Number and Nominal Composition	Hours Life	N	$\overline{A}\cdot 10^{-2}$	$\sigma\overline{A}\cdot 10^{-2}$	$\overline{\chi}$	$\sigma\overline{\chi}$	$\bar{I}_s$	$\sigma\bar{I}_s$	ΔT	T min	T max
1	24	6	21.5	11.9	0.93	0.07	0.029	0.013	130	760	1065
Pure Ni	102	6	9.6	13.3	0.93	0.13	0.017	0.008	135	840	1105
	248	6	4.7	1.6	0.89	0.07	0.013	0.006	145	865	1090
	490	6	4.8	5.6	0.93	0.11	0.009	0.005	120	840	1150
2	25	4	17.1	27.1	0.83	0.09	0.022	0.009	165	815	1100
Tensite	99	4	6.5	5.1	0.83	0.14	0.023	0.022	170	665	1105
1.34% Al, 0.015% Mg, 0.33%	265	4	38.2	36.5	0.90	0.17	0.043	0.028	165	690	1020
Si by weight	501	4	1.53	1.27	0.82	0.15	0.042	0.038	140	620	1080
3	24	4	9.6	5.5	0.94	0.05	0.009	0.010	170	895	1110
B-Ni	102	4	28.8	22.3	1.03	0.16	0.005	0.001	130	940	1115
0.5 Atomic % Boron	248	4	157.0	238.0	1.16	0.18	0.004	0.002	145	925	1190
	490	4	41.0	41.0	1.09	0.12	0.003	0.004	130	925	1175
4	25	3	15.0	8.9	0.98	0.08	0.009	0.005	160	850	1260
Co-Ni	99	3	9.6	1.9	0.92	0.05	0.019	0.015	160	820	1290
2.0 Atomic % Cobalt	265	5	34.4	48.4	0.92	0.125	0.017	0.013	165	815	1265
	501	3	2.9	1.5	0.77	0.014	0.045	0.032	165	735	1250
5	24	6	20.8	35.6	0.91	0.16	0.018	0.019	180	775	1105
Pt-Ni	100	6	3.7	2.5	0.81	0.12	0.017	0.009	180	840	1090
2.0 Atomic % Platinum	250	6	8.8	9.5	0.90	0.14	0.014	0.007	185	830	1095
	506	5	2.8	2.5	0.80	0.03	0.018	0.008	190	830	1105
6	25	6	28.0	39.0	1.03	0.14	0.004	0.003	155	930	1235
V-Ni	100	6	12.8	14.9	0.98	0.15	0.008	0.005	165	840	1190
2.0 Atomic % Vanadium	249	5	15.2	13.6	1.02	0.14	0.007	0.006	150	860	1250
	505	6	225.0	293.0	1.22	0.25	0.007	0.021	140	860	1250

$\overline{A}'$ = arithmetic mean (a.m.) of the emission constant in amp/cm²
$\sigma\overline{A}'$ = standard deviation (s.d.) of $\overline{A}'$
$\overline{\chi}$ = a.m. of the apparent work function in electron volts
$\sigma\overline{\chi}$ = s.d. of χ
$\bar{I}_s$ = a.m. of total thermionic emission at 800°K true in amp/cm²
$\sigma\bar{I}_s$ = s.d. of $\bar{I}_s$
N = number of tubes
$\overline{\Delta T}$ = a.m. of temperature range used in evaluating A' and χ in °K
T max = maximum temperature used, in °K
T min = minimum temperature used, in °K.

TABLE IV

Variation of Log A' with Temperature, Assuming Constant χ

Tube No.	126		127		128		129		130		131	
No. of Hours	log A'	T	log A'	T	log A'	T	log A'	T	log A'	T	log A'	T
24	4.52	600	4.15	705	4.36	645	4.26	625	3.88	790	4.27	685
	4.56	555	4.26	670	4.39	595	4.28	580	3.85	755	4.37	640
	4.56	520	4.12	630	4.39	560	4.38	540	3.80	765	4.42	600
	4.58	485	4.09	585	4.41	510	4.33	495	3.76	660	4.37	555
102	4.01	725	3.85	775	4.08	700	3.94	770	3.51	830	4.06	735
	4.08	685	3.85	735	4.08	655	3.97	730	3.49	790	4.12	700
	4.05	630	3.79	685	4.06	610	4.03	690	3.44	745	4.15	655
	4.07	580	3.37	625	4.14	565	4.00	640	3.44	700	4.10	605
248	3.92	755	3.70	815	3.55	755	3.92	780	3.53	815	4.09	740
	3.90	705	3.68	770	4.01	695	4.05	730	3.54	780	4.14	695
	3.94	650	3.75	725	4.04	650	4.68	685	3.48	725	4.19	645
	4.17	605	3.71	675	3.97	610	4.11	625	3.48	675	4.21	590
490	3.62	775	3.45	800	3.46	825	3.85	695	3.38	875	3.58	810
	3.54	725	3.43	755	3.40	775	3.88	650	3.46	840	3.57	770
	3.58	675	3.50	700	3.38	725	3.96	605	3.44	805	3.59	730
			3.50	660			3.95	565	3.44	765	3.61	685

T in °C
log A' in amp/cm²

Effects of Hydrostatic Pressure on Electron Flow in Diodes*

W. C. HAHN†, SENIOR MEMBER, IRE

Summary—**Inclusion of the hydrostatic pressure term, with constant temperature, in the electron force equation of a diode is shown to give a way by which the effect of Boltzman's relation for a retarding field disappears as the field becomes accelerating. Thus, one equation holds from the cathode surface through the potential minimum to the anode. As a result, one may deduce the usual temperature-limited emission formula for maximum current, the space-charge-limited characteristic, and the transition region from one to the other. Some remarks on this, from the standpoint of transport theory, are included.**

THERMIONIC EMISSION is ordinarily regarded as a problem in the evaporation of electrons from the surface of a hot metal. The argument proceeds along thermodynamic lines. As thermodynamics applies precisely to an equilibrium state, it is usual to calculate the current leaving the cathode for such a state and set the maximum emission equal to this value. It would seem reasonable that this would give a fairly close value; but, as Fowler[1] points out, the method gives no way of evaluating the error. Strictly speaking, one must use kinetic theory to discuss such problems of flow. The same thing applies, of course, to the space-charge-limited flow in diodes.

In applying kinetic theory to problems of electron flow in high-vacuum tubes, one must consider the effect of interactions between electrons. Little or no work has been done along this line, aside from the use of a smoothed charge density in Poisson's equation. As it stands today, the kinetic theory[2] consists essentially of the hydrodynamic equations with an extra equation giving the temperature. The components of the stress tensor and the heat conduction vector contain all the required information about statistics, etc. The usual plan of solution of these equations contemplates employing a series of approximations for these components; and in theory, at any rate, it is possible to appeal to the next approximation to get an idea of the order of magnitude of the error.

In this paper it will be shown that, in a very simple approximation, the effect of hydrostatic pressure is to limit the current flow over the potential barrier at the boundary of a metal, in much the same way as gas flow through an orifice is limited. The same equation can describe the flow in a space-charge-limited diode from the cathode through the potential minimum to the anode. Since it holds for both temperature-limited and space-charge-limited conditions, it can be used to give the transition between these conditions.

* Decimal classification: R138. Original manuscript received by the Institute, December 5, 1947. Presented, IRE Electron Tube Conference, New Haven, Conn., June, 1946.

† General Electric Research Laboratory, Schenectady, N. Y.

[1] Fowler, "Statistical Mechanics," University Press, Cambridge, England, 1936; p. 346.

[2] Chapman and Cowling, "The Mathematical Theory of Non-Uniform Gases," University Press, Cambridge, England, 1939; p. 51.

Interactions between Electrons

First, it may be profitable to consider exactly what is meant by interactions between electrons. In ordinary monatomic gases, the simplest theory of collision phenomena considers the molecules as smooth elastic spheres (such as rubber balls) which bounce off each other. The theory of such collisions leads directly to the concept of a mean free path. Results of this theory, as applied to gases, were only approximately verified by experiment. Actual forces between molecules may be considered electrical in origin (such as dipole or quadrupole forces), and, in this case, they may be considered to fall off as the vth power of the distance between them, so that the molecules do not touch during a collision. Furthermore, the nearest distance may vary with the relative velocity, etc. The sum of the charges being zero, the gas is still electrically neutral. By this means, very much better checks with experiment have been made. Considering electrons, the inverse square law is to be assumed, of course; but it is found that the collision integrals do not converge. This condition arises primarily because the inverse square law falls off so slowly with distance that, in effect, all of the electrons are always in collision with each other.

Suppose that one considers the electrons to be located in a regular manner throughout the volume and the whole to move as though they were fastened together. In this sense there are no collisions, and the interactions can be taken as the sum of the repulsions of each for all the others; i.e., one can smooth the space distribution of charge and calculate the force on each electron by Poisson's equation. This is already common practice. Now if the electrons have a random motion relative to these ordered positions, we will mean by interactions or encounters the additional forces (varying randomly with time) produced on an electron by such motions. It may be considered that, there being such a large number involved, the sum of the random forces produced by those a long way off is practically zero by cancellation. Thus the encounter forces on an electron are mainly the random forces produced by its nearest neighbors. One way to calculate these forces is to consider the theory of fluctuations of the space-charge forces.[3] This has not been done as yet. In fact, the only

[3] See chap. XX of footnote reference 1.

work applying to this situation seems to be that of Chapman,[2] who considered only interactions involving two electrons.[4] While these values may be considered indicative of the order of magnitude of the quantities involved, much remains to be done before it may be said that the theory is in reasonably good shape. In particular, it should be pointed out that some experimental work on measuring the required quantities is essential. In this paper, the effect of collisions is assumed to be such that the temperature is constant in the discharge, while any effects of viscosity are neglected.

Hydrostatic Pressure of Electrons

Suppose we consider a plasma of electrons whose mean velocity, normal to a plane, is v_0. There is, on the average, a density of n per unit volume; and each has, in addition, a random velocity relative to the mean, the probability of any velocity between u and $u+du$ being given by a Maxwellian distribution. In problems of this kind, the mean and random motions are kept distinct, just as one separates dc terms from ac terms in circuit analysis. The temperature is defined as $\frac{2}{3k}$ times the average of the random kinetic energy and constitutes a measure of it. Here k is the usual constant relating energy and temperature and has a value of 1.38×10^{-16} ergs per degree in the cgs system. Now one calculates the flow of kinetic energy per unit area, normal to the plane, caused by the electrons crossing it both ways. Dividing this by the average velocity with which they cross, gives the average energy associated with such movement. For the mean motion, we get $(nm/2)\cdot v_0^2$, and for the random motion, nkT. Now the derivative of this energy with respect to distance x, normal to the plane, gives the equivalent force F which must have acted to produce the energy change. Thus,

$$F = nm\frac{dv_0}{dt} + \frac{dnkT}{dx}. \tag{1}$$

This is the usual force required to accelerate the mass of the particles to the mean velocity v_0 plus an additional force required on account of the random motion. The quantity nkT is called the hydrostatic pressure. This force F is produced by the electric gradient E_x, so that

$$F = neE_x \tag{2}$$

(per unit area). Setting these equal and dividing by n to get an equation applying on a per-electron basis,

$$m\frac{dv_0}{dt} = eE_x - \frac{1}{n}\frac{dnkT}{dx}. \tag{3}$$

If one assumes that v_0 is zero, so that the plasma is in equilibrium, integrating this equation immediately gives Boltzman's relation for the density n. In this way one can see that, even though there is an electric gradient acting on the electrons, they have no average motion because the force generated by the varying hydrostatic pressure exactly balances the electrostatic forces on the average.

It is customary to use Boltzman's relation in a retarding field and to discard it when the field becomes accelerating. This creates a discontinuity at places where the direction of the electric gradient changes sign, such as a potential minimum. The restrictions on the validity of (3) are not with respect to the sign of E_x, but rather are implicit in the assumption of the velocity distribution and the value of T. From this standpoint, (3) should hold right through the potential minimum; and clear out to the anode, for that matter. Thus the application of Boltzman's relation and its disappearance become automatic and inherent in the equation.

In this study, it usually will be assumed that the temperature T throughout the whole discharge is constant at the cathode temperature. The rather remarkable success in explaining well-known experimental results justifies the statement that T cannot vary a great deal until one is beyond the potential minimum when the latter is close to the cathode.

The rest of the analysis will be made on the basis of a parallel-plane diode with the mean electron motion in the x direction only, esu units, and cartesian co-ordinates. These conditions with constant temperature T_0 give

$$m\frac{d^2x}{dt^2} = eE_x - \frac{kT_0}{n}\frac{dn}{dx}. \tag{4}$$

Temperature-Limited Emission

In addition to (4), there is the continuity equation

$$I_0 = en\frac{dx}{dt} \tag{5}$$

where I_0 is the anode current per square centimeter. Now differentiate (5) with respect to t and resubstitute (5):

$$-\frac{I_0}{en}\frac{d\log n}{dt} = \frac{d^2x}{dt^2} = -\left(\frac{I_0}{en}\right)^2\frac{d\log n}{dx}. \tag{6}$$

Using this in (4), there is

$$\frac{eE_x}{kT_0} = \frac{d\log n}{dx} - \left(\frac{I_0}{en}\right)^2\frac{m}{kT_0}\frac{d\log n}{dx}. \tag{7}$$

This equation, since $E=-(dV/dx)$, can be integrated to give

$$-\frac{eV}{kT_0} = \log\frac{n}{n_0} + \frac{m}{2kT_0}\left(\frac{I_0}{e}\right)^2\left(\frac{1}{n^2} - \frac{1}{n_0^2}\right) \tag{8}$$

where V is measured from the point where the electron density is n_0. Neglecting the last term, this is recognized

[4] A. Vlasov, "On the kinetic theory of an assembly of particles with collective interaction," *Jour. Phys.* (U.S.S.R.), vol. 9, no. 1, p. 25; 1945.

as Boltzman's relation. Thus the latter is changed, under conditions where there is a flow, by subtracting from $-(eV/kT_0)$ a term which easily can be seen from (5) to represent the kinetic energy of the flow in equivalent units. This is a relation of the Bernoulli type, as might be expected.

Now consider Fig. 1. On the right is plotted $-(eV/kT_0)$ versus x for a typical image force field with an added accelerating field.

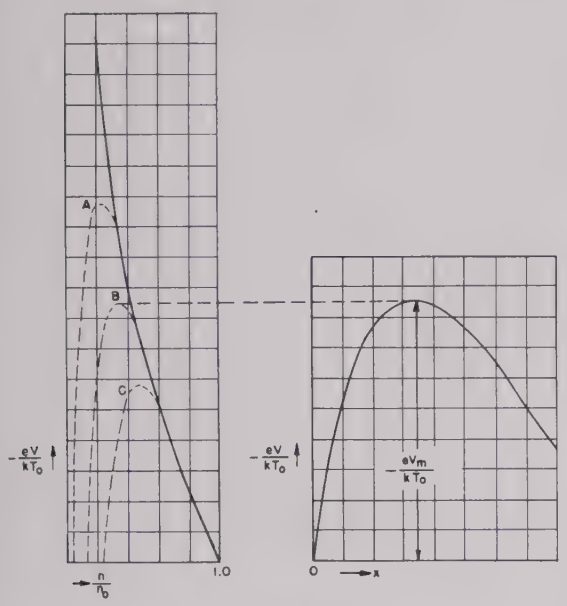

Fig. 1

The abscissa x is taken as zero at the surface of the metal lattice structure, and the maximum of the curve occurs where V is about equal to the thermionic work function V_m (neglecting any reduction of the maximum due to the external field). On the left is plotted equation (8). The solid line is the first term on the right of (8); and the dotted lines represent this with the second term subtracted, using various values of I_0. Considering curve A and noting that the same values of $-(eV/kT_0)$ must exist in both sets of curves simultaneously, it will be seen that one cannot go over the top of curve A because the maximum V is not high enough, and thus n returns to its original value of n_0. This corresponds to a metallic contact, not an electron tube, and will be discarded. For curve C, one cannot go over the top of the right-hand curve and must return down it. But obviously this requires that dx/dt must reverse, and thus this is not a physical condition. Finally, consider curve B where the maximum of the two curves are exactly the same. One may then go over the top of the image field V_m and, at the same time, go over the top of curve B so that n continually decreases. This is the only condition corresponding to a cathode; and, since V_m is fixed by the constants of the metal, the current I_0 is also fixed.

Mathematically, we may solve (7) for $(d \log n/dx)$ to get

$$\frac{d \log n}{dx} = \frac{\frac{eE_x}{kT_0}}{1 - \frac{m}{kT_0}\left(\frac{I_0}{en}\right)^2}. \tag{9}$$

If we assume that n continually decreases, and note that, from Fig. 1, E_x must go through zero; then at that point (the potential minimum) we must have

$$1 = \frac{m}{kT_0}\left(\frac{I_0}{en_1}\right)^2$$

or

$$I_0 = en_1\sqrt{\frac{kT_0}{m}} \tag{10}$$

where n_1 is the density at the potential minimum. This is precisely what we would get by differentiating for a maximum of curve B and using this simultaneously with the maximum V_m of the right-hand curve. Note that (9) and (10) are true for any potential minimum (such as occurs in space-charge-limited flow), and the present application for temperature-limited flow comes about by the identification of V_m with the thermionic work function of the surface. We may substitute (10) in (8) and get

$$-\frac{eV_m}{kT_0} = \log\left(\frac{I_0}{n_0 e\sqrt{\frac{kT_0}{m}}}\right) + \frac{1}{2} - \frac{1}{2}\left(\frac{I_0}{n_0 e\sqrt{\frac{kT_0}{m}}}\right)^2. \tag{11}$$

It will be found that the last term is negligible.

Solving for I_0,

$$I_0 = n_0 e\sqrt{\frac{kT_0}{2.718m}}\exp\left(-\frac{eV_m}{kT_0}\right). \tag{12}$$

Now V_m is customarily measured from the metal, and thus n_0 is the density at the surface of the metal. Fowler[5] gives this as

$$n_0 = \frac{2(2\pi mkT_0)^{3/2}}{h^3}. \tag{13}$$

This value is that which one would find at the metallic surface, if Maxwellian statistics held to this point, and is appropriate for this calculation. Its derivation takes into account the Fermi-Dirac condition actually present. Substituting (13) in (12), we finally arrive at

$$I_0 = AT_0^2\exp\left(-\frac{eV_m}{kT_0}\right) \tag{14}$$

[5] See p. 345 of footnote reference 1.

where the constant A has the value of 181 amperes per square centimeter. This is the standard equation derived by Dushman and Richardson and for which Fowler gives the value $A = 120$, based on the previously mentioned extension of equilibrium arguments. Actual experiments do not give A with great accuracy as a fundamental constant, its value depending to some extent on what one uses for V_m. As a matter of fact, considering that we have neglected viscosity and temperature variations, the simplicity of the derivation and the relative correctness of the result are startling.

It will be noted that, for space-charge-limited flow, the potential minimum occurs at some voltage greater than the thermionic work function. The same equation (14) can be used to determine this voltage from the assumed current. Thus, in this sense, the emission current is always limited by the same mechanism; and at high currents the potential minimum moves over so close to the metal that the externally applied gradient cannot appreciably affect its potential, and the emission becomes constant. Examination of the left-hand side of Fig. 1 will disclose that, until one gets very close to the potential minimum on the cathode side, Boltzman's relation holds very accurately. Beyond the potential minimum, n/n_0 goes on down. As the log n/n_0 is a slowly varying function of n, to a first approximation, one may set

$$-\frac{eV}{kT_0} \approx \frac{m}{2kT_0}\left(\frac{I_0}{n_0 e}\right)^2 \frac{n_0^2}{n^2} \tag{15}$$

by measuring V from a suitable zero point. Integration of this with Poisson's equation immediately gives the well-known 3/2-power law of the Langmuir-Child's equation. Thus (8) contains within it the two fundamental relations which distinguish the operation of a high-vacuum electron discharge device.

Space-Charge-Limited Flow

In the preceding section, it was shown that the flow has the usual characteristics before and after the potential minimum. Langmuir assumed that, in our notation, the flow was isothermal to the potential minimum and essentially adiabatic beyond, and derived the potential distribution for these assumptions by integrating Poisson's equation. It will be interesting similarly to integrate (8) with

$$\frac{d^2V}{dx^2} = -4\pi en \tag{16}$$

for purposes of comparison. This must be done numerically on account of the form of (8). Although the solution is continuous through the potential minimum, the boundary conditions are set, at this point, by (9) and (10), and thus one integrates each way from this point. In doing this, it is found, as previously mentioned, that Boltzman's relation holds from the cathode surface to a point very close to the potential minimum Thus we may disregard the image field, decreasing n by $\exp(-eV_m/kT_0)$, where V_m is the thermionic work function, and regard this as an equivalent value of n_0 at the surface.

The integration of (8) is conveniently performed by making the substitutions

$$s = x\sqrt{\frac{4\pi e^2}{kT_0}}\frac{I_0}{e}\sqrt{\frac{m}{kT_0}} \tag{17}$$

$$y = \left(\frac{I_0}{e}\right)^2 \frac{m}{kT_0}\frac{1}{n_2}, \tag{18}$$

which reduces (8) to

$$y = 1 + \log_\epsilon \dot{y} + 2\int_0^s ds \int_0^s \frac{ds}{\sqrt{y}} \tag{19}$$

and

$$-\frac{eV}{kT_0} = \int^s ds \int_0^s \frac{ds}{\sqrt{y}} \tag{20}$$

where we measure x and V from the potential minimum, and y is the ratio of the mean kinetic energy to that at the potential minimum. In Fig. 2 is plotted the resulting $(-eV/kT_0)$ versus s (full line). It will be noted that Langmuir's η is simply $(-eV/kT_0)$, and his ξ is related to our s by

$$\xi = (2\pi)^{1/4}s = 1.583s. \tag{21}$$

His curve is plotted in Fig. 2 as a dotted line. Estimates of the temperature variation, using Chapman's values of the coefficient of thermal conduction, indicate

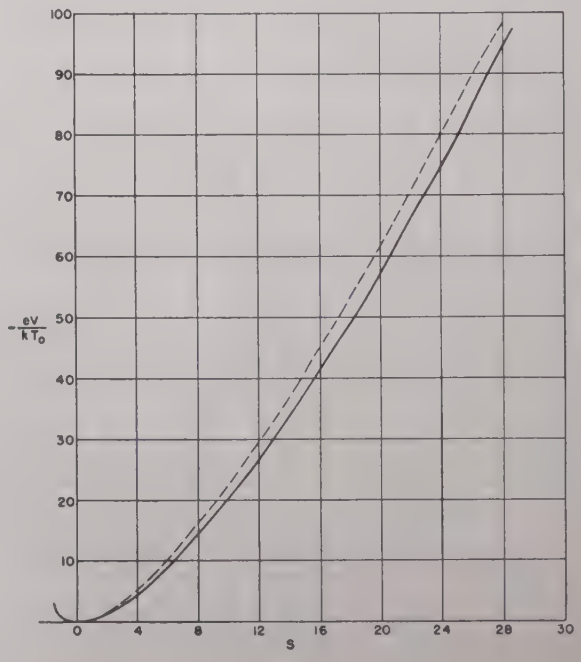

Fig. 2

a variation of temperature of the order of magnitude of about 5°, starting at 2500° at the cathode, between the metal and the end of the image field. Thus it seems probable that, in all cases, the discharge starts isothermally, or nearly so. On the other hand, consideration of the equation for temperature leads to the conclusion that, if the anode is an infinite distance away, somewhere the discharge becomes practically adiabatic. The changeover point may be on either side or immediately at the potential minimum, depending on the conditions (such as current, original temperature, etc.). Examination of (15) shows that, if the changeover point occurs subsequent to the potential minimum by a sufficient amount, the curve will be practically the full line in Fig. 2 (inasmuch as T_0 cancels out of (15) and does not affect the voltage-current curve). On the other hand, it is probable that, if the changeover is about at the potential minimum, Langmuir's curve is more nearly correct. In any case, the important point is that the difference in the resulting curves is small enough so that experiments would have to be made very carefully to distinguish between no interaction at all (dotted line) and a very considerable amount of interaction (full line). On the other hand, the temperature of the discharge is vitally affected by such differences in assumptions. From the point of view of the external characteristics of the tube, plate noise might suffer the largest change; and it would seem that calculations of this, especially for space-charge-limited tubes, should consider such effects.

Transition from Space-Charge- to Temperature-Limited Emission

As was remarked before, the transition from space-charge- to temperature-limited emission is accomplished by the potential minimum moving toward the cathode. Ultimately it will move into the image field far enough to be relatively unaffected by the applied gradient from the anode. Some slight effect will still remain, and calculations of this immediately give the Schottky effect in its customary form.

It is usual to assume the potential as that of a true image field having a maximum value of V_m, with a displaced origin of x to keep the gradient at the surface finite. Although this is admittedly approximate, it has given reasonably good results in quantum mechanical calculations.

To do this we note that, if subscript i is used for image fields,

$$eE_i = -\frac{e^2}{4(x-x_0)^2} \tag{22}$$

or

$$-\frac{eV_i}{kT_0} = \frac{e^2}{4kT_0(x-x_0)}, \tag{23}$$

and if the cathode is located at x_1, the displaced origin of the image field is at x_0 and the origin of x is at the potential minimum (so that both x_0 and x_1 are negative). Then

$$-\frac{eV_m}{kT_0} = \frac{e^2}{4kT_0(x_1-x_0)} \tag{24}$$

or

$$x_0 = x_1 + \frac{e}{4V_m}. \tag{25}$$

Here we are measuring V_i from the metal. If we measure it from the potential minimum, we should use

$$-\frac{eV_i}{kT_0} = \frac{e^2}{4kT_0(x-x_0)} + \frac{e^2}{4kT_0x_0} + \frac{e^2x}{4kT_0x_0^2}, \tag{26}$$

and the last term is an added constant field to give zero gradient at $x=0$. We may put this in the dimensionless form

$$-\frac{eV_i}{kT_0} = \frac{\gamma s^2}{(s_1+s_0)^2(s-s_1-s_0)} \tag{27}$$

where

$$s_0 = \frac{e}{4V_m}\sqrt{\frac{4\pi e^2}{kT_0}}\frac{I_0}{e}\sqrt{\frac{m}{kT_0}} \tag{28}$$

$$\gamma = \frac{e^2}{4kT_0}\sqrt{\frac{4\pi e^2}{kT_0}}\frac{I_0}{e}\sqrt{\frac{m}{kT_0}} \tag{29}$$

and s_0 and s_1 correspond to x_0 and x_1.

Thus we have the complete equation

$$y = 1 + \log_\epsilon y + \frac{\gamma s^2}{(s_1+s_0)^2(s-s_1-s_0)} + 2\int_0^s ds \int_0^s \frac{ds}{\sqrt{y}}. \tag{30}$$

With so many parameters, it is difficult to put a general solution in a useful form. For reasons which will become apparent, let us determine the maximum current at which the image force term is negligible. To do this, expand (30) in a Taylor's series in s around the point $x=0$. We have

$$y = 1 + s\sqrt{2\left(1-\frac{\gamma}{(s_1+s_0)^3}\right)} + \cdots. \tag{31}$$

Thus, if we define the constant β as

$$\beta = \frac{\gamma}{(s_1+s_0)^3} \tag{32}$$

then $\beta/2$ will be approximately the per-unit change in the coefficient of the first term of the series (31). Now if we assume β, remembering that s_1 must be negative

(and also β), we may find s_1 from (32). In doing this, we use saturation current for I_0 as a first approximation. Since s_1 is probably small (it is the distance of the cathode from the potential minimum), we may use the Taylor's expansion of (31) with $\gamma=0$ in (20), to get $(-eV_1/kT_0)$ from s_1. This is

$$-\frac{eV_1}{kT_0} = -s_1^2 + .236s_1^3 + \cdots \tag{33}$$

where, in effect, V_1 is the additional voltage, besides V_m, between the cathode and potential minimum caused by space-charge. From (14) we can see that the current under these circumstances is below the temperature-limited current by the factor exponent $(-eV_1/kT_0)$.

To put numerical values in the problem, assume $T_0=2500°$, $V_m=-4.55$, $A=120$ (amperes per square centimeter). Then we find that $s_1=0.183$ for $\beta=0.002$ (or a change in the series terms of about 0.001). This gives the current as 3.5 per cent less than saturation. If we choose $\beta=0.02$, then the current is only 0.7 per cent below saturation. In the latter case, we might expect a maximum change in anode potential of about 1 per cent due to the initial slope being changed. This is estimated by comparing the next to the last term of (30) with the full line of Fig. 2. The space charge will tend to reduce this to some extent, so that the actual anode voltage change will be something less than 1 per cent. From this we may conclude that the transition takes place, to all intents and purposes, within about 1 per cent change in current and voltage.

In actual tubes, the transition usually requires much more variation in current and voltage. This is not too surprising when we consider that the cathode surface in the problem was a plane on the scale of the depth of the image field (of the order of several times the distance between molecules). For actual surfaces, the roughness and variation in V_m will always introduce further rounding of the transition point. The above calculation must be regarded as approximate because we do not know the exact variation of the image field; and furthermore, it represents approximately the maximum degree of sharpness in the transition region which one could obtain with a perfectly flat and uniform cathode surface. This being so, there seems to be no need to calculate in more detail the exact shape given by (30).

Velocity Distribution

From the standpoint of the kinetic theory, information concerning the velocity distribution is all contained in the stress tensor and the thermal-conduction vector. In the first approximation, where viscosity and thermal-conduction effects are neglected, the velocity distribution is purely Maxwellian, being a function of the temperature only. If viscosity effects are present, they alter this condition by putting bumps or hollows on both sides symmetrically around the center. In the present instance we have heat conduction which produces an asymmetrical change in the distribution (bumps on one side and hollows on the other). Ordinarily, these distortions are small; but for an electron gas they may possibly become noticeable under some conditions. For the assumptions made here, the distortions approach zero; and, consequently, we have practically a pure Maxwell distribution, so far as these effects are concerned.

From a general standpoint, Burnett[6] has given a general orthogonal series for the velocity distribution. The coefficients for two terms are functions of the components of the stress tensor and thermal-conduction vector and apparently the only ones concerned with the average values used in mass flow. What the other coefficients mean has not been indicated, except that the series is general enough so that, for a given boundary condition, one could presumably specify an arbitrary velocity distribution somewhere and evaluate the coefficients from this. From this standpoint, it would seem that the actual velocity distribution would not have to be that given by Chapman's approximations, since they only represent certain terms in the over-all expansion. Thus one cannot conclude from this that a close approximation to the half-Maxwellian distribution assumed by Langmuir cannot exist in the gas.

Even though they be small, there are always some interactions; and one can reason that the gas will approach as closely to an equilibrium condition as the stresses due to its motion will allow. Since the gas from the cathode becomes Maxwellian long before it surmounts the potential barrier, it is reasonable to suppose that it has as nearly a complete Maxwellian distribution as the stresses will allow, using the usual approximations of the kinetic theory. This would argue very strongly against a discontinuous thing like a half-Maxwellian distribution.

Equivalent Mean Free Path

The use of collision integrals to describe encounters takes away any meaning from the term mean free path. The concept is, however, still useful in estimating possible errors from the use of any given approximation to the stress tensor, etc. For example, in the second approximation, one uses viscous forces derived from the derivatives of velocity with respect to the various co-ordinates. Now, if the velocity change is not reasonably linear in the length of a mean free path, then one might expect that it would be necessary to consider second derivatives, which requires Chapman's third approximation to the stress tensor. Ordinarily, such conditions are found in shock waves, capillary tubes, etc. The third approximation is so complicated that it is of little practical use; and if it is required, a radical change in the analytical approach is necessary. In the case of high-vacuum tubes,

[6] Burnett, "Mean motion in a nonuniform gas," *Proc. Math. Soc.* (London), vol. 40, p. 382; 1935.

it has long been considered that such is the case; and this condition led to the analysis in use at the present time.

The numerical value of the coefficient of viscosity, as given by Chapman, may be easily evaluated for electrons and comes out to be $3\times10^{-18}T^{5/2}$ (esu) where one uses 2500° and $n=10^{10}$ in evaluating the limits of integration. Now one can define an equivalent mean free path as that value which, in the usual free-path theory, would give the above coefficient of viscosity. This comes out to be

$$l = 1.07 \times 10^{4}\frac{T^{2}}{n}, \tag{34}$$

and at 2500° and $n=10^{10}$, $l=6.67$ centimeters. In connection with this formula, it is interesting to consult Langmuir's equation (229),[7] which gives the free path as the average distance traversed by an electron before suffering a deflection of θ degrees. Using this formula, where the average velocity of the electron relative to the swarm is taken equal to that corresponding to the prevailing temperature, one finds that the free path, given by (34), is that length in which the electron suffers an average deflection of 13.7°. Thus this value from Chapman is of a reasonable order of magnitude according to formulas given by Langmuir.

We mentioned previously that the discharge, beyond the potential minimum, ultimately must approach an adiabatic condition. Substituting this in (34), one sees that l will decrease as $T^{1/2}$, so that the equivalent mean free path ultimately decreases beyond the potential minimum. Physically, this can easily be seen to be a consequence of the fact that at lower temperatures the velocity of the electron relative to the swarm is lower, and it does not travel as far before it becomes deflected. This effect overcomes the natural lengthening of the path on account of the decrease of n. On the other hand, for isothermal conditions between the cathode and the potential minimum, l will decrease as we go back toward the cathode. In this case, one might say that somewhere near the potential minimum we have the largest value of l. A more accurate way of putting it would be to say that the maximum of l occurs at the changeover point between isothermal and adiabatic flow. Since we can evaluate n at the potential minimum, we can estimate l at this point, which should be not far from the largest value. But we do not know T. The assumptions made in this paper would make l very much larger than the tube dimensions. However, since the tube noise decreases very markedly as we go from the temperature-limited to space-charge-limited condition, one might expect the corresponding temperature at the potential minimum to drop as well. Since l varies as T^2, it is not out of the question that it may, under some conditions, become much smaller than tube dimensions. In addition, it must be considered that the above calculations are based on encounters between two electrons only; and multiple encounters will reduce l still more. Thus the effect of temperature and multiple encounters must be evaluated before this question of mean free path in a diode can actually be settled.

[7] Langmuir and Compton, "Electrical discharges in gases; Part II, Fundamental phenomena in electrical discharges," *Rev. Mod. Phys.*, vol. 3, p. 219; April, 1931.

Conclusions

The kinetic theory consists of a series of approximations, each presumably more accurate than the last, to the actual conditions in a moving gas. In the present study, we have seen that a relatively simple approximation, used with the concept of potential barrier at the surface of a metal, can succeed in describing, in one equation, the phenomena of temperature-limited emission, space-charge-limited conditions, and the transition between these. Furthermore, the theory may introduce, in a natural manner, the concept of a varying temperature which possibly may be used to explain the reduction of plate noise under space-charge-limited conditions, and some characteristics of magnetrons. There is good reason to believe that progress in any branch of the art depends on being able to make an analysis in terms of a series of simple concepts, each of which can be understood and accepted by itself. The author suggests that these two problems have resisted analysis for a long time, for a lack of an additional concept of this kind.

All of the above phenomena, of course, can be calculated from other points of view, still obtaining the correct answer. There is, however, a vast body of processes normally used in dealing with gases in practical cases, among which are the concepts of variable temperature, isothermal and adiabatic expansion, etc., which the kinetic theory attempts to explain. By putting the electron-gas on a basis of this kind, one opens up many avenues of analysis which would not normally be apparent. In other words, in dealing with interaction integrals and mass flow equations all at one time, one may not "see the woods for the trees."

The author wishes to acknowledge the many helpful discussions and encouragement of John Blewett, formerly of this Laboratory.

Precision Measurement of Electrical Characteristics of Quartz-Crystal Units*

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Summary—**Extensive manufacture and use of quartz-crystal units for radio communication, instrumentation, control, and detection purposes was responsible for new investigations to improve all phases of piezoelectric units and devices. A small part of the total work included development of circuit arrangements and instruments for accurately measuring the electrical constants of crystal units. This paper describes precision methods of high-frequency radio measurements as used successfully on quartz-crystal units and applicable to other types of units having *Q*'s and impedance ranges up to several million or higher.**

I. Introduction

AS A RESULT of the enormous demand for quartz-crystal units in the recent war, many new methods of production, measurement, and testing were studied and put to use. Precise measurements of many characteristics of crystal units were made, but data concerning the dynamic electric impedance were supplied largely in an arbitrary and indirect manner.[1,2]

The electrical characteristics of a crystal unit, i.e., the impedance, the equivalent reactance, and the equivalent resistance at any point of operation may be measured and specified entirely apart from any external circuit with which the crystal unit is used.[3–5] From these data the behavior of a crystal unit in any specific external circuit whose characteristics are known may be predicted. Although the great advantages of using this kind of data as the basis for a standardization of crystal units are described at length by others,[3,5] it may briefly be noted that this application would eliminate the multitude of comparison test sets and the associated problems of calibration and maintenance required by today's indirect methods of specification, and should lead to more exact methods of measurement and performance rating in terms of those parameters likely to be most useful to the radio and electronics design engineer.

This paper reviews means of expressing the equivalent electrical circuit of a quartz-crystal unit. It describes measurement methods and techniques in which a generator of continuously adjustable frequency of stability comparable to that of a crystal unit permits the use of such laboratory instruments as rf bridges and *Q* meters to determine the crystal unit's electrical parameters. It shows how the techniques may be used for exploring secondary responses in a crystal unit to determine its usefulness as an element in filter circuits and to investigate causes of instability in oscillator circuits. The data are presented in simple graphical form favoring differentiation between a "normal" crystal unit and one remaining constant under limited conditions. Finally, correlation is shown between data based on the more fundamental crystal-unit characteristics and data derived from the more familiar "activity" and "stability" of a particular oscillator.

This work was done in connection with development of precision laboratory methods of measuring high-*Q* components.

II. Equivalent Circuit Representing a Normal Mode of Vibration of a Quartz-Crystal Unit

Piezoelectric crystals were introduced into the radio-frequency field to meet the need for a resonant circuit (series or parallel) having constant parameters and high *Q*. A fundamental requirement was that the crystal unit be the electrical equivalent of a single resonant (or antiresonant) circuit. The unit that for practical purposes meets this requirement is called a normal one. Crystal units with interfering responses, or closely spaced multiple responses, are apparently equivalent to electrical circuit meshes with a number of mutually coupled resonant circuits. These are more complex units, and should be treated separately.

The equivalent circuit of a normal crystal unit is shown in Fig. 1(a). It has one resonant frequency f_r and one antiresonant frequency f_a for any particular value of shunting capacitance C_t. The impedance across its terminals R_p is illustrated by means of the susceptance with reactance diagrams of Fig. 1(c) and (d). R_s is small as compared with ωL, and can be neglected in these diagrams.

The susceptance B_{abc} of the L_1, C_1 series branch is added to the susceptance B_t of the shunt capacitance C_t. C_t is the total shunt capacitance across the crystal plate, and is usually designated as C_0 when the capacitance external to the crystal unit is zero. The resultant

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† Central Radio Propagation Laboratory, National Bureau of Standards, Washington 25, D. C.

[1] G. M. Thurston, "A crystal test set," *Bell Labs. Rec.*, vol. 22, pp. 477–480; 1944.

[2] W. E. McNatt, "Test set for quartz crystals," *Electronics*, vol. 18, pp. 113–115; April, 1945.

[3] Karl S. Van Dyke, "The standardization of quartz-crystal units, PROC. I.R.E., vol. 33, pp. 15–20; January, 1945.

[4] Geoffrey Builder, "A note on the determination of the equivalent electrical constants of a quartz-crystal resonator," *A.W.A. Tech. Rev.*, vol. 5, pp. 41–45; April, 1940.

[5] C. W. Harrison, "The measurement of the performance index of quartz plates," *Bell Sys. Tech. Jour.*, vol. 24, pp. 217–252; April, 1945.

B_e is obtained in Fig. 1(c) as the resultant termination reactance X_e in Fig. 1(d). It is evident from the diagram that, as C_t is increased in value, the slope of B_t will increase, and f_a will approach f_r. It is also apparent that X_e is positive between f_r and f_a. The equivalent circuit at frequencies in this range is, therefore, as shown in Fig. 1(b), where R_e is the resistive component including the effect of the series resistance R_s and all other losses of the unit.

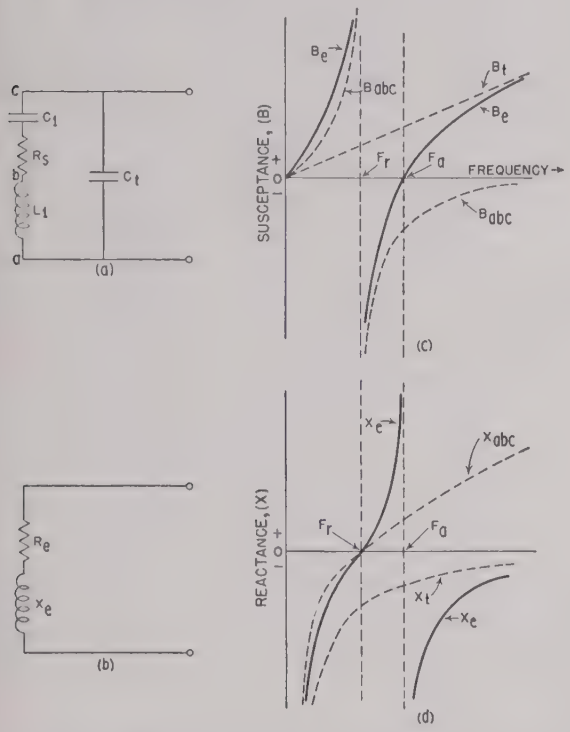


Fig. 1—(a) Equivalent circuit of a quartz-crystal unit. (b) Effective impedance of a crystal unit between f_r and f_a. (c) Susceptance diagram of a crystal unit. (d) Reactance diagram of a crystal unit.

When such a circuit is tuned to antiresonance with an external capacitor, the termination impedance is resistive, and is equal to

$$R_p = Q_e X_e = \frac{X_e^2}{R_e}. \tag{1}$$

The termination impedance of the equivalent circuit of Fig. 1(b) (often referred to as the Performance Index, or *PI*) may also be expressed in terms of the shunt capacitance and the series resistance, as follows:

At any antiresonant frequency f_a (where $f_a > f_r$), the termination impedance is, for small values of R_s, equal to that of a tapped antiresonant circuit, and is resistive; namely,

$$Z_p = R_p = Z_x \frac{C_1^2}{(C_1 + C_t)^2}$$

where

$$Z_x = Q_x \frac{1}{\omega_a C_e} = \frac{1}{R_s(\omega_a C_e)^2}.$$

Q_x is the Q of the antiresonant circuit, and

$$C_e = \frac{C_1 C_t}{C_1 + C_t}.$$

Now,

$$R_p \doteq Z_x \frac{C_e^2}{C_t^2} = \frac{1}{R_s(\omega_a C_t)^2} = \frac{X_t^2}{R_s}. \tag{2}$$

And, from (1) and (2),

$$\frac{X_t^2}{R_s} \doteq \frac{X_e^2}{R_e}. \tag{3}$$

R_p may be plotted versus C_t as an independent parameter on log-log co-ordinate paper in the form of a straight line with a negative 2-to-1 slope. This follows from (2), since

$$\log R_p = \log\left[\frac{1}{\omega^2 R_s C_t^2}\right] = \log\left[\frac{1}{\omega^2 R_s}\right] - 2 \log C_t.$$

R_s is assumed constant and the percentage change in ω with C_t negligible. Experimental evidence shown later supports these assumptions. Hence,

$$\log R_p = K_1 - 2 \log C_t. \tag{4}$$

Similarly, the antiresonant frequency increments $(f_a - f_r)$ may be plotted on log-log paper as a straight-line function of C_t with a negative 1-to-1 slope. This may be shown as follows (see Fig. 1(a)):

$$\omega_r^2 = \frac{1}{L_1 C_1}. \tag{5}$$

The antiresonance frequency is determined from the interrelation:

$$\omega_a^2 = \frac{1}{L_1 \dfrac{C_1 C_t}{C_1 + C_t}} = \frac{C_1 + C_t}{L_1 C_1 C_t}.$$

From this and (5),

$$\frac{\omega_a^2}{\omega_r^2} = \frac{f_a^2}{f_r^2} = \frac{(C_1 + C_t)C_1}{C_1 C_t} = 1 + \frac{C_1}{C_t}$$

or

$$\frac{C_1}{C_t} = \frac{f_a^2}{f_r^2} - 1 = \frac{(f_a + f_r)(f_a - f_r)}{f_r^2}.$$

For practical purposes, $f_a + f_r = 2f_r$.
Hence,

$$(f_a - f_r) \doteq \frac{1}{2} \frac{f_r C_1}{C_t} \tag{6}$$

or

$$\log (f_a - f_r) \doteq \log (\tfrac{1}{2} f_r C_1) - \log C_t.$$

Since f_r and C_1 are constant,

$$\log (f_a - f_r) \doteq K_2 - \log C_t. \qquad (7)$$

These straight-line characteristics ((4), (7)) of normal crystal units are demonstrated in Figs. 2, 3, and 4. Slopes of 2-to-1 and 1-to-1 were drawn for the straight-line sections in the figures. The position of each measured point shows how closely the theoretical curves are followed. Curvatures at high R_p values were caused by holder losses, as evidenced by the same characteristics measured, and plotted in Fig. 5, with the crystal removed from the holder.

It might seem at this time as if a single point on each of these R_p and $(f_a - f_r)$ versus C_t curves, through which the engineer could draw the appropriate 2-to-1 and 1-to-1 lines, would convey sufficient data for design purposes. And this could almost be done if the crystal unit in question were known to be normal, and if the

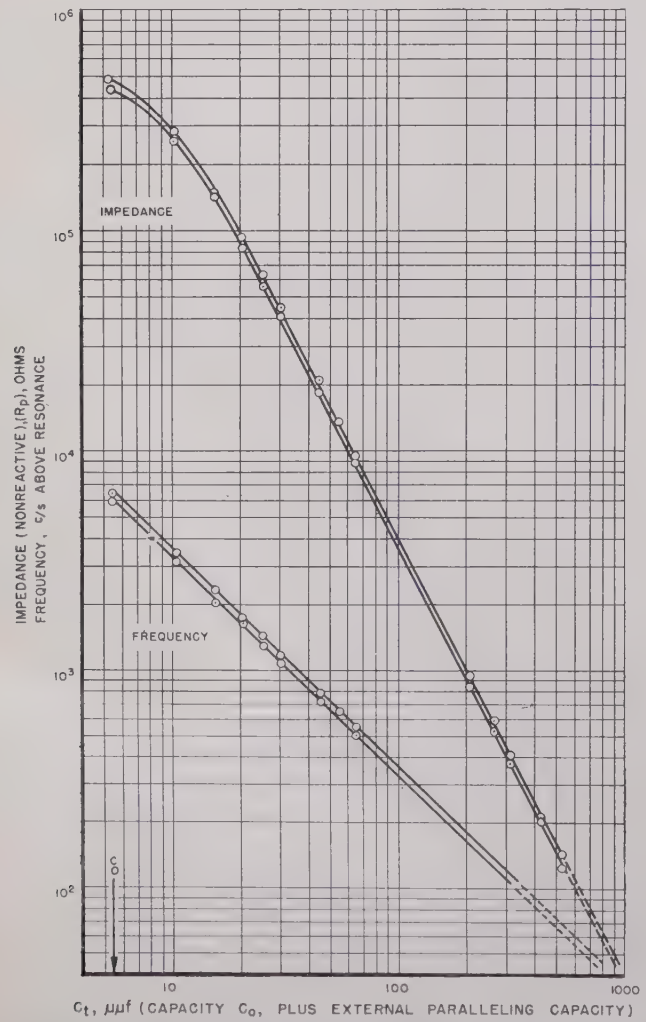


Fig. 2—Electrical characteristics of two 8.7-Mc wire-mounted crystal units with metal-film electrodes. These exhibit the 2-to-1 and 1-to-1 slopes derived from theory.

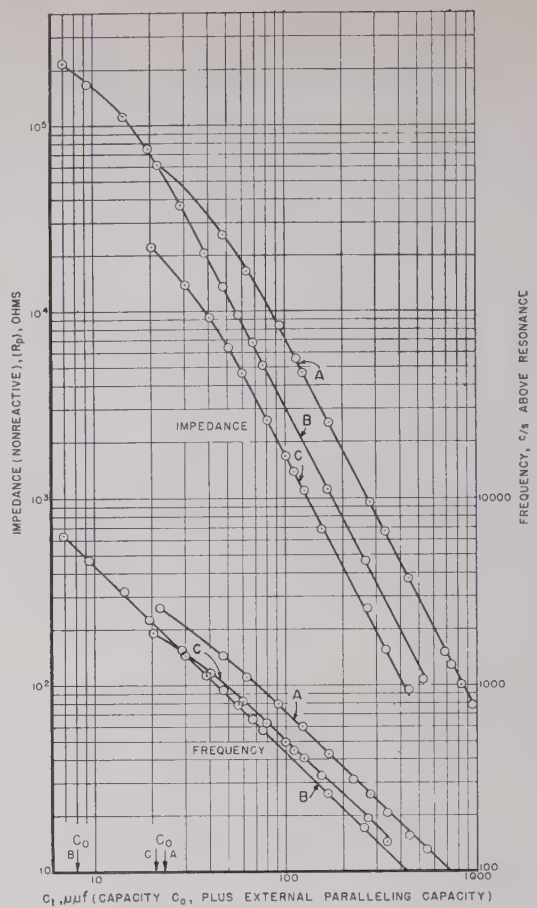


Fig. 3—Electrical characteristics of "normal" crystal units of different physical construction. (a) CR-1 type; air-gap, pressure-mounted. (b) FT-243 type; metal-film, spring-clip mounting. (c) FT-243 type; air-gap, pressure-mounted.

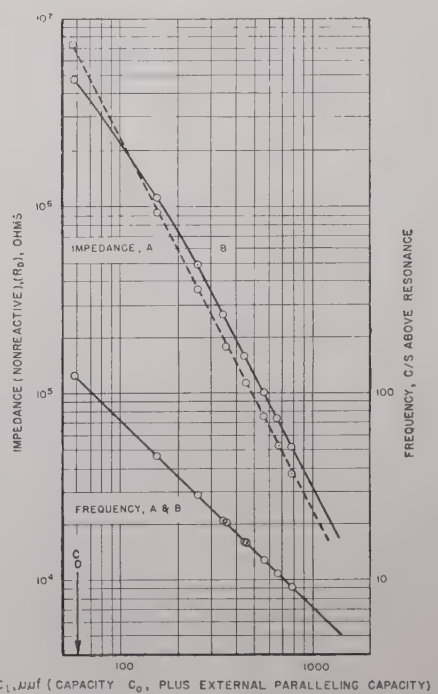


Fig. 4—Electrical characteristics of two different 100-kc quartz crystal units; *GT*-cut, metal-film electrodes, wire-mounted crystal. Unit A had a low-loss holder.

engineer were supplied with such additional data as the values of C_0 and either f_r or f_a at C_0. Still lacking, however, would be the effect of holder losses, and the frequency spectrum of the crystal unit showing the magnitude and disposition of secondary responses and data on parameter variation with temperature.

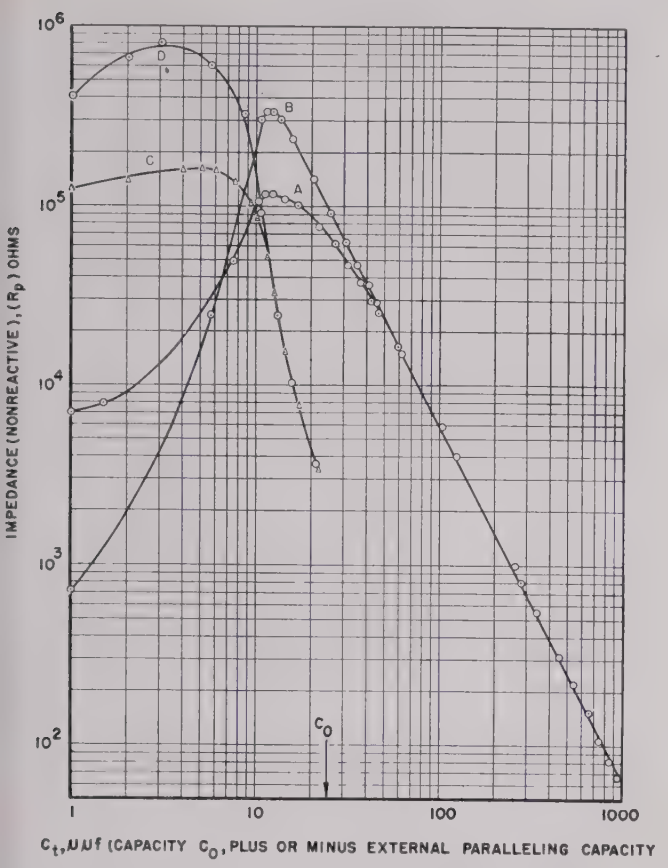


Fig. 5—Impedance measurements of the fundamental and adjacent responses on both sides of antiresonance of an air-gap, pressure-mounted crystal unit. The effect of holder losses on the high R_p values of a crystal unit is shown. Fundamental response: (a) Lossy holder. (b) Low-loss holder. Secondary response: (c) Lossy holder. (d) Low-loss holder.

III. Measurement Methods

Those parameters investigated in these measurements, which have not yet been defined and are not presented in Fig. 1, are:

ΔC_t = an increment of C_t

$\Delta f = f_a - f_r$

R_h = resistance shunting the quartz crystal (equivalent to holder losses)

$Q_e = X_e/R_e$ = the effective Q of the crystal unit

$Q_x = (2\pi f L_1/R_s) = (1/2\pi f C_1 R_s)$ by definition, where f is the nominal frequency of the crystal unit. is usually referred to as the Q of the crystal.

Some useful expressions of parameters in terms of directly measurable quantities are derived as follows:

$$C_1 \doteq \frac{2\Delta f C_t}{f_r} = \text{equation (6) rearranged.} \tag{8}$$

$$Q_x = \frac{1}{4\pi\Delta f C_t R_s} \text{ from definition of } Q_x \text{ and (8); noting that,} \tag{9}$$

since $\Delta f \ll f_r$ any frequency between f_a and f_r can be used.

Measurements of f_a (to determine Δf) are usually made at $C_t = C_0$. Then,

$$Q_x = \frac{1}{4\pi\Delta f C_0 R_s} \tag{10}$$

$$L_1 = \frac{1}{(2\pi f_r)^2 C_1}. \tag{11}$$

The effect of holder losses reduce R_p to

$$R_p' = \frac{R_h R_p}{R_h + R_p}. \tag{12}$$

A. Q-Meter Measurements of Quartz-Crystal Unit Characteristics

The well-known Q-meter principle makes use of an antiresonant circuit arrangement as shown in the table (Fig. 6). For our purposes, constant rf voltage was applied directly to the 0.04-ohm resistance element in the Q-meter tank with the Q meter's internal oscillator disconnected by leaving the band switch in an intermediate position.

L_Q, C_Q (Fig. 6) is tuned for antiresonance at the frequency at which X_e, R_e are to be measured. The crystal unit is connected in parallel with C_Q. Now the latter must be readjusted by ΔC_Q to restore the antiresonance at the Q-meter tank, and

$$X_e = \frac{1}{\omega\Delta C_Q}. \tag{13}$$

The voltage across the antiresonance tank is a measure of the Q of the circuit. Connecting the crystal unit in parallel with the tank reduces the original Q as a result of the shunt effect of R_e. From the two values of Q,

$$R_e = \frac{1.59 \times 10^8 \left(\frac{C_{Q1}}{C_{Q2}} Q_1 - Q_2\right)}{f C_{Q1} Q_1 Q_2} \tag{14}$$

where C_{Q1} is the capacitance, in micromicrofarads, required to tune the Q-meter tank without the crystal unit, but at the crystal frequency; C_{Q2} is the capacitance, in micromicrofarads, required to tune the Q-meter tank with the crystal unit in parallel with it; Q_1 is the original Q of the tank; Q_2 is the reduced Q of the tank; and f is the nominal frequency of the crystal unit in kc.

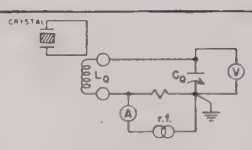
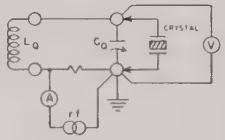
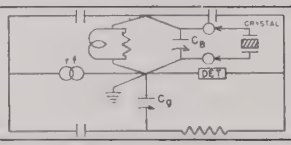
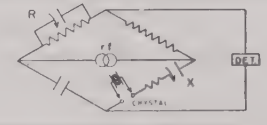
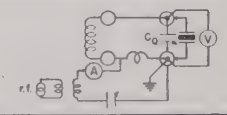
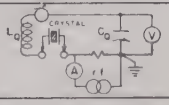
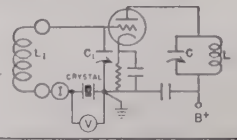
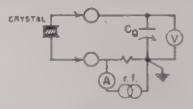
PARAMETER	RANGE	INSTRUMENT	CIRCUIT ARRANGEMENT (FUNDAMENTAL SCHEMATIC)	PROCEDURE	PRECAUTIONS	ACCURACY, CONSERVATIVE ESTIMATE
STATIC CAPACITY C_0, AND PARALLELING CAPACITY ΔC_t	0 TO 30 $\mu\mu f$	"Q" METER	SEE f_s	CALIBRATE C_0 VS. DIAL SETTING. $C_0 = C_{0_1} - C_{0_2}$	MEASURE C_0 AT r.f. FOR WHICH CRYSTAL IS INACTIVE. POSITION AND SHIELDING OF CRYSTAL UNIT IS IMPORTANT	± 0.1 $\mu\mu f$
	30 TO 150 $\mu\mu f$					± 0.2 $\mu\mu f$
	150 TO 2000 $\mu\mu f$					± 5 $\mu\mu f$
	100 TO 1100 $\mu\mu f$	TWIN-T IMPEDANCE-MEASURING CIRCUIT	SEE R_p	CALIBRATE CAPACITANCE DIAL		± 2 $\mu\mu f$
SERIES-RESONANCE FREQUENCY, f_r	"Q" METER	"Q" METER		ADJUST L_0C_0 FOR MAXIMUM V. ADJUST r.f. FOR MINIMUM V.	USE SHORTED TERMINALS OR SMALLEST LOOP FOR INDUCTIVE COUPLING TO CRYSTAL UNIT	± 1.5 IN 10^6
	R-F BRIDGE	R-F BRIDGE	SEE R_s AND R_s (HIGH LEVEL)	ADJUST r. f., AND RESISTANCE ARM OF BRIDGE FOR BALANCE	SHIELD WELL. NO F.M. IN OUTPUT OF r-f GENERATOR	± 1.5 IN 10^3
ANTIRESONANCE FREQUENCY, f_a	"Q" METER	"Q" METER		TUNE "Q" METER TO f_r. PLACE CRYSTAL UNIT IN PARALLEL WITH C_0. INCREASE r.f. FOR MAX V. REMOVE CRYSTAL UNIT AND RETUNE "Q" METER. REPLACE CRYSTAL UNIT AND READJUST r.f. FOR MAX V.	FOR MAXIMUM PRECISION ADJUST C_0 FOR MAX. V. USE LARGE L/C RATIO IN "Q" METER	± 2.5 IN 10^6
	TWIN-T CIRCUIT	TWIN-T CIRCUIT	SEE R_p (LOW LEVEL)	ADJUST r. f., AND C_0 ARM OF BRIDGE FOR BALANCE	SHIELD WELL	± 2.5 IN 10^6
EQUIVALENT SERIES RESISTANCE R_e, AND REACTANCE, X_e	0 TO 1000 OHMS	R-F BRIDGE	SEE R_s	ROUTINE	SHIELD WELL. USE LOW r-f INPUT TO BRIDGE	± 0.2 OHMS ± 2%
	0 TO 500 OHMS (AT 10 Mc.)					± 1 OHM ± 1%
ANTIRESONANCE IMPEDANCE, R_p (LOW LEVEL)	5000 TO 10,000 OHMS	"Q" METER	SEE f_a	SUBSTITUTION METHOD ADJUST r.f. FOR MAXIMUM V. AT EACH ΔC_t	FOR MAXIMUM PRECISION ADJUST C_0 FOR MAX. V.	± 5% OR BETTER
	10,000 TO 100,000 OHMS					± 2%
	100,000 TO 5,000,000 OHMS					± 5% OR BETTER
	1000 TO 18,000 OHMS	TWIN-T CIRCUIT		ROUTINE	SHIELD WELL. USE LOW r-f INPUT TO BRIDGE	± 2.5 %
	0 TO 1000 OHMS	R-F BRIDGE		ADJUST r.f., AND RESISTANCE ARM OF BRIDGE FOR BALANCE WITH CALIBRATED SILVER-MICA CONDENSERS IN PARALLEL WITH CRYSTAL UNIT	SHIELD WELL. USE LOW r-f INPUT TO BRIDGE. NO F.M. IN OUTPUT OF r-f GENERATOR	± 0.1 OHMS ± 1.5 %
R_p (HIGH LEVEL)	1000 TO 100,000 OHMS	SPECIAL HIGH LEVEL "Q" METER		SUBSTITUTION METHOD. USE 1 TO 5 WATT r-f INPUT	PERMIT CRYSTAL UNIT TO REACH TEMPERATURE EQUILIBRIUM	± 5 %
SERIES-RESONANCE RESISTANCE, R_s	0 TO 1000 OHMS	R-F BRIDGE	SEE R_p (LOW LEVEL)	SEE f_r	SEE f_r	± 0.1 OHM ± 1 %
	1000 TO 10,000 OHMS	"Q" METER	SEE f_a	SUBSTITUTION METHOD	USE GOOD L_0 COILS AND LOW L_0/C_0 RATIOS	± 5%
	0 TO 10 OHMS			SUBSTITUTION METHOD		± 2.5 %
R_s AND f_r (HIGH LEVEL)	0 TO 100 OHMS AND ABOVE	SPECIAL CIRCUIT ARRANGEMENT		ADJUST L,C FOR MAXIMUM I, AND E_p FOR DESIRED I VALUE. ADJUST C_1 FOR MINIMUM V. $R_s = \frac{V}{I}$	CALIBRATE I AND V. SEE R_p (HIGH LEVEL)	NOT FULLY INVESTIGATED
R_p	5,000 TO 5,000,000 OHMS	"Q" METER WITH LOW C_0		ADJUST r.f. FOR MAXIMUM V. AT A VALUE HIGHER THAN f_r. $R = \frac{Q}{\omega C_0}$	V IS MAXIMUM WHEN $C_0 = C_0$ AND FREQUENCY = $f_r + \frac{\Delta f}{2}$	NOT FULLY INVESTIGATED

Fig. 6—Tabulation of measurement methods and the accuracies attained.

The values of R_p and R_s may also be derived directly in terms of the Q's observed:

$$R_p = \frac{1.59 \times 10^8 Q_1 Q_2}{f C_{Q1}(Q_1 - Q_2)} \tag{15}$$

$$R_s = R_p' \left[\frac{(Q_1 - Q_2)C_{Q1}}{Q_1 Q_2 (C_{Q_2} - C_{Q1})}\right]^2 \tag{16}$$

where the symbols are as stated above.[6]

R_p measurements from values of 5 million down to several thousand ohms were readily made with the Q meter using the above formulas. It was found, however, that a direct substitution of calibrated rf resistors in place of the crystal unit (across the Q-meter tank) eliminated time-consuming computations, with no sacrifice in accuracy.

Each of a number of coils was resonated at the desired frequency and its Q_2 values (see (14)) plotted on semilog paper against rf resistors (on the logarithmic co-ordinate) causing the drop in Q. Measurements of R_p at any frequency within a given crystal-unit response were made. At the fixed frequency, the Q-meter tank (with crystal unit in shunt) was resonated by tuning the capacitor C_Q to a maximum Q_2 indication. Measured Q_2 values were converted directly into corresponding R_p values from the resistance (R_p) versus Q_2 calibration curves. The over-all accuracy (conservatively estimated and tabulated in Fig. 6) is limited by the accuracy to which the rf values of the resistors used for calibration are known, and by the reproducibility of Q-meter readings. The inductance and the Q of a specific tuning coil determine the resistance range for greatest accuracy.

To measure the series-resonance frequency, an unshielded coil was resonated in the Q meter at a frequency near that of the crystal unit. The crystal unit was short-circuited and placed near the low-voltage end of the coil. The Q-meter frequency was varied until a sharp dip indicated the f_r of the crystal unit.

Measurements of f_a, when $C_t = C_0$, were made with the crystal unit in parallel with the Q-meter coil. The coil was first resonated at the nominal frequency of the unit. After the insertion of the crystal, the frequency was varied until the Q indicator rose to a maximum. With the crystal unit again removed, the coil was resonated at constant frequency by adjusting C_Q. Again the crystal unit was placed in the circuit and the frequency adjusted to a maximum Q indication. These adjustments were repeated until a constant frequency value (f_a) was obtained. At this point $C_t = C_0$. For highest precision a final adjustment of C_Q to a maximum Q indication was made at constant frequency. It will be recognized that the Q indication (Q_2) yielded the impedance R_p of the crystal unit for this operating point, as previously described.

[6] At high R_p values, R_p' (equation 12) must be used to correct for holder losses.

To measure f_a, when $C_t \neq C_0$, the Q-meter capacitance was changed from the $C_t = C_0$ setting by the ΔC required to bring the operating point to the desired frequency within the crystal response, and the frequency was varied to the new f_a indicated by the peaking of the Q indicator. Again the Q_2 value yielded the R_p of the crystal unit for this new operating point between f_a and f_r.

The Q meter was used for measurements of R_s directly. The crystal units were connected in series with the tank coil for low R_s values (up to 10 ohms), and in parallel with the tank for values higher than 1000 ohms. An rf impedance bridge was found to be more suitable for measurements of R_s from 1 to 1000 ohms.

Values of R_p at crystal-unit voltages approaching those in a Miller- (or Pierce-) type oscillator were measured with a Q-meter circuit specifically constructed for the purpose (Fig. 6; see R_p high level). Stable frequency voltage was injected into the LC tank by link-coupling the output of an auxiliary amplifier into a small section of the tank inductance. Sensitive thermoelements and potentiometers were used for current and Q indicators. Care was taken in the measurements to avoid frequency-response shift as the temperature of the crystal varied with the current through it. The technique was modified to permit presetting the voltage levels desired. Since there was no apparent difference in R_p values at high and low voltages (Fig. 7), there seems to be little need for high-voltage measurements, and the procedure is not elaborated upon.

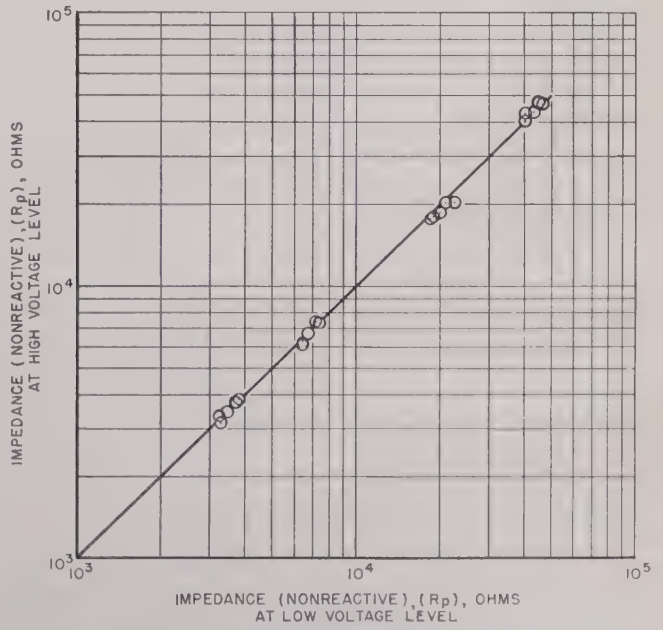


Fig. 7—Correlation between R_p values measured at high and at low rf voltages across the crystal units. Five units were measured with four different values of shunting capacitance.

It should be observed that, when a crystal unit is connected in parallel with an antiresonant circuit tuned

approximately to the crystal frequency, there are present two antiresonance peaks of the circuit impedances.[4] The lower peak is not influenced by the frequency response of the crystal unit, and appears at a separation many times the width of $f_a - f_r$. Hence, it does not affect the measurements.

The Q meter was singularly useful in locating and measuring secondary responses. The Q-meter tank was tuned to a frequency just below the nominal frequency of the crystal unit. The frequency was then increased continuously, with the crystal unit in parallel with the tank. Voltage dips were observed at f_r points, and peaks at f_a points in the frequency spectrum. Individual responses were closely explored by returning the tank (crystal unit removed) and repeating the sweep of frequency (crystal unit replaced) to determine more exactly the values of f_r and f_a. The process of returning the tank and of readjusting the frequency was repeated to a final unchanging frequency.

Tank coils of high inductance were effective in locating f_a points, while low-inductance tank coils were more effective in locating f_r's. For most precise measurements, different coils were used as conditions warranted.

It was found enlightening to plot the relative position of the secondary responses on semilog paper as shown in Fig. 8. For practical purposes, it might be sufficient to interconnect the R_s and R_p points of each response

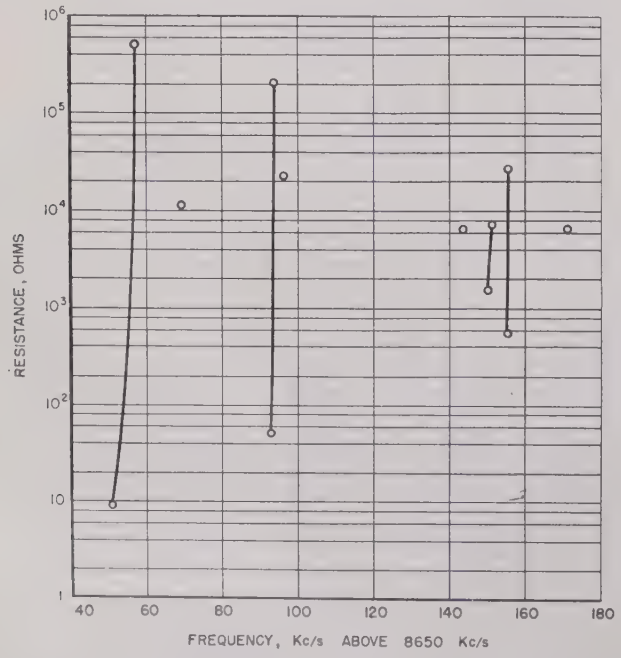


Fig. 8—Frequency-response spectrum of a metal-film-electrode, wire-mounted crystal unit. The curves connect measured R_p and R_s values for each response. The single points represent R_p values of responses whose R_s values are uncertain.

with a straight line, for the curvature involved is negligible relative to the frequency scale employed. This approximation may be justified analytically when considering the variation of R_e within a single response, between f_r and f_a.

Designating the total series reactance of the equivalent circuit of Fig. 1(a) as

$$X_1 = \omega L_1 - \frac{1}{\omega C_1}, \quad \text{and} \quad X_0 = \frac{1}{\omega C_0},$$

the termination impedance of the unit is

$$Z = X_0 \frac{X_0 R_s + j[R_s^2 + X_1(X_1 + X_0)]}{R_s^2 + (X_1 + X_0)^2}.$$

Then

$$|Z| = X_0 \frac{\{X_0^2 R_s^2 + [R_s^2 + X_1(X_1 + X_0)]^2\}^{1/2}}{R_s^2 + (X_1 + X_0)^2}.$$

The equivalent series values of R_e and X_e are then

$$R_e = \frac{X_0^2 R_s}{R_s^2 + (X_1 + X_0)^2}, \tag{17}$$

and

$$X_e = \frac{X_0[R_s^2 + X_1(X_1 + X_0)]}{R_s^2 + (X_1 + X_0)^2}. \tag{18}$$

For practical purposes, at f_a the reactive component of Z vanishes, and $X_1 + X_0 = 0$; therefore,

$$Z_a \doteq R_p \doteq \frac{X_0^2 R_s}{R_s^2} = \frac{X_0^2}{R_s} \doteq R_e' \tag{19}$$

where R_e' is the value of R_e at f_a.

At f_r, X_1 is zero, and

$$|Z| = X_0 \frac{[X_0^2 R_s^2 + R_s^4]^{1/2}}{R_s^2 + X_0^2} = \frac{X_0 R_s}{(X_0^2 + R_s^2)^{1/2}}.$$

For values $X_0 \gg R_s$, $Z \doteq R_s$. Similarly, at f_r, X_1 is zero, and, from (17),

$$R_e = \frac{X_0^2 R_s}{R_s^2 + X_0^2} \doteq R_s. \tag{20}$$

It is apparent, therefore, that for responses having series-resistance values considerably lower than X_0, plotting R_e versus frequency (Fig. 8) would closely represent a quantitative variation of the parameter of the crystal unit within the limits of R_s and R_p. For responses with high series-resistance values, this presentation of a response is correct at the R_p and R_s points only. In the curves shown, the R_p and R_s values representing the top and bottom points of each response are the measured values of these parameters; the intermediate points have qualitative significance only.

The Q meter was used to locate the responses. The R_p and R_s values were then measured with Q meter, twin tee, or rf bridges, depending on the magnitude of the parameter. Some of the relatively small secondary responses (R_s values above 1000 ohms) behaved in a

manner that made the measurement of R_p uncertain. Further study of this behavior will be required, and these responses are here represented by a single point of measured R_s. Other responses at, and below, the threshold of detectability with the Q meter have been neglected as insignificant in this study.

B. Twin-Tee Circuit and Impedance Bridge

A twin-tee admittance-measuring circuit was used for R_p values of 1000 to 10,000 ohms. Resistance values computed from the admittances obtained were checked often against measurements of rf resistors maintained as standards. Increments of C_t applied across the crystal units (either by varying the internal capacitor of the instrument or by connecting external capacitors across them) were known to high accuracy at the operating frequencies.

It is possible to measure X_s and R_s of a crystal unit directly with the twin tee. However, the relatively limited range of measurable X_s at corresponding R_s values renders the instrument useful only for checking a few points obtained with the Q meter. Its use, therefore, yields little additional information.

An rf impedance bridge was used to measure R_s, R_s and X_s values from 1000 down to a few ohms, and R_s values down to 100 ohms. In fact, R_p values as low as 10 ohms were actually measured with 8.7-Mc crystal units. However, the validity of the latter was uncertain.

In measuring R_s and R_p with the bridge, the frequency and resistance arms were varied until a minimum deflection on the null balance indicator was obtained. R_s was measured more rapidly by first determining f_r with the Q meter, as described above. R_p values when $C_t \neq C_0$ were measured with external capacitors connected in shunt with the crystal-unit terminals. R_s and X_s values were obtained at any frequency by adjusting the resistance and reactance arms for a null balance.

The major requirement in the application of the twin tee and rf bridge was a source of stable frequency, adjustable in small increments. Good wave form without frequency modulation was found essential for a sharp null balance.

The equipment shown in the block diagram of Fig. 9 was used for 8.7-Mc crystal-unit measurements. A standard frequency of 100 kc was injected into a frequency multiplier, the products of which were mixed with the output of a continuously adjustable stable-frequency oscillator. The output of the mixer was then amplified to the desired level.

Frequency could be adjusted, measured, and maintained to better than 5 parts in 10^7 over a period of time required to complete any one measurement. Regular frequency-measuring equipment incorporating a radio receiver as a detector, a stable audio-frequency oscillator to determine the beat frequency between the adjustable oscillator and a harmonic of the standard source, and an oscillograph were used throughout the measurements. A second radio receiver was used as a null indicator for the twin tee and rf bridge.

Special circuits were developed and used to measure independently and directly the values of the series-resonant resistance R_s and series-resonant frequency f_r. Fig. 6 lists the methods, accuracies, and range of parameters measured.

IV. A Complex Crystal Unit

A great difficulty encountered in this investigation was that of locating and segregating a group of quartz-crystal units whose characteristics could be measured, and the results reproduced to an accuracy near that of the measuring equipment. Stability tests of R_s, of R_p, and of "activity" were made over a period of weeks, and only the most highly stable units were used for measurement.

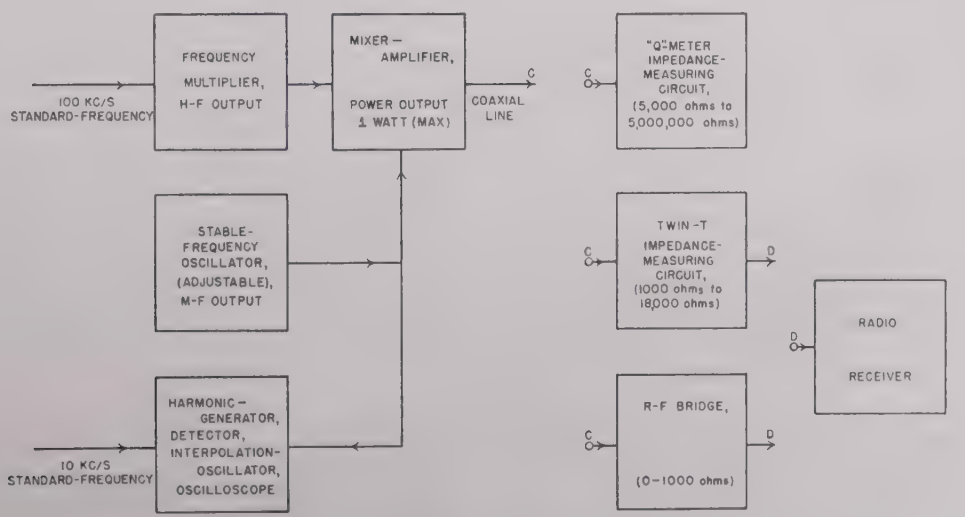


Fig. 9—Block diagram of the apparatus used.

Among those in the select group, there were a few relatively stable with respect to R_s and to "activity," but with R_p (and Δf) versus C_t characteristics which deviated considerably in slope from the expected 2-to-1 and 1-to-1 ratios. A typical example is shown in Fig. 10.

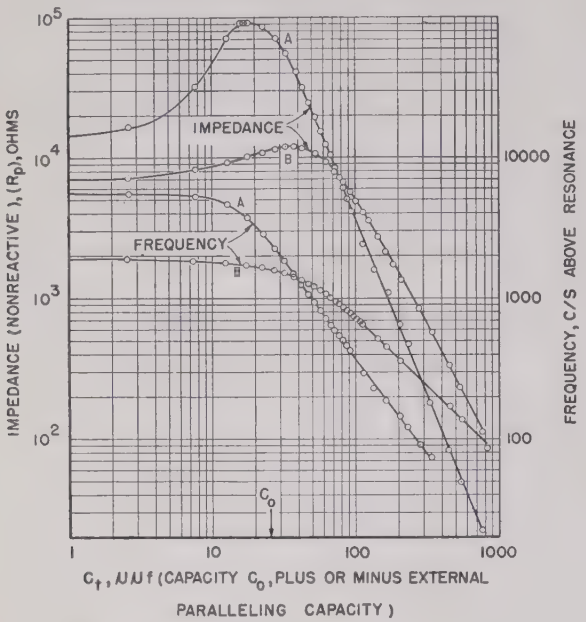


Fig. 10—Electrical characteristics of two interfering responses of a "complex" crystal unit. (a) f_r: 8697.87 kc. (b) f_r: 8700.40 kc.

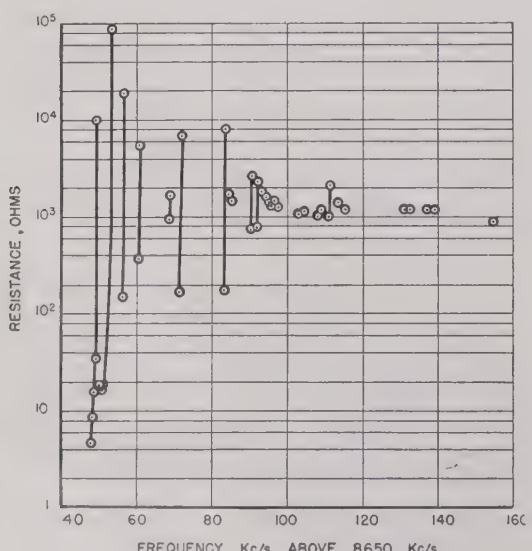


Fig. 11—Frequency-response spectrum of a "complex" crystal unit. The large number of responses should be compared with the relatively few in Fig. 8. The position and magnitude of the antiresonant impedances of the first two responses are indicative of the behavior shown in Fig. 10, and described in the text.

At C_t values between C_0 and 75 $\mu\mu$f, the first response (A) has higher R_p's than the second; and the crystal should operate at the frequency of the former. Above 75 $\mu\mu$f, the R_p values of the second response are higher; and here the crystal should operate on the second response. This frequency jump from one response to the next was observed in a Miller-type oscillator at the stated C_t value.

It is probable that the presence of many an interfering response will not show up in test oscillators in use at present. This particular one was detected in other measurements: abnormally high R_s, and an unusually wide Δf. The frequency spectrum of this particular crystal unit is shown in Fig. 11. It demonstrates the proximity and the relative values of R_p, R_s, and Δf of all responses; and this frequency spectrum should be compared with the spectrum of the normal crystal unit as shown in Fig. 8.

The role of the second response as an interfering one was again indicated by a decrease in R_e as the frequency was increased from f_r. From the minimum, reached several hundred cycles above series resonance, R_e then increased in a more normal manner. This is not intended to indicate a specific manner of behavior of a complex crystal-unit response, but to point out another practical means of exploring such behavior. Further study of this type of crystal should be undertaken as a separate problem, employing methods of measurement described here.

V. Correlation Between Measured Characteristics and Oscillator Performance of Crystal Units

It has been shown in a number of publications[5,7,8] that the activity of a crystal in an oscillator is a function of the equivalent dynamic resistance R_p (PI) of the crystal unit. The higher the Q of a crystal unit, the greater its stability and effectiveness in electric circuits. The relative merits of crystal units may be stated, then, in terms of these parameters.

To verify this, activity, (I_g), and R_p measurements were performed on a number of crystal units of varying degrees of stability. Fig. 12 shows thirteen such units with three values of C_t and their corresponding R_p's measured at voltages approximating those across the crystal unit in a Miller-type oscillator. R_p was measured immediately after an I_g measurement to prevent any change in characteristics as a result of handling. To further minimize differences, the activity was later reduced by lowering the oscillator plate voltage, while the R_p measurements were made at the low-voltage levels of the Q meter. These results agree with those in Fig. 12. There is a good correlation between high- and low-activity crystals and their respective R_p values. Moreover, there is good correlation between variation

[7] I. E. Fair, "Piezoelectric crystals in oscillator circuits," *Bell Sys. Tech. Jour.*, vol. 24, pp. 161–216; April, 1945.

[8] M. Boella, "Performance of piezo-oscillators and the influence of the decrement of quartz on the frequency oscillations," Proc. I.R.E., vol. 19, pp. 1252–1274; July, 1931.

in activity of individual crystals and corresponding variation in their R_p values.

Still another correlation exists between the R_s of a crystal unit and its activity in a Miller-type oscillator at various temperatures. Variations in R_s were measured with the crystal unit under test placed in a temperature-controlled cabinet and connected to the impedance bridge through a half-wavelength coaxial cable. Corrections were made for losses in the cable. No difficulties were encountered in the application of the coaxial cable to these measurements.

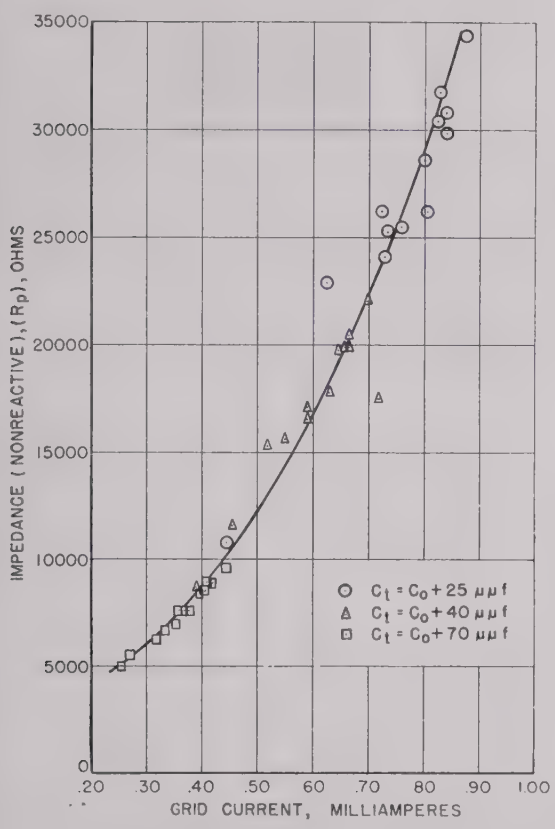


Fig. 12—Correlation between R_p and activity (grid current I_g) of thirteen pressure-mounted crystal units.

Since I_x activity is a function of R_p, it is also a function of R_s and of C_t (2). Observations were made, therefore, on the effect of temperature on C_0. As expected, no detectable variations were found. It may be assumed, then, that the activity variations were caused entirely by changes in R_s.

VI. Additional Observations in Crystal-Unit Measurements

Temperature effects on the relative position of crystal-unit responses and their R_p, R_s, and $f_a - f_r$ values were measured by the methods described. This type of measurement may be of value in the study of activity dips.

Another graphical presentation of the effect of temperature on the quartz-crystal unit is shown in Fig. 13. In curve A, a relatively high crystal current resulted in a resonance-frequency turning point at an ambient temperature of 3°C. Actually, the high crystal current brought the enclosed crystal plate to a temperature of about 44°C (the nominal turning point), even though the ambient indicating thermometer read 3°C. Curve B shows the resultant shift in turning point to an ambient of 15°C as the crystal assembly was removed from its enclosure and exposed to free ventilation. In curve C,

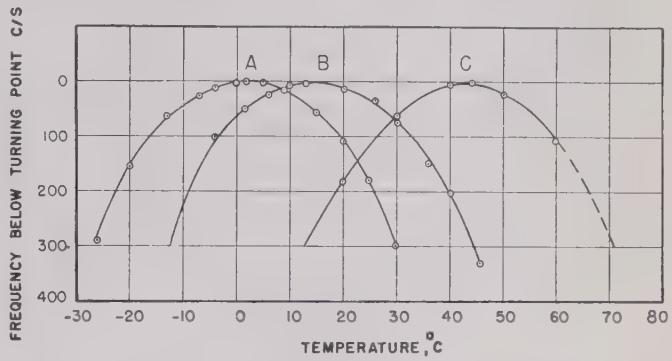


Fig. 13—Effect of operating conditions on the temperature and frequency turning point at resonance of an 8.7-Mc crystal; *BT*-cut, metal-film electrodes, wire-mounted. (A) Crystal in plastic holder, measured with relatively high crystal current. (B) Conditions same as (A), but crystal removed from holder. (C) Crystal in holder again, measured at negligible crystal current.

the crystal current was made negligible, and a full 44°C ambient temperature was necessary to bring the crystal plate to its frequency turning point.

Thus any predetermination of frequency turning point with temperature by a manufacturer or a design engineer must take into consideration at least the approximate amplitude of oscillation expected of the crystal unit and the mechanical design of the crystal-plate holder and electrodes, insofar as heat dissipation is concerned. This is in keeping with the observation that the graphical curves suggested for use in design work should be accomplished by temperature data for a more complete expression of the characteristics of a quartz-crystal unit.

VII. Conclusion

It was shown that comprehensive data on electrical characteristics of quartz-crystal units may be obtained with rf bridges and Q meters and a source of constant cw frequency. Electrical characteristics of fundamental or of secondary responses may be measured equally well by the methods described.

Graphical presentation of measured parameters as straight-line functions on log-log co-ordinate paper was suggested as particularly suitable for design purposes. Examples of crystal behavior were given to demonstrate this presentation.

Contributors to the Proceedings of the I.R.E.

Howard L. Andrews was born on October 27, 1906, at Kingstown, R. I. He received the B.S. and M.S. degrees in physics from Brown University in 1927 and 1928, respectively, and the Ph.D. degree in physics in 1931 from the same institution. From 1931–1934 Dr. Andrews was an instructor in physics at Brown University, and during 1934–1937 he was associated with the Rockefeller Foundation as research associate in psychology. In 1937 he joined the staff of the United States Public Health Service, and was commissioned an officer in 1945. In this capacity, he has done work in high-frequency oscillation and the biophysical aspects of drug addiction.

H. L. Andrews

During the war, Dr. Andrews engaged in biophysical research on problems relating to climatology and physiology, and since 1946 has been very active in the field of atomic energy. He was also a member of the radiological safety staff of Operation Crossroads at Bikini. Dr. Andrews received letters of commendation for his war work from Secretary of Defense Forrestal, Vice Admiral W. H. P. Blandy, and Rear Admiral C. A. Swanson.

Dr. Andrews was a Charles A. Coffin Fellow of the General Electric Company in 1928–1929, and is a member of the American Physical Society, the American Association for Advancement of Science, Sigma Xi, and the National Committee on Radiation Protection. He has published numerous papers in physical and biological journals.

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Walter P. Crossley was born in Brooklyn, N. Y., on December 31, 1926. During the war years he studied pre-engineering subjects at Drew University under the Navy training program, and is now a physics major at the Polytechnic Institute of Brooklyn.

W. P. Crossley

He came to the Advanced Development Laboratory of Sylvania Electric Products Inc. in January, 1947, and is working on vacuum systems, life-test apparatus, and calculations and details pertaining to thermionic tube development.

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W. D. George (A'33) was born in Spartanburg, S. C., in 1907. He attended the Georgia School of Technology and received the B.S. degree in 1929. He entered the radio section of the National Bureau of Standards in 1929, where he was engaged in research, development, and tests on radio standards, materials, and instruments. This included work on standards of frequency, and time and extension of accuracy and services broadcast by radio station WWV.

W. D. George

Since 1946 Mr. George has been Chief of the high-frequency standards section of the Central Radio Propagation Laboratory of the National Bureau of Standards. He is a member of the American Geophysical Union and the International Scientific Radio Union.

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W. C. Hahn (A'36–SM'45) received the B.S. degree from the Massachusetts Institute of Technology in 1923. Following a student engineering course at the General Electric Company, he was sent to the Chicago office of this firm until 1932. From 1933 to 1944 Mr. Hahn was in the engineering general department at the Schenectady works. Since then he has been in the Research Laboratory.

W. C. Hahn

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George W. Hees was born in New York, N.Y., on July 27, 1918. He received the A. B. degree in chemistry and physics from Brooklyn College in 1939, and is now doing graduate work at the Polytechnic Institute of Brooklyn.

George W. Hees

He was employed as a physicist at the Aircraft Radiation Laboratory until 1942, and until 1944 was an engineer with the Army Service Forces on the procurement of vacuum tubes. From 1945 to 1946 he directed the activities of a group of Signal Corps inspectors of the ASF. He has been in the Advanced Development Laboratory of Sylvania Electric Products Inc. since June, 1946, and is doing work on the fundamental properties of emission phenomena.

Harold Jacobs was born on November 21, 1917, at Port Chester, N. Y. He received the A.B. degree from The Johns Hopkins University in 1938, and continued work at New York University, where he received the M.A. and Ph.D. degrees.

Harold Jacobs

Dr. Jacobs was employed by the RCA Manufacturing Company at Lancaster, Pa., from 1942 to 1945, when he came to the Advanced Development Laboratory of Sylvania Electric Products Inc. He is currently working on problems concerning surface phenomena, including cathode surface emission in gas and vacuum tubes. He is a member of the American Physical Society.

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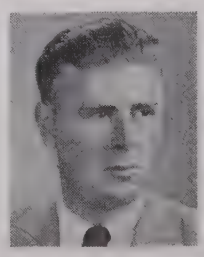
R. E. Lapp was born in Buffalo, N. Y., on August 24, 1917. He attended Canisius College from 1936 to 1938 under a New York State Scholarship, and University of Chicago from 1938 to 1940 under an Entrance Scholarship and a Divisional Honor Scholarship. He was awarded the B.S. degree in physics at the University of Chicago in 1940, and did graduate work at that University until 1943 as a Henry Strong Educational Foundation Fellow. He re-received the Ph.D. degree in 1946.

A specialist in cosmic-ray research, mass spectroscopy, and scientific administration, Dr. Lapp was associated with the Manhattan Project during the war as division director and assistant laboratory director of the Metallurgical Laboratory, and, until October, 1946, of the Argonne National Laboratory. He was a member of the radiological safety group of Operation Crossroads at Bikini, and a scientific advisor on atomic energy to the War Department General Staff. Dr. Lapp is now the executive director of the Committee on Atomic Energy of the Research and Development Board, and a consultant to the Atomic Energy Commission.



R. E. Lapp

Dr. Lapp is co-author of the book "Radiological Defense," published by the National Military Establishment, is also co-author of a forthcoming text, "Nuclear Radiation Physics," and has contributed to the *Physical Review* and the *Bulletin of the Atomic Scientists*. He holds membership in Sigma Xi, Phi Beta Kappa, the American Physical Society, the American Association for Advancement of Science, and the American Institute of Physics.

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T. G. Mihran

T. G. Mihran (S'43) was born in Detroit, Mich., on June 28, 1924. He received the A.B. degree in electrical engineering from Stanford University in June, 1944. The following two years were spent in the Navy, where Mr. Mihran's radio technician duties took him to Japan as a member of the occupation forces. Returning to Stanford following his discharge, he received the M.S. degree in June, 1947, and is now engaged in his second year of graduate work. During his senior and first graduate year, he was engaged in part-time work on ionosphere, field-strength, and direction-finding projects.

Mr. Mihran is a member of Phi Beta Kappa, Tau Beta Pi, and an associate member of Sigma Xi. He is currently studying under a Fortescue fellowship.

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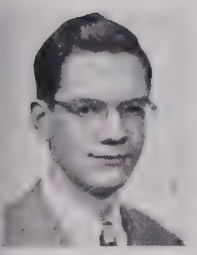

Donald R. Rhodes

Donald R. Rhodes (S'44–A'47) was born in Detroit, Mich., on December 31, 1923. He received the B.E.E. and N.Sc. degrees from The Ohio State University in 1945 and 1948, respectively. Mr. Rhodes has been a research associate of the Antenna Laboratory of the Ohio State University Research Foundation in Columbus, Ohio, from 1945 to the present, with the exception of a ten-months period at the U. S. Navy Electronics Laboratory in San Diego, Calif.

Mr. Rhodes is a member of Sigma Pi Sigma, Eta Kappa Nu, and the American Physical Society.

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Reuben Scolnik

Reuben Scolnik was born in Lewiston, Maine, on October 27, 1917. He received the B.S. degree from Bates College in 1939, and performed graduate study at Columbia University during 1940 and 1941. In 1942 Mr. Scolnik became associated with the optics division of the National Bureau of Standards; he transferred to the radio division in 1944, remaining there until May, 1947, when he joined the Naval Research Laboratory, optics division. At present he is engaged in research in physiological optics.

Mr. Scolnik is a member of Phi Beta Kappa.

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M. C. Selby

M. C. Selby (SM'47) was born in Jaruga, Ukraine, in 1898. An honor graduate in preparatory education and past student of the University of Odessa and Polytechnicum of Prague, he received the B.S. degree in electrical engineering from the Carnegie Institute of Technology in 1929, and the M.S. degree in physics from the Polytechnic Institute of Brooklyn in 1939.

In 1929 and 1930 Mr. Selby was employed by the Westinghouse Electric Corporation, where he had his first electronic experience on one of V. K. Zworykin's research projects. From 1930 to 1941 his activities continued in the field of radio-electronics and electronic development projects with several concerns, the greater part of the time with the Emerson Radio and Phonograph Corporation.

Since 1941, Mr. Selby has been a physicist at the National Bureau of Standards, where he is at present in charge of several research and development projects on high-frequency precision measurements and absolute standards.

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F.L.H.M. Stumpers

F. L. H. M. Stumpers was born at Eindhoven, the Netherlands, on August 30, 1911. He entered the Philips Laboratories in 1928, leaving in 1935 to study physics at Utrecht University. After receiving his degree in 1938 he returned to the Philips Laboratories, and since 1939 he has been engaged in the radio research department.

Delft University awarded Dr. Stumpers the degree of Doctor of Technical Science in 1946 for a dissertation on frequency modulation.

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F. Tihelka was born in Ujpest, Hungary, on November 1, 1915. In 1933 he became associated with the Hungarian Ericcson Company Ltd., and in 1939 he joined the development laboratory of the Telephone Factory Company Ltd., in Budapest, a subsidiary of the International Telegraph and Telephone Corporation. He is chiefly engaged in the design of amplifiers, audio-oscillators, and electrical wave filters.

F. Tihelka

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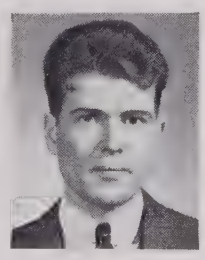

R. M. Whitmer

Robert M. Whitmer (A'29) was born in Battle Creek, Mich., on June 14, 1908. He attended the University of Michigan, receiving the B.A. degree in 1928 and the Ph.D. degree in 1939, both in physics. He spent three and one-half years with the Bell Telephone Laboratories working on directive antennas, ultra-high frequencies, and electron ballistics. He also spent a year with the Philco Corporation, engaged in television systems work.

As an instructor in physics at Purdue University, Dr. Whitmer assisted in constructing a cyclotron, and did experimental and theoretical research in electron diffraction and thermodynamics. During the war, he was a staff member in the M.I.T. Radiation Laboratory. For a time he was in charge of the development of radar beacons, and later he did theoretical research on waveguide problems. He also spent some time in Europe studying the techniques of bombing accuracy assessment and German military scientific research. Dr. Whitmer is now an associate professor of physics at Rensselaer Polytechnic Institute, Troy, N.Y.

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G. Willoner

G. Willoner was born in Budapest, Hungary, on April 10, 1913. He received the Dipl. Ing. degree in electrical engineering in 1936, and the doctorate in engineering from The Vienna Technical University in 1946. Since 1936 he has been employed in the development laboratory of the Telephone Factory Company Ltd., in Budapest, a subsidiary of the International Telegraph and Telephone Corporation. Dr. Willoner is chiefly engaged in the design of amplifiers, audio oscillators, and electrical wave filters.

Correspondence

A Note on the Effect of a Capacitor Shunting a High Impedance*

In obtaining impedance versus frequency characteristics of antennas, as well as circuits having similar impedance characteristics, rf bridges such as the General Radio 916-A have been commonly used. Unfortunately, impedances of many antennas have reactive and resistive components which are outside of the range of the bridge. This is particularly true for some aircraft antennas in the region of antiresonance (maximum-resistance region). In such cases it is possible to bring the impedance into the range of the bridge by using a capacitor in shunt with the antenna. With a shunting element, the critical values, such as antiresonant frequency, magnitude of antiresonant resistance, frequencies at and magnitudes of reactance extremes, are badly obscured and cannot be easily determined.

Nielsen[1] has presented a convenient set of graphs which makes it possible to determine the true impedance from the values of the equivalent series circuit elements. For the particular rf bridge under discussion, the GR 916-A, the capacitive values as required by Nielsen's graphs cannot be obtained directly, and all reactances read on the bridge must be divided by the frequency in megacycles.

All critical values can be shown to bear simple relationships to the readable quantities. Let us consider the circuit of an antenna with a shunting capacitor and its series equivalent circuit. The quantities D_S, D_C, D_R are bridge dial readings for the isolated shunting capacitor, equivalent series capacitance, and equivalent series resistance, respectively. D is negative for capactitive reactances.

In terms of the bridge readings, the unknown impedance Z_A is equal to

$$Z_A = R_A + jX_A$$

$$= \frac{D_S}{f} \frac{fD_RD_S - jf^2D_R^2 + jD_C(D_S - D_C)}{f^2D_R^2 + (D_S - D_C)^2}. \quad (1)$$

Equating imaginary terms, the condition for antiresonance is given by

$$D_C^2 - D_CD_S + f^2D_R^2 = 0. \quad (2)$$

Solving for D_C, and if

$$D_S^2 > (4fD_R)^2, \quad (3)$$

the antiresonance frequency is one which makes D_C satisfy the following relationship:

$$D_C = D_S - \frac{f^2D_R^2}{D_S}. \quad (4)$$

If the antenna is a "high-Q" circuit, D_R is very small, and

$$D_C \doteq D_S. \quad (5)$$

This shows that the frequency for antiresonance is one which makes the equivalent series capacitance almost equal to the shunting capacitance. This frequency can be easily found.

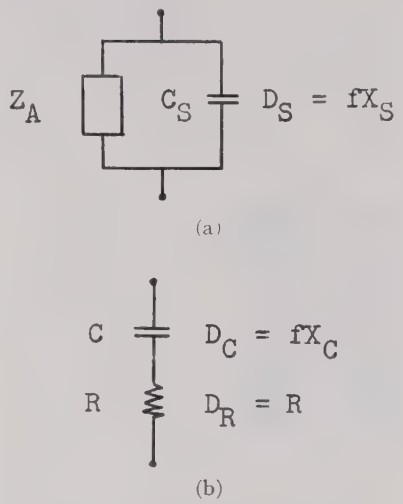

Fig. 1.—(a) Unknown impedance with shunting capacitor. (b) Equivalent series circuit.

The antiresonance resistance is

$$(R_A)_{\text{anti}} = \frac{D_S^2}{f^2D_R} \frac{1}{1 + \frac{f^2D_R^2}{D_S^2}}, \quad (6)$$

and, for a high-Q circuit,

$$(R_A)_{\text{anti}} \doteq \frac{D_S^2}{f^2D_R}. \quad (7)$$

For a high-Q circuit of constant parameters, the phase angle of the impedance is equal to 45 degrees at the reactance extremes near antiresonance. Equating the real and the imaginary components of Z_A,

$$fD_RD_S = \pm f^2D_R^2 \mp D_C(D_S - D_C). \quad (8)$$

Solving for D_C, and if

$$D_S^2 > 16fD_R(fD_R - D_S), \quad (9)$$

the frequencies for the reactance extremes are those which make D_C satisfy the following relationship:

$$D_C = D_S \pm fD_R - \frac{f^2D_R^2}{D_S} \quad (10)$$

$$\doteq D_S \pm fD_R. \quad (11)$$

It is a simple matter to find the frequencies which will give the desired values of D_C. The magnitudes of the reactance extremes are equal to approximately one-half of the antiresonance resistance. Both inequalities (3) and (9) are easily satisfied when measuring impedances of a high-Q circuit.

The determination of the critical values involves the quantities fD_R and $f^2D_R^2/D_S$. Both quantities can be plotted as straight-line contours on a log-log graph paper having f and D_R as the two axes. For a set of similar measurements, only one D_S is usually necessary to get the critical values. Such a graph can be found to be very convenient. It is usually found that f and D_R are slowly varying quantities through the antiresonance region.

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* Received by the Institute, May 18, 1948.

[1] R. L. Nielsen, "Charts for simplifying high-impedance measurements with the radio-frequency Bridge," Proc. I.R.E, vol. 31, pp. 372–379; July, 1943.

The Maximum Directivity of an Antenna*

Although I cannot point to the mathematical form of the limitation, I feel that the integral approximation of a function which is quoted by H. J. Riblet[1]

$$\left| g(t) - \int_{-a}^{a} f(x)e^{ixt}dx \right| < \epsilon \quad (1)$$

cannot be used without some restriction on the nature of $g((t)$ in relation to the maximum value of x. It is sufficiently puzzling to find that an antenna system of limited dimensions can give unlimited directivity, but another application of the same formula gives an even more startling result. Let $g(t)$ be the amplitude versus time function of a wave form of arbitrary shape, and in place of x write $\omega(=2\pi$ times frequency). We then have

$$\left| g(t) - \int_{-a}^{a} f(\omega)e^{i\omega t}d\omega \right| < \epsilon. \quad (2)$$

If no restriction is placed on the relationship between the form of $g(t)$ and the value of a, this means that a pulse of any degree of narrowness can be represented by a limited frequency spectrum extending from $\omega = -a$ to $\omega = a$. Another startling result appears to be that the resolving power of an optical instrument would not be limited by its aperture if appropriately designed.

The empirical and qualitative approach to (1) is that if $g(t)$ is a function displaying sharp curvature, i.e., the higher-order differential coefficients of $g(t)$ are of substantial magnitude, x must be allowed to extend to large values (i.e., a must be large), so that values of e^{ixt} include those representing high "frequencies." Empirically, it appears that too small a value of x in e^{ixt} cannot be compensated by any form whatsoever of the function $f(x)$ so as to represent rapid changes in $g(t)$. Can mathematical theory either prove this empirical assumption or demonstrate a numerical example to the contrary?

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* Received by the Institute, June 2, 1948.

[1] H. J. Riblet, "Note on the maximum directivity of an antenna," Proc. I.R.E, vol. 36, pp. 620–624; May, 1948.

Correspondence

A Discussion of the Maximum Directivity of an Antenna*

In his recent paper on this subject,[1] H. J. Riblet concludes by raising the question as to why no aperture-type antenna which has a directivity greater than that based on the assumption of uniform current distribution has ever been built, especially since calculations may be made which indicate that such an antenna is theoretically possible.

I believe that I am correct in stating that the answer to this important question has already been found as a result of research studies conducted at the Hughes Aircraft Company, and independently by L. J. Chu at the Massachusetts Institute of Technology. Although the principles involved have been made public on a few previous occasions, it appears that a brief discussion in answer to Mr. Riblet's paper may be in order.

As Mr. Riblet suggests, ohmic losses are a factor in limiting the performance of "superdirective" antennas. However, critical tolerances and narrow bandwidth are even more important limitations. The extreme rapidity with which these limitations take effect as an antenna is made even slightly superdirective is the singular feature of this problem, and is the reason for the nonappearance of such antennas in the field of practical design. As an example of these principles, calculations were made for antennas with linear-array-type patterns of high directivity; although, to simplify the problem it was assumed that the antenna designer had an entire spherical portion of space at his disposal, rather than a single linear source distribution. In such a calculation, the diameter of the sphere corresponds to the broadside aperture which is such an important parameter in the customary design formulas. Thus, it is found that an antenna designed within a sphere of 50 wavelengths diameter can be relied upon to produce a beam whose width is slightly in excess of 1 degree. If the beamwidth is held constant and the diameter of the sphere reduced, the antenna must necessarily become superdirective. The result of this process, starting with a 50-wavelength sphere, is shown in Table I.

TABLE I

Diameter of Sphere which Encloses Antenna	Approximate Q
45λ	5×10^{2}
40λ	5×10^{10}
35λ	10^{22}
30λ	10^{38}
25λ	10^{49}

By Q is meant the ratio of the energy stored in the intense portion of the induction field which immediately surrounds the antenna (if the design were optimized) to the energy radiated per electrical radian. Although ohmic losses may exceed radiation by a considerable factor, the Q ratio based on the latter still stands as a fundamental limitation, since radiation is the effect desired. It is left to the reader to draw his own conclusions about critical tolerances and other undesirable features which are concomitant with such tremendous quantities of circulating energy.

The mathematical basis for the foregoing remarks is available by either of at least two approaches. The first of these depends upon the fact that any antenna pattern consists of a series of solutions to the spherical wave equation, and that the behavior of these solutions is well known. For example, a highly directive axially symmetric pattern consists of a series of spherical waves, each of which varies transversely as an associated Legendre polynomial, and radially according to a spherical Hankel function. The directivity is related to the order of the highest-ordered wave which must be included. An observer studying the life history of a given wave, say that of order n, passes two important spheres on an imaginary trip from infinity to the origin. The first of these occurs at a radius equal to $n^2\lambda/2\pi$, and might properly be called the asymptotic sphere because it roughly marks the end of the region where the Hankel function may be regarded merely as $\exp(-jkr)/kr$. Going inward, the observer passes what might be called the cutoff sphere at a radius equal to $n\lambda/2\pi$. Within this sphere the Hankel function ceases to resemble a traveling wave at all; the amplitude of its imaginary part rises sharply, while that of its real part goes to zero. The wave itself is in much the same position as a wave in a guide below cutoff, since there is insufficient room on the periphery of the sphere to contain its transverse field pattern. To launch a spherical wave from a source within its cutoff sphere is a very impractical project, since it requires that the wave amplitude (and consequently the stored energy) in the neighborhood of that source be tremendous in comparison to the amplitude of the wave after it has emerged from the cutoff sphere. The implications of these concepts in terms of antenna design are obvious. If an antenna is supposed to produce a pattern of a certain directivity, it must produce spherical waves of various orders up to and including a certain order n. It follows, then, that this antenna cannot be contained within a sphere whose radius is smaller than $n\lambda/2\pi$ without suffering the consequences. It turns out that an axially symmetric pattern containing all wave orders up to and including n is associated with a beamwidth in degrees of $160/n$; hence, the limitation imposed by the cutoff sphere is exactly the same as that based upon the assumption of uniform current distribution. In other words, the design criteria which have always been used in connection with high-resolution aperture-type antennas have been given a new and conclusive validity by the operation of the principles just discussed.

Another example of the cutoff principle is found in considering radiation from capacitors and inductors which are small compared with wavelength. Thus, a small parallel-plate capacitor attempts to radiate a dipole field, which is none other than the spherical wave of order unity. The cutoff radius for this wave is simply $\lambda/2\pi$. If the radius of the capacitor plates and the separation between them is comparable with $\lambda/2\pi$, it is found that the capacitor is on the border line between a storage device and a radiating device. However, as its dimensions are made smaller the storage function very rapidly becomes predominant, and if there were no losses except radiation, the Q would approach infinity as the dimensions approach zero. The rapidity of this effect, noticeable even at order unity, is magnified many times when higher orders are considered.

The second approach to the superdirectivity problem consists in assuming a fixed number of isotropic elements (in a linear array, for example) and a fixed over-all length, and solving for the respective excitations subject to the constraint that the directivity shall be maximum. Fortunately the processes may be arranged in such a way that only linear equations are involved, and therefore a solution can always be obtained. If the number of elements assumed is such that the spacing is less than one-half wavelength, the directivity will be found to be greater than that predicted by the usual aperture formula. If the number of elements is made to approach infinity, the directivity likewise approaches infinity. However, when the calculations are critically examined, it is found that adjacent elements frequently have very large excitations of opposite sign. In such an array the elements are never completely in phase; directivity is a result of the fact that for some angles of observation the elements are more nearly in phase than for others. If reasonable physical characteristics are ascribed to the elements, it may be shown that the stored energy (in comparison with radiated power) increases very rapidly, both as the original assumed length and as the factor of directivity improvement are increased. Thus, on the basis of this approach alone, it is possible to show that arrays longer than a few wavelengths with a directivity improvement of more than a few per cent are impractical.

It is hoped that these remarks will speed the process of popularizing some very fundamental principles of radiation theory. If properly applied, these principles cannot fail to have a beneficial effect upon antenna design, even if that effect is only a delineation of the limitations involved.

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* Received by the Institute, June 3, 1948.
[1] H. J. Riblet, "Note on the maximum directivity of an antenna," PROC. I.R.E., vol. 36, pp. 620–624; May, 1948.

Institute News and Radio Notes

Executive Committee

May 4, 1948

Student Branch Petitions. Mr. S. L. Bailey moved that the following petitions for Student Branches be approved and the branches established:

University of Arizona	IRE-AIEE
Polytechnic Institute of Brooklyn	IRE
Princeton University	IRE-AIEE
California State Polytechnic College	IRE

National Research Council. The National Research Council has requested the IRE to appoint an IRE representative to the NRC for a term of three years, beginning July 1, 1948, such appointment to become effective if a change is made to allow continuing representation for each society. Executive Secretary Bailey was instructed to notify President Shackelford of the name of the last representative of the IRE on the NRC. President Shackelford will appoint a scientist of standing as IRE representative on the conditional basis as requested.

June 2, 1948

Student Branches. The following Student Branch matters were discussed and action taken:

a. *Student Branch Petitions.* Dr. Baker moved that petitions for the formation of Student Branches at the following recognized schools be approved:

Columbia University	IRE-AIEE
Illinois Institute of Technology	IRE
John Carroll University	IRE
Massachusetts Institute of Technology	IRE-AIEE
Newark College of Engineering	IRE
Oregon State College	IRE
South Dakota School of Mines and Technology	IRE
University of Maryland	IRE-AIEE
University of North Dakota	IRE-AIEE
University of Toledo	IRE-AIEE
University of Wisconsin	IRE
University of Wyoming	IRE
Utah State Agricultural College	IRE

(Unanimously approved.)

b. *Student Branch Constitutions.* Dr. Baker moved that the Student Branch Constitutions submitted by the following recognized schools, which are patterned after the model Constitution previously approved by the Board of Directors, and which include all requirements contained in the model Constitution, be approved:

Columbia University Student Branch
Northwestern University Student Branch

(Unanimously approved.)

Petition for Formation of Audio Group. The following petition for an Audio Group, signed by the required number of Members and Fellows, was submitted for approval:

"TO THE PROFESSIONAL GROUP COMMITTEE AND THE BOARD OF DIRECTORS OF THE INSTITUTE OF RADIO ENGINEERS

We, the undersigned Members and Fellows of the Institute, respectfully request the formation of an audio group within The Institute of Radio Engineers. We believe that a group concerned especially with the technology of communication at audio frequencies and with the audio-frequency portion of radio systems is extremely desirable. If such a group is formed, we intend to be active members of it."

Dr. Sinclair moved that the petition for formation of an Audio Group be approved, subject to an explicit statement that Board approval will be called for, and that the scope is defined only as of the present and may be subject to later revision. (Unanimously approved.)

Joint Technical Advisory Committee

1. Creation of a Joint Technical Advisory Committee for the purpose of advising governmental agencies, such as the Federal Communications Commission and other professional and industrial groups, on technical aspects of radio, television, and electronic problems, was announced today jointly by the Radio Manufacturers Association and The Institute of Radio Engineers. The JTAC reports to the two Boards of Directors who have delegated the responsibility to B. E. Shackelford, President of The Institute of Radio Engineers, and to W. R. G. Baker, Director of Engineering of the Radio Manufacturers Association, as direct contacts.

2. Members of the committee were appointed by the respective Boards of Directors of RMA and IRE. The members were chosen, according to W. R. G. Baker and Benjamin E. Shackelford, "on the basis of professional standing," irrespective of the organizations to which they belong or the companies by which they are employed.

3. Philip F. Siling, Chief Engineer of the RCA Frequency Bureau, of Washington, D. C., was appointed first chairman of the new committee as a representative of IRE. The vice-chairman is Donald G. Fink, editor of *Electronics*, New York, N. Y., who is a representative of RMA.

4. The other six members of the eight-man committee are: Ralph Bown, of the Bell Telephone Laboratories, Murray Hill, N. J.; Melville Eastham, of the General Radio Co., Cambridge, Mass.; John V. L. Hogan, of the Interstate Broadcasting Co., Inc., New York, N. Y.; E. K. Jett, a former commissioner of the FCC and now director of the Baltimore Sun's radio and television operations; Haraden Pratt, of Mackay Radio, New York, N. Y.; and David B, Smith, of the Philco Corp., Philadelphia Pa. Laurence G. Cumming, IRE Technical Secretary, is nonmember secretary of the committee.

5. One of the first tasks of the committee will be to gather authoritative information on the availability of equipment and propagation characteristics of the ultra-high frequencies in preparation for a television hearing called by the Federal Communications Commission to start September 20.

6. RMA and IRE decided to establish JTAC as a central and authoritative policy advisory group in the technical field because of the increasing number of far-reaching problems confronting the industry and governmental regulatory agencies as a result of rapid developments in television, FM broadcasting, and other radio and electronic services. The formation of JTAC follows a suggestion of FCC Chairman Wayne Coy that the industry provide FCC with authoritative technical information on the adaptability of the frequencies above 216 Mc for television broadcasting.

7. The objective of JTAC, as stated in its charter, is "to obtain and evaluate information of a technical or engineering nature relating to the radio art for the purpose of advising government bodies and other professional and industrial groups.

"In obtaining and evaluating such information," the charter continues, "the JTAC shall maintain an objective point of view. It is recognized that the advice given may involve integrated professional judgments on many interrelated factors, including economic forces and public policy."

JTAC will supplant the Radio Technical Planning Board in its relations with the FCC and other governmental agencies, and the RTPB has been dissolved.

Other functions of the RTPB are being taken over by various technical committees of the IRE and RMA Engineering Department, and both RMA and IRE committees will be called upon from time to time by JTAC for information.

Where a qualified technical group does not exist, JTAC has authority to appoint an ad hoc committee to study and report on particular subjects, after which it will be disbanded.

Specific duties of JTAC are outlined in the charter as follows:

(a). To consult with government bodies and with other professional and industrial groups to determine what technical information is required to insure the wise use and regulation of radio facilities.

(b). To establish a program of activity and determine priority among the problems selected by it or presented to it in view of the needs of the profession and the public.

(c). To establish outlines of the information required in detailed form. These outlines will be submitted to qualified groups, as hereinafter defined, who shall study the requirements and supply the required information.

(d). To sift and evaluate information thus obtained so as to resolve conflicts of fact, to separate matters of fact from matters of opinion, and to relate the detailed findings to the broad problems presented to it.

(e). To present its findings in a clear and understandable manner to the agencies originally requesting the assistance of the Committee.

(f). To make its findings available to the profession and the public.

(g). To appear as necessary before government or other parties to interpret the findings of the Committee in the light of other information presented.

Members of JTAC are appointed for two years, and the chairmanship will be alternated each year between RMA and IRE representatives.

IRE-RMA Meet in Rochester

The 1948 Rochester Fall Meeting of members of The Institute of Radio Engineers and the Radio Manufacturers Association's Engineering Department will be held November 8, 9, and 10 at the Sheraton Hotel in Rochester, N. Y. The meeting will be sponsored by the Rochester Fall Meeting Committee, headed by Virgil M. Graham, who has served as chairman since its inception twenty years ago.

Calendar of COMING EVENTS

Electronic Technicians' Town Meeting, New York City, Sept. 27–29

1948 West Coast Convention of the IRE, Los Angeles, Sept. 30–Oct. 2

IRE-URSI Meeting, Washington, D.C., Oct. 7–9

1948 General Meeting of the AIEE, Milwaukee, Wis., Oct. 18–22

Optical Society of America Meeting, Detroit, Mich., Oct. 21–23

Society of Motion Picture Engineers Convention, Washington, D. C., Oct. 25–29

1948 Conference on Electrical Insulation, National Research Council, Washington, D. C., Oct. 27–29

National Electronics Conference, Chicago, Nov. 4–6, 1948.

Electronic Technicians' Town Meeting, Boston, Nov. 15–17

American Physical Society Meeting, Chicago, Nov. 26–27

IRE-RMA Rochester Fall Meeting, Rochester, N. Y., Nov. 8–10

1948 Southwestern I.R.E. Conference Dallas, Tex., Dec. 10–11

American Physical Society Meeting, New York City, Jan. 27–29, 1949

March 7–10, 1949 IRE National Convention, New York City

Utilization of 475 to 890 Mc Band for Television Broadcasting

On September 20, 1948, the FCC will hold a hearing, Docket 8976, on the utilization of the band 475–890 Mc for television broadcasting. The issues to be considered are as follows:

1. To obtain full information concerning interference to the reception of television stations operating on channels 2 through 13 resulting from adjacent channel operation of other services, from harmonic radiations, and from man-made noise.
2. To receive such additional data as may be available since the close of previous hearings (Dockets 6651 and 7896) concerning the propagation characteristics of the band 475 to 890 Mc.
3. To obtain full information concerning the state of development of transmitting and receiving equipment for either monochrome or color television broadcasting, or both, capable of operating in the band 475 to 890 Mc.
4. To obtain full information concerning any proposals for the utilization of the band 475 to 890 Mc or any part thereof, for television broadcasting and the standards to be proposed therefor.

At the request of the Joint Technical Advisory Committee, members of the Commission staff have prepared the following list of detailed questions:

1. What is the present state of development of equipment in the band 470 to 890 Mc, in regard to (a) transmitters, tubes, and components; (b) receivers and components; (c) antennas, transmission lines, and related equipment for transmission and reception?
2. How much experimental work has been undertaken in television systems in this band, with respect to field operation (transmitter hours operated, number and distribution of receivers, and propagation tests) and laboratory work (development of receivers, transmitters, and tubes)?
3. What consideration has been given to the costs of television systems for this band, particularly to the reduction of receiver costs, and the transfer of cost burdens to the transmitter?
4. What areas of service might be expected in this band, based on the following assumptions: (a) a particular system, using one of the following typical bandwidths: 6 Mc, 13 Mc, 20 Mc; (b) radiated power, available now and expected to be available, say, 10 years in the future; (3) receiver sensitivity, and (4) at each of the following typical frequencies: 475, 600, and 890 Mc?
5. What co-channel and adjacent-channel separations would be appropriate under the assumptions made in item 4, above?
6. How many channels would be available in the band 475–890 Mc, on the assumptions of item 4, above, and how might they be allocated among the 140 metropolitan districts of the United States?

It is requested that any individual or organization having information related to these questions, communicate directly with the Secretary of JTAC, L. G. Cumming, The Institute of Radio Engineers, 1 East 79 St., New York 21, N. Y.

AUSTRALIAN-AMERICAN REPRINT PLAN

The membership of The Institute of Radio Engineers will doubtless find pleasure and encouragement in the following editorial appearing in the April, 1948 issue of the *Proceedings of the Institution of Radio Engineers, Australia:*

"International Goodwill. . .

"Several months ago we received a letter from Dr. Alfred N. Goldsmith, Editor of the PROCEEDINGS OF THE I.R.E. in the United States, expressing the goodwill of the Board of Directors and Officers of that Institute towards the Institution of Radio Engineers, Australia.

"In furtherance of this thought it was suggested that we give consideration to a proposal whereby the Institute of Radio Engineers, might be permitted to reprint in the PROCEEDINGS OF THE I.R.E. papers originally appearing in the *Proceedings of the Institution of Radio Engineers, Australia*. In return the American IRE offered reciprocal rights to the Australian Institution.

"It was proposed that the arrangement be made in the first instance for an experimental period, after which it would be determined whether any modification of the plan was desirable. It was not suggested that all or even a large proportion of the papers appearing in either journal would be reprinted in the other, but that papers published originally by one Institution, which were of special interest, would in this way be made available to the other.

"The proposal was considered by the Publications Board and Council of our Institution, and it is with pleasure that we announce that details have now been completed and ratified by both sides, and that steps have been taken to place the plan in operation for the current year.

"Both Institutions are now free to reprint any papers selected by their editorial groups without obtaining special permission, but in each case will advise the other Institution of the intention to reprint the paper, and will include a suitable acknowledgement of the source and of the courtesy involved.

"The Publications Board looks forward to the operation of the plan with enthusiasm, not only because of the direct benefit members will derive from the reprinting of papers of outstanding interest, but because of the plan's significance as an indicator of the goodwill and friendly attitude of each society towards the other.—*Murray H. Stevenson*"

Mr. Stevenson is the Chairman of the Publications Board of our Australian sister society.

We have been authorized to state that the Board of Directors and Officers of the IRE agree with the thoughts and heartily reciprocate the sentiments expressed by Mr. Stevenson on behalf of the Institution of Radio Engineers, Australia; and that they too anticipate mutual benefits to the membership of both societies as the result of the plan outlined above.—*The Editor.*

ATTENTION, AUTHORS!

Donald B. Sinclair, Chairman of the Technical Program Committee for the 1949 IRE National Convention, requests that authors of papers to be considered for presentation submit the following information to him as soon as possible:

Name and address of the author, title of the paper, and sufficient information about the subject matter to enable the reviewing committee to assess its suitability for inclusion in the Technical Program.

Although it will not be necessary to submit a paper in its entirety. Chairman Sinclair urges authors to prepare the necessary material promptly and mail it to him at 275 Massachusetts Avenue, Cambridge 39, Mass. The last possible date for acceptance of material relative to Convention papers is December 1, 1948.

AIEE-IRE Conference on Electronic Instrumentation

The AIEE-IRE Conference on Electronic Instrumentation in Nucleonics and Medicine will be held in the AIEE auditorium at 29 West 39 Street, New York, N. Y., on November 29 and 30 and December 1. A program of over 20 papers has been tentatively scheduled.

On the first day, a group headed by Dr. W. A. Geohegan will discuss biological requirements and present design practices in amplifiers and recording devices.

Dr. G. W. Dunlap will be chairman of the November 30 session, at which the following topics will be presented: biological requirements for radioactive isotope measurements; stable isotope measurement; nucleonics, electron multiplier, and crystal counters; nucleonics and health protection instrumentation; and biological effects of radiation and health protection.

On the last day, under Dr. H. H. Goldsmith's leadership, the group will consider geiger and proportional counters; ionization and cloud chambers, and stabilized high-voltage supply for them; photographic emulsion; and high-speed counting techniques.

IRE-URSI Meeting

A second joint meeting of the American Section of the International Scientific Radio Union (URSI) and The Institute of Radio Engineers will be held in Washington on October 7, 8, and 9. The morning session on Thursday, October 7, will be held jointly with District 2 AIEE.

The program will, as usual, be devoted to the more fundamental and scientific aspects of radio and electronics. A booklet listing the program of titles and abstracts will be available for distribution before the meeting. Correspondence should be addressed to Newbern Smith, Secretary, American Section, URSI, National Bureau of Standards, Washington 25, D. C.

IRE West Coast Convention

The West Coast Convention of the IRE, to be held September 30 to October 3, in conjunction with the West Coast Electronic Manufacturers Association, will be the major electronic event of the West this year. Opening with registration in the Biltmore Hotel, it will be followed by sessions in the Embassy Auditorium, South Grand Avenue at Ninth Street, Los Angeles, Calif.

The first day, Thursday, two simultaneous afternoon sessions will be given, "Broadcasting and Allied Arts," headed by Bernard Walley; and "Computers," headed by A. R. Willson. Both sessions will start with a speech of welcome offered by Walter Kenworth. At the broadcasting session, papers will be presented on "A Low Cost Program Switching System," by I. Gifford and A. P. Chesney; "Antenna Input Systems for Television Receivers," by D. E. Foster; "Operation of AM Broadcast Transmitters into Sharply Tuned Antenna Systems," by W. H. Doherty; and "Stratovision," by C. E. Nobles.

At the same time, J. L. Barnes will speak on "The Outlook for Electronic Computers" at the computer session, followed by "Input and Output Equipment for Electronic Computers," by C. H. Page; "Electronic Techniques Applied to Analog Methods of Computation," by D. G. McCann, C. H. Wilts, and B. M. Locanthi; and "Design and Use of the Reevac: A General Purpose Electronic Digital Computer," by Samuel Lubkin. Thursday evening there will be an Audio Symposium, with E. S. Naschke acting as chairman.

The morning session on Friday will feature "Measurements and Propagation" under the leadership of O. A. Steele. The following papers will be presented: "A New Type of Direct Reading RF Phase Meter for Low-Level Signals," by M. K. Goldstein; "The Determination of the Shunt Resistance of Cavity Resonators by Means of an Electrical Network Analyzer," by F. W. Schoot and K. R. Spangenberg; "A Method of Obtaining the Product of Two Voltages," by M. A. H. El-Said; and "Propagation Measurements at High Radio Frequencies over Flat Desert Terrain," by J. P. Day and L. C. Trolese.

L. E. Reukema will be chairman of the afternoon session on "Electronic Devices." R. H. Delano will offer a paper on "Signal-to-Noise Ratios of Linear Detectors," after which "A Mass Spectrometer Designed for Industrial Use," by C. E. Berry, R. L. Sink, and Carl Spaulding will be presented; as well as "Problems in the Design of Megawatt Output Klystrons for Pulsed Operation," by Marvin Choderow and E. L. Ginzton; and "Application of Microwave Spectroscopy to Determination of Interatomic Distances in Molecules," by D. H. Coles.

The final session will cover "Systems and Navigational Aids" on Saturday morning under the chairmanship of C. N. Tirrell. Papers will be offered on "Systems Engineering Aspects in Military Communications," by W. S. Marks; "The VHF Omnidirectional Range," by the CAA; "Design of a Radar Set for Commercial Airlines," by F. G. Suffield; "Design of Antenna for Optimum Directivity," by T. T. Taylor; and "Bandwidth in Communication Systems," by W. G. Tuller.

MEMBERSHIP EXPIRATION DATE

The date of membership expiration appears in each case as part of the address on the wrapper of every issue of the PROCEEDINGS, should any member wish to refer to it.

Ionospheric Radio Propagation

The Central Radio Propagation Laboratory of the National Bureau of Standards has prepared a new book on "Ionospheric Radio Propagation" (NBS Circular 462), which is available for $1.00 from the Government Printing Office, Washington, D. C. The book contains 209 pages and 207 illustrations, and includes chapters on the theory of radio wave propagation, measurement techniques, structure and variations of the ionosphere, maximum usable frequencies, ionospheric absorption and skywave intensity, radio noise and required field intensity, and lowest required radiated power and lowest useful high frequency.

Those parts of the old IRPL Handbook the accuracy of which has stood the test of time are offered with such modifications as are necessary for clearer presentation and inclusion of later theory. In addition, the new book contains information of great practical and operational value heretofore not generally available to the public, concerning methods of calculation of incident field intensity, required field intensity, lowest required radiated power, and lowest useful high frequency.

NEW EDITORIAL REQUIREMENT

Papers submitted for consideration by the editorial board of the PROCEEDINGS shall henceforth indicate in the early portion of the paper or in a footnote, when such statement is relevant, whether the author is employing cgs or rationalized mks terminology in the paper itself. Although mks terminology is preferred, its use will not be a condition precedent to acceptance of the paper.

Industrial Engineering Notes[1]

International Treaty Ratified

The U. S. Senate ratified the Telecommunications Convention and Radio Regulations resulting from the Atlantic City meeting last summer. In a report to the Senate from the Foreign Relations Committee, Senator White (Rep., Me.) stated that no government agency opposes ratification of the treaty, and "I know of no American commercial interest which has raised its voice against ratification."

[1] The data on which these NOTES are based were selected, by permission, from "Industry Reports," issues of June 21 and 25, and July 2, 9, and 16, published by the Radio Manufacturers' Association, whose helpful attitude in this matter is hereby gladly acknowledged.

Mine Radio Communication Tests

Experiments on radio communication in mines—between the surface and underground workings, and between widely-separated points underground—were reported in detail in Report of Investigations 4294, "Applicability of Radio to Emergency Mine Communications," published by the Publications Distribution Section of the Bureau of Mines, 4800 Forbes Street, Pittsburgh 13, Pa.

During the tests the Bureau used low-frequency radio waves to transmit the human voice through ground strata and also via trolley wires, pipes, and other metallic installations. The report, copies of which are available without charge, is part of a continuing study of emergency mine communications and covers progress of the work during 1947.

Recent FCC Rulings

The FCC has announced a new action (Mimeograph No. 21376, available from the Secretary of the FCC, Washington 25, D. C.) to supplant the present General Mobile Radio Service with **three new classes of mobile service** on a regular basis: Land Transportation Radio Services, Domestic Public Mobile Radiotelephone Services, and Industrial Radio Services. . . . The FCC amended its rules to provide for a **graduated scale of television programming** during the early license period of a station: During the first 18 months the program operating schedule must be not less than 2 hours daily in any 5 broadcast days per week and not less than a total of 12 hours per week. From 18 to 24 months the regulations remain the same, but there must be at least 16 hours per week; 24 to 30 months, 20 hours per week; 30 to 36 months, 24 hours per week. After 36 months, the station must broadcast not less than 2 hours in each of the 7 days of the week, and at least 28 hours per week . . . Telanserphone, Inc., of New York, was granted a construction permit by the FCC for a Class 1 experimental radio station to test the feasibility of a **radio paging service for doctors.** It proposes a one-way transmission system to reach doctors carrying small portable receivers. Experiments will be conducted in the 72–76-Mc portion of the spectrum, provided that no interference is caused to television channels 4 and 5. . . . The FCC abolished **special temporary authorizations** in connection with standard broadcast station operation. The Commission noted a general trend by AM stations to use STA's to operate beyond the hours for which they are licensed, many resorting to this practice over extensive periods of time, with the result that service "by full-time stations is suffering considerable degradation."

FCC Appoints Woman Member

For the first time since its creation in 1933, the FCC has a woman member. Miss Frieda B. Hennock was appointed for a seven-year term to succeed Clifford J. Durr.

Low-Powered FM Stations

The FCC proposed to amend its rules and regulations to permit noncommercial educational broadcast stations to broadcast with power of 10 watts or less, and to waive other requirements in order to promote this type of service. The U. S. Office of Education favors the licensing of low-power FM transmitters for school systems, largely because of the low cost of equipment required to provide satisfactory service to limited areas. This type of station would be authorized in the 88 to 82-Mc band now allocated to the noncommercial educational broadcasting service, and could be received on ordinary FM sets.

New Television Developments

Television reception interference caused by automobile ignitions can be corrected or minimized if motor-car manufacturers equip their automobiles with proper suppressors on spark plugs, distributors, etc., according to the most recent of a series of tests conducted by the RMA Engineering Department in co-operation with the Society of Automotive Engineers. These tests demonstrated that automobile ignition systems must not exceed 35 microvolts per meter if they are not to interfere with normal television reception. "The use of special spark plugs with built-in ignition suppressors reduced the radiation from the ignition system by an appreciable amount," the committee said. "In the two tests conducted with these spark plug suppressors, the tolerable interference moved from a distance of 200 feet from the antenna to approximately 70 feet." . . . **Regular television stations now on the air** have increased to 30, with 104 construction permits outstanding, and 299 applications still pending. Three new stations have been added: WNHC-TV in New Haven Conn.; WNAC-TV in Boston, Mass.; and WPIX-TV, New York, N. Y.

Radio-Television Equipment Production and Distribution

Television receiver production continued to climb in May, reaching a weekly production rate 38 per cent greater than the weekly average of the first quarter of this year. May's television set production by RMA members totalled 50,177 for an average of more than 12,500 receivers weekly, and brought total television set production by RMA members to 214,543 for the five months of 1948.

A total of 354,000 television receivers were in use throughout the U. S. on June 15, according to estimates of Audience Research Inc., headed by Dr. George H. Gallup. Of this number 314,000 are installed in private homes, and 40,000 in bars or taverns. Estimates show that a minimum of 1,100,000 additional families will acquire television sets within the next 12 months, making a total of 1,500,000 sets in operation by June, 1949. Dr. Gallup also estimated that an additional 5,400,000 families would be in the market for television sets if the average price were $200 instead of today's average price of $400.

Sales of radio and television equipment, including electron tubes, totaled $40,351,820 during the first quarter of 1948, according to RMA tabulations. Almost half of these sales went to the U. S. Government, and $12,875,196 of the federal purchases were for radar equipment. Production of cathode-ray receiving tubes of the type used in television sets showed the greatest gain over the corresponding period of 1947. Sales of transmitting and communications equipment of all types during the first quarter of 1948 fell below the $56 total reached in the first quarter of 1947, because of a drop in government procurement from about $40 million to $18 million. Broadcast transmitting equipment sales were about equal during the first quarters of 1948 and 1947, the 1948 quarterly report totalling $6,725,385. FM transmitting equipment sales by RMA members during the first quarter of this year aggregated $1,615,204; AM sales $667,435. Studio equipment sales for both AM and FM stations was $1,193,060; antenna equipment, $558,577. Sales of television transmitting equipment amounted to $1,682,615; miscellaneous broadcast transmitting equipment sales, $369,048. General communications equipment sales added up to $2,878,198 in the first quarter of this year, and marine communications and navigation equipment sales $1,389,317. Sales of aviation communications and navigational equipment totalled $683,101. All types of electron tube sales, including cathode ray, accounted for $10,536,935 of the $40 million total for transmitting equipment and accessories.

588 FM Stations Now on Air

A total of 593 FM stations and 22 noncommercial educational FM outlets are now in operation. Conditional grants number 109, and 788 construction permits were authorized. New FM stations have begun broadcasting in the following states: *Ala.*, Sylacauga (SWYO); *Ark.*, Jonesboro (KBTM-FM); *Ga.*, Atlanta (WABE), Cedar town (WGAA-FM); *Ill.*, Alton (WOKZ-FM), Chicago (WXRT); *Ind.*, Fort Wayne (WKJG-FM and WFTW-FM); *Kans.*, Topeka (WREN-FM); *Mich.*, Hillsdale (WVMH), Pontiac (WCAR-FM); *Miss.*, Gulfport (WGCM-FM), Meridian (WMCX-FM); *Ohio*, Hamilton (WMOH-FM); *Pa.*, Erie (WLCU-FM); *Tex.*, Temple (KIEM-FM); and *Va.*, Arlington (WARL-FM).

May Excise Collections

May collections of the 10 per cent excise tax on radios and phonographs and their component parts dropped below the April and May, 1947 levels, according to a report released by the Bureau of Internal Revenue. Collections during May totalled $4,740,786.10, as compared with $5,714,409.84 in April, 1947, and $6,374,539.98 in May, 1947.

90 Per Cent of Canadian Homes Have Radio

Results of a survey conducted by the Dominion Bureau of Statistics showed that in August, 1947, 90 per cent of the homes in Canada, numbering 2,818,000, had radios, compared to 78 per cent in 1941. About 8 per cent, or 249,000 homes, were estimated to have more than one radio.

RMA Plans Industrial Mobilization

Anticipating military requirements for radio and electronics equipment and components that may possibly reach a billion dollars, the RMA Board of Directors at the twenty-fourth annual RMA convention at Chicago in June developed an industrial mobilization plan for the radio and electronics industry which will distribute contracts among manufacturers, both large and small, equitably, and will also speed production. RMA President Max F. Balcolm has appointed an Industrial Mobilization Policy Committee to present the plan at once to appropriate Government officials in Washington, and to request immediate revision of Government procurement policies before contracts are let on the new rearmament program.

One of the first objectives of the RMA Industrial Mobilization Committee will be to persuade the military services to centralize and co-ordinate the procurement of radio and electronic equipment and components by placing all purchase contracts under the direction of a four-man committee which would be composed of representatives of each of the three services—Army, Navy, and Air Force—and one civilian industry representative.

World War II Electronics Expenditures

The final report submitted by the Chairman of the War Production Board to the President shows that the armed services spent more money on communications and electronic equipment during the last war than for "guns and fire control." A total of $10.6 billion was spent for communications and electronic equipment, while $9.7 billion was spent for guns and fire-control apparatus.

RMA Standards Keep Up-to-Date

With the aim of keeping abreast of wartime and more recent technical developments in the radio and electronics industry, the RMA Engineering Department has issued 21 recommended engineering standards during the past year, and has 40 more in progress. A total of 174 committees, comprising hundreds of the industry's foremost engineers, are engaged in the study.

Most of the 21 standards issued during the past year by the RMA were new; only a few were revisions of pre-war standards. Since 4 standards were developed by the Joint Electronic Tube Engineering Council, they were subsequently reprinted as joint standards of RMA and the National Electric Manufacturers Association.

The RMA Engineering Department also carried on extensive tube type designations and registrations for the radio industry during the year. A total of 154 tube type designations were registered, and 197 were reserved.

RMA Membership Now 325

The RMA membership increased to 325 members at the Association's convention in Chicago. Thirteen new members were elected by the Board of Directors, and five memberships were terminated.

Sections

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G. E. Van Spankeren San Martin 379 Buenos Aires, Arg.	BUENOS AIRES	A. C. Cambre San Martin 379 Buenos Aires, Arg.
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G. P. Hixenbaugh Radio Station WMT Cedar Rapids, Iowa	CEDAR RAPIDS	W. W. Farley Collins Radio Co. Cedar Rapids, Iowa
K. W. Jarvis 6058 W. Fullerton Ave. Chicago 39, Ill.	CHICAGO September 17	Kipling Adams General Radio Co. 920 S. Michigan Ave. Chicago 5, Ill.
C. K. Gieringer 3016 Lischer Ave. Cincinnati, Ohio	CINCINNATI September 14	F. W. King RR 9 Box 263 College Hill Cincinnati 24, Ohio
F. B. Schramm 1764 Wickford Rd. Cleveland 12, Ohio	CLEVELAND September 23	J. B. Epperson Scripps-Howard Radio 306 Prospect St. Berea, Ohio
C. J. Emmons 158 E. Como Ave. Columbus 2, Ohio	COLUMBUS October 8	L. B. Lamp 846 Berkeley Rd. Columbus 5, Ohio
L. A. Reilly 989 Roosevelt Ave. Springfield, Mass.	CONNECTICUT VALLEY September 16	H. L. Krauss Dunham Laboratory Yale University New Haven, Conn.
J. G. Rountree 4333 South Western Blvd. Dallas 5, Texas	DALLAS-FT. WORTH	J. H. Homsy Box 5238 Dallas, Texas
George Rappaport 132 East Court Harshman Homes Dayton 3, Ohio	DAYTON September 16	C. J. Marshall 1 Twain Place Dayton 10, Ohio
C. F. Quentin Radio Station KRNT Des Moines 4, Iowa	DES MOINES-AMES	F. E. Bartlett Radio Station KSO Old Colony Bldg. Des Moines 9, Iowa
A. Friedenthal 5396 Oregon Detroit 4, Mich.	DETROIT September 17	N. C. Fisk 3005 W. Chicago Ave. Detroit 6, Mich.
E. F. Kahl Sylvania Electric Products Emporium, Pa.	EMPORIUM	R. W. Slinkman Sylvania Electric Products Emporium, Pa.
W. H. Carter 1309 Marshall Ave. Houston 6, Texas	HOUSTON	J. C. Robinson 1422 San Jacinto St. Houston 2, Texas
R. E. McCormick 3466 Carrollton Ave. Indianapolis, Ind.	INDIANAPOLIS	Eugene Pulliam 931 N. Parker Ave. Indianapolis, Ind.
Karl Troeglen KCMO Broadcasting Co. Commerce Bldg. Kansas City 6, Mo.	KANSAS CITY	Mrs. G. L. Curtis 6005 El Monte Mission, Kan.
R. W. Wilton 71 Carling St. London, Ont., Canada	LONDON, ONTARIO	G. H. Hadden 35 Becher St. London, Ont., Canada
Walter Kenworth 1427 Lafayette St. San Gabriel, Calif.	LOS ANGELES September 21	R. A. Monfort L. A. Times 202 W. First St. Los Angeles 12, Calif.

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L. A. Hopkins, Jr. 629 Permanent Quarters Sandia Base Branch Albuquerque, N. M.	NEW MEXICO	T. S. Church 637 La Vega Rd. Albuquerque, N. M.
J. W. McRae Bell Telephone Labs. 463 West Street New York 4, N. Y.	NEW YORK October 6	R. D. Chipp 99 Morse Ave. Bloomfield, N. J.
C. G. Brennecke Dept. of Electrical Eng. North Carolina State College Raleigh, N. C.	NORTH CAROLINA-VIRGINIA	C. M. Smith Radio Station WMIT Winston-Salem, N. C.
W. L. Haney 117 Bourque St. Hull, P. Q.	OTTAWA, ONTARIO September 16	G. A. Davis 78 Holland Ave. Ottawa, Canada
A. N. Curtiss Radio Corp. of America Camden, N. J.	PHILADELPHIA October 7	C. A. Gunther Radio Corp. of America Front & Cooper Sts. Camden, N. J.
E. M. Williams Electrical Engineering Dept. Carnegie Institute of Tech. Pittsburgh 13, Pa.	PITTSBURGH October 11	E. W. Marlowe 560 S. Trenton Ave. Wilkinburgh PO Pittsburgh 21, Pa.
O. A. Steele 1506 S.W. Montgomery St. Portland 1, Ore.	PORTLAND	F. E. Miller 3122 S.E. 73 Ave. Portland 6, Ore.
A. V. Bedford RCA Laboratories Princeton, N. J.	PRINCETON	L. J. Giacoletto 9 Villa Pl. Eatontown, N. J.
K. J. Gardner 111 East Ave. Rochester 4, N. Y.	ROCHESTER September 16	Gerrard Mountjoy Stromberg-Carlson Co. 100 Carlton Rd. Rochester, N. Y.
E. S. Naschke 1073-57 St. Sacramento 16, Calif.	SACRAMENTO	W. F. Koch California Div. of Forestry Sacramento 14, Calif.
G. M. Cummings 7200 Delta Ave. Richmond Height 17, Mo.	ST. LOUIS	C. E. Harrison 4530A W. Papin St. St. Louis, Mo.
C. L. Jeffers Radio Station WOAI 514 W. Lynwood San Antonio, Texas	SAN ANTONIO	H. G. Campbell 233 Lotus Ave. San Antonio 3, Texas
C. N. Tirrell U. S. Navy Electronics Lab. San Diego 52, Calif.	SAN DIEGO October 5	S. H. Sessions U. S. Navy Electronics Lab. San Diego 52, Calif.
L. E. Reukema Elec. Eng. Department University of California Berkeley, Calif.	SAN FRANCISCO	W. R. Hewlett 395 Page Mill Rd. Palo Alto, Calif.
W. R. Hill University of Washington Seattle 5, Wash.	SEATTLE October 14	W. R. Triplett 3840—44 Ave. S.W. Seattle 6, Wash.
F. M. Deerhake 600 Oakwood St. Fayetteville, N. Y.	SYRACUSE	S. E. Clements Dept. of Electrical Eng. Syracuse University Syracuse 10, N. Y.
W. M. Stringfellow Radio Station WSPD 136 Huron St. Toledo 4, Ohio	TOLEDO	M. W. Keck 2231 Oak Grove Pla. Toledo 12, Ohio

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G. P. Adair 1833 "M" St. N.W. Washington, D. C.	WASHINGTON	H. W. Wells Dept. of Terrestrial Magnetism Carnegie Inst. of Washington Washington, D. C.
J. C. Starks Box 307 Sunbury, Pa.	WILLIAMSPORT	R. G. Petts Sylvania Electric Products, Inc. 1004 Cherry St. Montoursville, Pa.

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F. A. O. Banks 81 Troy St. Kitchener, Ont., Canada	HAMILTON (Toronto Subsection)	E. Ruse 195 Ferguson Ave., S. Hamilton, Ont., Canada
A. M. Glover RCA Victor Div. Lancaster, Pa.	LANCASTER (Philadelphia Subsection)	C. E. Burnett RCA Victor Div. Lancaster, Pa.
H. A. Wheeler Wheeler Laboratories 259–09 Northern Blvd. Great Neck, L. I., N. Y	LONG ISLAND (New York Subsection)	M. Lebenbaum† Airborne Inst. Lab. 160 Old Country Rd. Box 111 Mineola, L. I., N. Y.
L. E. Hunt Bell Telephone Labs. Deal, N. J.	MONMOUTH (New York Subsection)	G. E. Reynolds, Jr. Electronics Associates, Inc. Long Branch, N. J.
J. B. Minter Box 1 Boonton, N. J.	NORTHERN N. J. (New York Subsection)	A. W. Parkes, Jr. 47 Cobb Rd. Mountain Lakes, N. J.
A. R. Kahn Electro-Voice, Inc. Buchanan, Mich.	SOUTH BEND (Chicago Subsection)	A. M. Wiggins Electro-Voice, Inc. Buchanan, Mich.
R. M. Wainwright Elec. Eng. Department University of Illinois Urbana, Illinois	URBANA (Chicago Subsection)	M. H. Crothers Elec. Eng. Department University of Illinois Urbana, Illinois
W. A. Cole 323 Broadway Ave. Winnipeg, Manit., Canada	WINNIPEG (Toronto Subsection)	C. E. Trembley Canadian Marconi Co. Main Street Winnipeg, Manit., Canada

Books

Loran: Long Range Navigation, edited by J. A. Pierce, A. A. McKenzie, and R. H. Woodward

Published (1948) by the McGraw-Hill Book Co., 220 W. 42 St., New York 18, N. Y. 467 pages, 8-page index, xiv pages, 198 figures. 6¼×9¼. $6.00.

This book, Volume IV in the Massachusetts Institute of Technology's Radiation Laboratory series, describes the Loran system and the history of its development. Very few portions require more than an elementary knowledge of radio transmitters, receivers, and pulse techniques to be properly understood. Even though eleven authors contributed, the text appears to have an integrated continuity.

Part I deals entirely with the Loran system of navigation. Included are an introductory chapter describing the principal forms of radio navigation and other chapters on the history and principles of Loran and the propagation of the radio wavelengths involved. Chapter 4, entitled "Future Trends," seems to be somewhat out of place in an otherwise factual and objectively written treatise. A cautious, well-thought-out forecast of possible future trends might well be included in a text such as this, but in the chapter under discussion the author appears to have built one supposition upon another in an empirical fashion seldom found in a truly scientific work.

Part II, entitled "Loran Equipment," is relatively complete in its coverage and well-illustrated, but is, of necessity, brief in its description of individual devices. The 12-page bibliography will enable a reader interested in further detail to obtain more information.

Stuart W. Seeley
Radio Corporation of America
711 Fifth Ave.
New York 22, N. Y.

Microwave Receivers, edited by S. N. Van Voorhis

Published (1948) by the McGraw-Hill Book Co., Inc., 330 W. 42 St., New York 18, N. Y. 611 pages, 6 page index, xviii. 430 figures. 6¼×9¼ inches. $8.00.

The worth and extent of the Massachusetts Institute of Technology Radiation Laboratory Series of books is now well known. "Microwave Receivers," volume 23 of that series, is intended to acquaint the reader with the large amount of information gathered, and the various techniques employed, by the members of the Radiation Laboratory in the solution of the problems they encountered in designing and constructing receivers for the microwave region. It covers work done not only at the Radiation Laboratory, but at other research laboratories as well. The book begins with a discussion of noise in receivers in general and points out the extreme importance of fluctuation noise in microwave receivers. The various components that are used in such a receiver are then taken up in detail. The reader is informed of the various difficulties to be encountered in the design of such things as TR tubes, converters, local oscillators, intermediate-frequency amplifiers, detectors, etc., and with the various methods of avoiding those difficulties. The concluding chapters contain detailed descriptions of several different microwave receivers, illustrating a large variety of circuits and techniques.

In general, the various topics discussed throughout the book are presented in a clear, concise, and readable manner. However, nineteen authors participated in the preparation of this book, and, as a result, there are some repetitions. The fact that the situation is not much worse illustrates the excellent job of editing that has been done in preparing this volume. Those sections dealing with noise, noise figure, radio-frequency amplifiers, and intermediate-frequency amplifiers, are especially good.

The years which have elapsed since World War II have seen the opening of the microwave region to use not only by the communication industry but to other services as well. Since the receivers described in this book were intended primarily for radar use, the present-day microwave engineer will encounter many problems that are not discussed in this book, as well as finding some material not applicable to his work; but he will find a large amount of material that he can use directly, and from the rest he will get many helpful leads. This book is recommended to him without reservation.

Karl G. Jansky
Bell Telephone Laboratories, Inc.
Holmdel, N. J.

IRE People

Frank Rieber (M' 41–SM' 43), physicist and inventor, died recently in New York. Born in Placerville, Calif., Mr. Rieber received the B. S. degree in physics from the University of California at Berkeley in 1915. For the next eight years he worked in San Francisco, principally on X-ray equipment, inventing the first satisfactory stabilized filament control. During World War I he was a secretary of the California War Inventions Committee and a member of the Submarine Defense Commission.

In the nineteen-twenties, Mr. Rieber developed a radio altimeter system, seismograph instruments for oil search, and several new electronic and mechanical measuring devices. Moving to Los Angeles in 1930, he continued with innovations in sound track and thin plastic disk recording. He shifted his laboratories to New York in 1940. When the Second World War broke out, he became a consulting engineer on war contracts and for various agencies of the War and Navy Departments. Some of his military inventions were recording devices for airplane control towers, a magnetic submarine detector, an instrument for measuring the muzzle velocity of shells, and a method of determining by sound the location of enemy guns.

At his death Mr. Rieber had been working on a sensitive instrument for scientific and industrial measurement which he called the Vibroton. This can measure almost any physical displacement and convert the data into electrical impulses which can then be relayed to a recording apparatus or control mechanism at a remote point.

Mr. Rieber was a member of the American Institute of Physics, the American Geophysical Union, the American Meteorological Society, the Seismological Society of America, and other scientific organizations.

❖

The General Electric Company's Charles A. Coffin award—the highest honor bestowed by the Company—has been given to William F. Goetter (A'41), Henry P. Thomas (A'34–M'44–SM'47), Kenneth C. DeWalt (A'29–SM'45), and Robert B. Dome (A'27–M'38–SM'43–F'40) for work of outstanding merit during 1946 and 1947 on transmitter and broadcasting developments.

Mr. Goetter and Mr. Thomas, who design and develop radio transmitting equipment, were cited "for outstanding skill and ingenuity in designing and developing a new line of frequency-modulation broadcasting transmitters." Mr. DeWalt, a designing engineer in the tube division, won his award for transmitter-tube production accomplishments. Mr. Dome, advanced development section engineer in the receiver division, was honored for his invention of a new circuit system for communications equipment which makes possible systems much simpler and less costly than present techniques.

William R. Ahern (S'41–A'41–M'47), formerly engineer in the television equipment division of the General Electric Company, was recently appointed engineer in the facilities section of the American Broadcasting Company's engineering department in New York.

Mr. Ahern was graduated from the Worcester Polytechnic Institute in 1939 with the degree of B.S., and two years later he received the M.S. degree from the same school. In that year he joined the General Electric Company, working during the war as airborne radio engineer in the electronics section.

❖

George F. Platts (A'30–M'37–SM'43), who recently joined the Clippard Instrument Laboratory, Cincinnati, Ohio, as executive vice-president, was formerly general manager of the electric products division of the McQuay-Norris Manufacturing Company, St. Louis, Mo.

Mr. Platts is a graduate of the University of Cincinnati's College of Electrical Engineering. In June, 1941, he went on active duty with the Navy, serving as priorities officer for the Cincinnati district, with the rank of lieutenant, j.g. During the war, he was assigned to various duties in connection with the No. 2 secret weapon project, the VT (proximity fuze), and received a commendation ribbon from the Secretary of the Navy for his services. He now holds the rank of Commander in the United States Naval Reserve.

A former chairman of the Cincinnati Section, Mr. Platts was also formerly secretary-treasurer of the Cincinnati Technical and Scientific Society Council.

❖

Electrodesign, a new electronics group, has recently been formed by three IRE members—H. H. Schwartz (S'38–A'39–M'45), C. A. Rosen (A'44), and L. A. Geddes (A'47). Offices will be at 1178 Phillips Place, Montreal, Que., Canada. In addition to supplying, servicing, and modifying standard electronic equipment, the group will also design and build new equipment to meet special needs.

Mr. Schwartz is a former associate of the Northern Electric Company, and the Canadian Marconi Company. Mr. Rosen was previously with Canadian Fairchilds Engineering Department. Mr. Geddes is Consulting Engineer to the Montreal Neurological Institute.

❖

Maxwell K. Goldstein (A'30–SM'46) has recently joined the staff of the Research Divisions of the Office of Naval Research, Navy Department, Washington, D. C. He will act as Electronics Consultant to the Director of Naval Sciences.

Dr. Goldstein took his doctorate at the Johns Hopkins University in 1933, and has since had extensive experience in electronics. Before joining the Naval Research Laboratory in 1939, he was successively Radio Project Engineer for the Signal Corps Aircraft Laboratory and Advanced Radio Design Engineer for the Civil Aeronautics Authority. As Head of the Naval Research Laboratory's Radio Direction Finders Activity he received the Distinguished Civilian Service Award. He was subsequently promoted to Head of the Airborne Systems Engineering Section, and later to Head of the Navigation Section. In his new post, Dr. Goldstein will undertake important program analyses in fields associated with electronics.

❖

Bertram F. N. Israel (A'47) of the Royal Australian Air Force recently was presented by President Truman with one of this nation's highest military awards, the Medal of Freedom with bronze palm, for "meritorious services which aided the United States in the war against Japan in New Guinea."

Colonel Israel is one of Australia's leading radio engineers in civilian life, and is manager of one of the largest radio firms in that country. He also has lectured frequently on many phases of radio and its development, since he is a consulting engineer for the Commonwealth's radio system.

During the war he worked far ahead of the combat units to set up mobile aircraft warning outposts that enabled the United States to seize air superiority from the Japanese. The President's citation states that "by his efforts Squadron Leader Israel was largely responsible for the establishment of an aircraft warning system which was instrumental in wresting air superiority from the enemy and in the successful defense of New Guinea and Australia."

❖

Irving Greene (A'47), previously Manager of the Sound and Communications Department of Heins & Bolet, was recently appointed Manager of the Sound and Communications Division of Sun Radio & Electronics Company, New York, N. Y.

During the war, Mr. Greene served for three years in the Pacific in the Air Force Signal Section. On his discharge he joined the Langevin Company as test engineer, in which capacity he was responsible for extensive sound installation work.

❖

Harvey W. Smith (A '34) has joined the engineering staff of the Lenkurt Electric Co., San Carlos, Calif., where he will be in full charge of the design and construction of transformers, both for carrier application and custom manufacture.

Previous to his employment at Lenkurt, Mr. Smith had been an independent consultant, as well as chief engineer at the Peerless Electrical Products Co. and the Robert M. Hadley Co., both of Los Angeles, and the Glen-Roberts Co. at Oakland. He also has been a member of the engineering staffs of Bendix Aviation, Ltd. and Electric Research Products, Inc., Hollywood, and Audio Products Co., Burbank, Calif.

John C. Petkovsek

Chairman, Beaumont–Port Arthur Section, 1947–1948

John C. Petkovsek was born on July 13, 1921, in Beaumont, Tex. After studying at the Texas Agricultural and Mechanical College, he entered the employment of a Beaumont bank, leaving in 1941 to join the Sun Oil Co. In 1942 he enlisted in the U. S. Navy, where he completed radar training at Texas A. & M. College and at Ward Island, Corpus Christi, Tex. During the war he worked on an aviation radar project and earned a radiotelephone first class license.

Returning to the United States after several months of service overseas, Mr. Petkovsek entered the University of the South in 1943, transferring later in the same year to the Georgia School of Technology, from which he received the B.E.E. degree (communications) in 1946. In the fall of that year he was appointed electrical engineer at the Magnolia Petroleum Co.'s Beaumont Refinery. There he has participated in projects involving unusual applications of mobile radio and of complex interoffice communications.

An amateur radio enthusiast, Mr. Petkovsek operates his own station, W5KVG, and is a member and past officer of the Sabine District Amateur Radio Club. He is also a member of the ARRL and of the Texas Society of Professional Engineers.

Mr. Petkovsek first joined The Institute of Radio Engineers as a Student in 1945. In 1947 he transferred to Associate grade, and became a Member later in the same year. During the first part of 1947 he served on the Membership Committee of the Houston Section of the IRE, resigning in November, 1947, to become the first Chairman of the newly formed Beaumont-Port Arthur Section, of which he was one of the principal founders.

George P. Adair

Chairman, Washington Section, 1948–1949

George P. Adair was born on December 8, 1903, in Rancho, Tex. After receiving the B.S. degree in electrical engineering from the Texas Agricultural and Mechanical College, he joined the radio department of the General Electric Co., where he remained until 1929, when he became associated with the Straus-Bodenheimer Co. of Texas.

In 1931 Mr. Adair joined the staff of the Federal Radio Commission, transferring to the Federal Communications Commission when it was formed in 1934. Ten years later he was appointed chief engineer of the FCC, and served in that capacity until 1947, when he left to open his own offices as a consulting radio engineer.

During the war, Mr. Adair worked with the OWI in setting up the International Broadcast Service, and he was active in the Board of War Communications and related work. He also served on a number of Government Committees, and was the FCC's observer on the Radio Technical Planning Board. A U. S. delegate to the 1945 Conference on Frequencies for the Liberated Countries of Europe, Mr. Adair was also technical advisor to the second North American Regional Broadcasting Agreement Conference and to the preliminary engineering conference for the third in 1946. He holds a number of U. S. and foreign patents.

Mr. Adair is a member of the Radio Advisory Board of Stephens College, a lifetime honorary member of the International Municipal Signal Association, and an observer on the RMA Television Systems Committee. Joining the IRE as an Associate in 1942, Mr. Adair became a Senior Member in 1944 and a Fellow in 1947. He is Chairman of the Washington Section of the IRE, and vice-chairman of the Mid-Atlantic Regional Committee of the IRE.

College Research to the Aid of "Small Business"*

STANFORD C. HOOPER†, FELLOW, IRE

IN 1863, THE CONGRESS granted a charter to the Academy of Sciences, and in 1918, President Woodrow Wilson recognized its subsidiary, the National Research council, as the agency to assist the military services in research problems.

In 1934, a meeting was held in the Navy Department by request of the president of the Council to discuss the feasibility of better co-operation between the Navy and industry research through the good offices of the Council. It was stated by the officers of the Council that such co-operation might lead to greatly superior fighting power for our fleet. Dr. R. A. Millikan was chairman at that time, and Admiral Standley represented the Navy Department as Chief of Naval Operations.

I attended the meetings and within the year was assigned to duty as the first technical director of the newly created Technical Division, and chairman of the Research Co-ordinating Committee of the Navy.

During the ensuing four years, this Committee became a separate entity, and commissioned officers were assigned to exclusive duty as research officers in each bureau. The Committee assembled lists of research and development problems from the bureaus, from the fleets, and from suggestions by the public, and processed these as individual projects.

A separate research budget was arranged in each office, and research officers became active in traveling the country over to familiarize themselves with industrial and college research. I, myself, visited over one hundred laboratories during this time. I found myself becoming an index for research men and projects. Regretfully, I found that the capacity of the human brain is very limited in its ability to retain large stores of knowledge.

Among other achievements of these activities was the formation of a Scientific Reserve, and the enrollment as commissioned officers of thirty-odd college experts, one from each fundamental branch, as recommended by the National Research Council.

In cases wherein a Navy bureau was at a loss for knowledge as to how to proceed with a research problem, one of these officers was asked to prepare a study thereon. At that time, there were only two development projects arranged with colleges (except for eight handled through the Bureau of Aeronautics).

This procedure led to close ties between the Bureau and the colleges; and in four years, there were seventy-five such contracts. This was the background of the beginning of the great movement of college professional men into the Navy technical field.

At that time, the War Department had no system for co-ordination of research and development, but later adopted a similar system. Then, with the war, came the Office of Scientific Research, headed by Dr. Vannevar Bush, a university man, universally recognized as the peer of leaders in this organization; and following the Armistice there came into existence the Joint Army and Navy Research and Development Board, the child of the OSRD, still guided by much the same personnel.

Now, in 1948, broad and public recognition has been given by the Government to the vast importance of research in industry and to the value of training in research at the colleges. This viewpoint culminated in the passage of a bill in the 90th Congress to create a National Science Foundation. This original bill, somewhat revised, will likely become a statute in 1948, and will thus provide a real foundation for government encouragement of technical developments for the future.

The funds to be appropriated by Congress, in accordance with the new projected law, will be to promote research for our industry, security, and health. They will be allotted principally to colleges, to industrial laboratories, and through the military departments to their own and industrial laboratories.

A great responsibility will come to colleges with the allotment of their tasks for research. They will do much excellent work, make important discoveries, and perhaps develop apparatus of great value. By virtue of the results they will obtain from this source, they will collectively become masters of modern science, each in its own chosen field; and their research workers will be more highly qualified and renowned as teachers. Their achievements will become of value to industry, to defense, and to the betterment of our lives generally. In order to spread the results of basic research from this source, the government will encourage the scientist to publish his findings.

By far the most valuable products of colleges will be in training and teaching, which have always been their mission—training of scientific men and women ready to fit into the industrial and other noncollege laboratories as part of the life of the nation.

We will become increasingly proud of the work of these products of new college training and of the new techniques in America which they will build.

The leaders among our industries which pioneered the nation's outstanding mass-production lines are suffering from the lack of scientific personnel to push new ideas to fruition. They await the new graduates impatiently, and all of us, especially as we pass middle life, become almost desperate in our hopes for ever faster advances, particularly in the medical world.

Not only do we look to the scientific graduates to fill the needs in laboratories and factories, but there is the responsibility to formulate college policy in government appropriations for research, and the allotment of scientific manpower, in such manner that the college laboratories will not compete with business laboratories. A proper balance must be maintained with teaching, which is always the prime mission of the college. But the college laboratories should assist industry by a supply of graduates, and in close co-operation with business. Otherwise, American business will lose a strong prop of efficient research. I recommend much serious thought on this point.

From a military viewpoint, it must be realized that the time for invention and the production of new laboratory models is between wars. When war comes, it is the time for whipping together all knowledge and getting it into finished designs and quantity production overnight. Not that we should discontinue our research on outbreak of war—it should be concentrated and pushed desperately. But I venture to say that nine-tenths of the new principles and inventions which were put to practical use during Wold War II were developments listed by our small combined committees before 1942. Take, for example, radar and the atom bomb. Although there was a good deal of putting pieces together after 1942, and vast numbers of men were employed, certainly the individual chips were mostly "in the basket" before war was declared.

The radar had been tested, but many new types and improvements had to be designed. For years atoms had been smashed, and we were on the very verge of producing the chain reaction.

It would be very interesting to me to see exactly what happened as a result of our original seventy-five projects with universities, and, in fact, all of the projects our original Committee had on its list. We had only about four million dollars annually for research, for all Navy development activities at that time. And I venture to say that these problems were so well along in 1942 that these became the basis for quantity production of our top apparatus which helped win the war.

Another very important feature arising from government-supported research will be its influence on small and independent manufacturing firms, compared with the large scale or so-called "monopoly" type.

Soviet literature challenges the ability of our democratic and elective systems to maintain our competitive system against actual monopoly. The Soviet points to a number of cases where our elected representatives and officials have allegedly bowed to the will of "big business" and organized pressure groups in order to obtain re-election. We have all witnessed apparent instances of this sort of thing. It is one practice sometimes waved aside as "politics."

* Decimal classification: R010. Original manuscript received by the Institute, April 20, 1948. An editorial revision based on an address made by Admiral Hooper on February 5, 1948, at Drury College, Springfield, Mo., upon receipt of the degree of LL.D.

† U.S. Navy (Retired), Washington, D. C.

But we might well be alarmed by the progress of "big business" at the expense of "independent business," and particularly because of the increasingly frequent reports by the antitrust officials and small-business committees stating that, notwithstanding all our efforts, monopoly trends appear to be spreading.

Yet, with it all, we have profound faith in the future and know that the American public is far ahead in the enjoyment of equal rights, progress in technique, proper privileges, good health, reasonable comfort, and some luxury; in fact, very much ahead. We see what happens under the fascist and communist systems, and we know we want none of such bitter oppression and poverty.

It should be stressed that what happens to the "know-how" and patents from government-supported research and development work will mean much to the success of independent and "small-business" concerns; and we cherish the thought that we may have lots of "presidents" and "directors" in even our small towns, rather than "district representatives" and "local agents." The former terms sound democratic and call for respect, while the latter somewhat smack of regimentation and remote control.

If we can somehow use the college developments primarily to help small and independent business most, rather than "big business" (which already is strong and able to employ inventors almost without limit, and to take on all nature and size of business without much patent protection or mortgaging of its private property), we may go far in offsetting whatever undue advantage "big business" may possess. This is especially true as regards the patent of the newly developed device or system.

The "attic inventor" in a small room with one droplight, and with no money to spend in "time off," has the maximum incentive to produce. He is lying awake nights trying to get new ideas and to design a workable model. The poverty of his life is a mighty incentive which has brought into this great land of ours the myriad of inventions which have formed the background of our great production and wealth. God gives us our original ideas and zeal, and the poor and lonely are often much nearer to God.

The patent system itself is an important part of our incentive. But for the fact that we wish to make money, or become famous, or desire personal achievement, we might let such ideas pass; but no—we struggle to put them into practical form. We save, and scrape, and make a crude model, and perhaps sell the patent to the "small-business" competitor.

"Small business" is frequently more progressive and less unwieldy than "big business." It has relatively less overhead and greater skill. It has made many of the real advances. It is apt to be on the alert for new products, and more anxious to get them on the market with greater speed, possibly because it lacks the sluggish overhead and red tape of "big business." It is the *competitor* in the competitive system—our own "American Way."

Whether independent or from a university laboratory, an inventor will often take his new idea or model to the "small-business" firm before he does to "big business." He knows that the comparatively self-satisfied large concern may not be so much on the alert for new treasures as will be the small one. That, again, is competition.

The small competitor knows that he has protection through the patent granted by the United States Government, and so he or some other "small company" can put capital into it and into the machinery necessary to market the products therefrom. Accordingly, we must preserve and protect the independent inventor and, by all means, the patent system.

"Big business" sometimes intimates that the day of the independent noncompany inventor is over. It will never be over. I have heard it said that the great inventions in the present century have come from great company laboratories rather than individuals. Where they come from does not matter; it is always the individual who makes the invention, even though often several cooperate. Frequently the large laboratory makes this co-operation possible, and thus contributes to the public welfare.

In the electronic field, which I knew from its earliest days, nearly all the inventions originally came from the individual inventor. Even if the Government pays for a new development, we must not let the ownership of the patent reside other than with the individual inventor or the company for which he works, although the Government must retain certain protection for its own purposes.

Unless we do this, the patent system will disintegrate, the individual inventor will lose his incentive, and the independent and "small business" will lose its protection.

It will require grouping of small independent competitors, each in a field complementary but not competitive to others in the group, to stand successfully against powerful "big business" wherein, in one company with many divisions, anything and everything is made, and nearly unlimited funds and large-scale methods of merchandising are possible.

If in some way these groups of small companies become able to profit by obtaining a good share of the newly trained scientists and can co-operate with the colleges for patent protection of development made in college laboratories, it may be one of the greatest contributions to offset the claim that our democratic civilization cannot hold the line against excessive concentration of business activities.

"Big business," with its powerful mass production, is essential to the public welfare and to our national defense, and "small business" is needed as the vigorous competitor to guarantee the permanence of the "American Way." Well-thought-out co-operation between college and business laboratories is, therefore, particularly vital to our national well-being, in peace and war.

Correction

W. H. Huggins has called to the attention of the editors the following corrections to his paper, "Multifrequency Bunching in Reflex Klystrons," which appeared on pages 624–631 in the May, 1948, issue of the PROCEEDINGS OF THE I.R.E.:

(1) in equations (8) and (9), replace X_k by $_jX_k$.

(2) In equations (9) and 1(1), product is to be taken *only* over k different frequencies; $\prod_{k=1,2,3\ldots}$

(3) Second line after equations (28) should read "equation (28) together with (27)."

(4) First line before equations (30), " . . . (28) may be replaced," etc.

(5) Fourth line before Fig. 4, replace (30) with (33).

(6) Third line after equation (41), replace (38) with (41).

(7) In Table I, replace 0.025 with 0.625, and 1.025 with 1.625.

Technical Aspects of Experimental Public Telephone Service on Railroad Trains*

N. MONK†, AND S. B. WRIGHT†, SENIOR MEMBER, IRE

Summary—**Telephone service was made available experimentally in 1947 to passengers aboard certain railroad trains operating between New York, N. Y., and Washington, D. C. Communication between these trains and the telephone network is effected through radio stations of the urban mobile telephone services of the Bell System. This paper describes component parts of the train telephone system, results of radio coverage tests on the routes involved, and devices employed to control two-way transmission. Special features of the installations which differ from previous mobile installations and some results of the experimental operation are pointed out.**

I. Introduction

THE IDEA OF communicating with moving trains has intrigued engineers for many years. As early as 1887, an experimental two-way telegraph system employing wires paralleling the track was tried out for a brief period in the United States. Following World War I, the availability of radio enabled the railroads to experiment with telephone train communication for operational purposes, and such systems are now in daily use. An experimental passenger telephone system using wayside wires and carrier frequencies was operated in Canada in 1929, but was later abandoned.

For over 20 years the Bell System has co-operated with the railroads in the search for practical methods of providing passenger telephone service. With the advent of general mobile telephone service in the United States, means became available for initiating an experimental service of this type. Accordingly, the Bell System extended its communication network last August on an experimental basis to include travelers on certain railroad trains operating between New York, N. Y., and Washington, D. C. The purpose of this experimental telephone service is to obtain practical experience, results of which will be available for consideration in connection with future planning of methods and facilities required to meet expanding public needs.

Radio frequencies near the center of the vhf range—above about 100 Mc—are especially attractive for communicating with moving trains for the same reasons that make them useful for general mobile telephony. These frequencies are transmitted effectively over land to distances somewhat beyond "line-of-sight," yet they are not plagued by sporadic long-distance transmission experienced at lower vhf frequencies. The availability of a succession of urban mobile telephone systems between New York and Washington, operating at frequencies in the 152- to 162-Mc range, favored the use of these frequencies for the initial experiments. The train installations are thus mobile units of urban mobile telephone systems in the several areas through which the trains are hauled. There are, however, important differences between the train installations and those normally made in automobiles. One of these is the elimination of the "push to-talk" feature, so that passengers use the telephone in normal fashion.

Further experiments are planned in connection with highway mobile telephone systems operating at frequencies in the 30- to 44-Mc range, looking toward the possibility of using these systems for railroad telephone service. Other possibilities for such communication include carrier frequencies below about 200 kc between inductors on the train and wayside conductors (inductive system).

II. System Components

The routes selected for the initial service trials are those of the Baltimore & Ohio Railroad and the Pennsylvania Railroad between New York and Washington. At the New York end, the service area ends just west of the Hudson River, since the B&O train terminal is at Jersey City, and the Pennsylvania tracks are carried under the Hudson in tunnels which act as shields to radio transmission.

The separate routes of the two railroads differ somewhat in the type of terrain traversed and in obstructions such as cuts and intermediate tunnels. Also, overhead wires are used for electrical propulsion over the Pennsylvania route, while the B&O route is, for the most part, clear of such wires. Thus, a variety of conditions is encountered.

Three railroad cars are equipped for service. One is carried on the north- and southbound Royal Blue train of the B&O. Another is normally on the Pennsylvania's southbound Speaker and northbound Congressional, and the third is carried on the Pennsylvania's southbound Congressional and northbound President.

Parts of the two rail routes are in working range of the following urban mobile telephone systems[1] operated by the telephone companies indicated.

Urban Mobile System	*Telephone Company*
Newark, N. J.	New Jersey Bell Telephone Company
Philadelphia, Pa.	Bell Telephone Company of Pennsylvania
Baltimore, Md.	The Chesapeake & Potomac Telephone company of Baltimore City
Washington, D. C.	The Chesapeake & Potomac Telephone Company

* Decimal classification: R533. Original manuscript received by the Institute, February 11, 1948; presented, 1948 National IRE Convention, March 22, 1948, New York, N. Y.

† Bell Telephone Laboratories, Inc., New York 14, N. Y.

[1] H. I. Romnes and R. R. O'Connor, "General mobile telephone system," presented, AIEE Midwest General Meeting, Chicago, Ill., November 6, 1947.

The general arrangement of facilities for each of these systems is shown schematically in Fig. 1. Connections are made from a land telephone through the local telephone exchange and over a trunk circuit to a mobile-service operator who has access to the urban mobile circuit. This begins with a control terminal having branch lines to a 250-watt phase-modulated radio transmitter, and to several fixed or land receivers located at points suitable for reception from the 20-watt mobile transmitters. Transmission in the two directions is at radio frequencies which differ by approximately 5 Mc. This frequency separation, together with the geographical separation between the land transmitting and receiving antennas, is sufficient to prevent the land receivers from being affected by the land transmitter. On the train, where the separation between transmitting and receiving antennas is limited to less than a car-length, a double-tuned coaxial stub filter is inserted between the receiving antenna and the receiver to suppress transmission from the associated transmitter. The train equipment also includes the necessary control and telephone signaling equipment. Selective calling is provided, since many mobile units normally share the use of a given pair of frequencies.

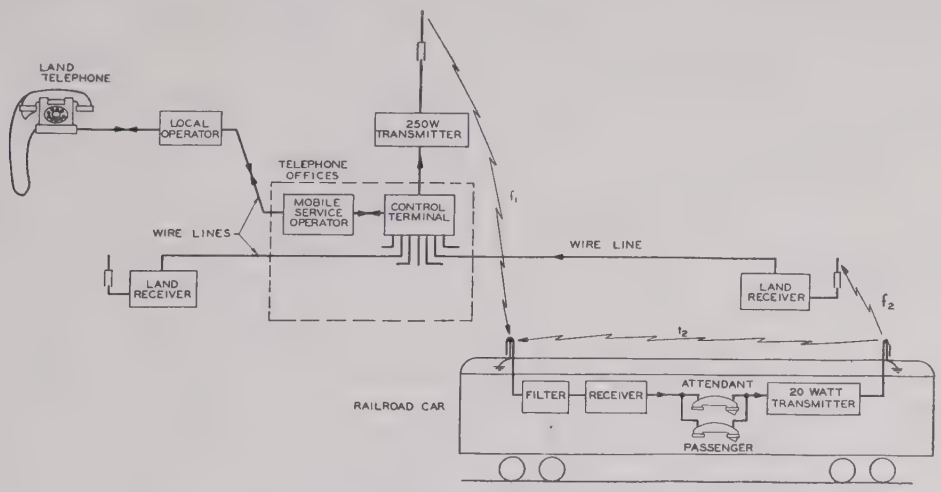

Fig. 1—Diagram showing system components.

III. Radio Coverage

Test runs were made over both routes in both directions to determine the coverage provided by the existing urban mobile systems. The tests included measurements of radio noise, transmitted frequency and power, received carrier intensity, and judgment of telephone speech transmission by experienced observers. Some tests were also made of selective signaling from the control terminals to the trains.

The method used to measure radio noise on the train employs a receiver and standard audio-frequency circuit noise meter which have been calibrated previously by a carrier input suitably modulated by speech signals. Carrier from a local oscillator in place of carrier from the distant land transmitter is supplied to the antenna exposed to the noise, and the measurement consists of determining the amount of this carrier power which, from the calibration, is known to provide a just-satisfactory speech-to-noise ratio. This amount of radio power is referred to as "required carrier."

In general, the radio noise measured on the train was less on the B&O route than on that of the Pennsylvania, and over the former route was at nearly all times at, or close to, set noise. In nonindustrial areas the noise on the Pennsylvania was about 10 db higher, and consisted mainly of spurts of sharp crackle lasting from three to ten seconds, perhaps caused by variations of the contact between the overhead wire and the pantograph of the electric locomotive. Occasionally, also, a few spurts of noise were detected while passing electric trains. On the Pennsylvania route, noise in the order of 20 db above set noise was occasionally noted in the industrial areas such as Newark and Philadelphia. This was of no consequence, however, since the train is close to the land transmitters when in these areas, and the carrier input to the receiver is consequently well above the required value.

An example of the received carrier intensity from the Philadelphia land transmitter compared to the required value as measured on the Pennsylvania route in the vicinity of Philadelphia is shown in Fig. 2. From this figure it may be seen that satisfactory transmission can be obtained with a carrier power at the receiver input of approximately 130 db below 1 watt, which permits a range for commercial telephone service of approximately 35 track miles north and over 40 track miles south of the Philadelphia Thirtieth Street Station. It should be noted, however, that these figures are for the direction of transmission from land to train. To obtain the range for two-way service, the train-to-land transmission must be considered, as well. This may differ from that in the other direction, since the fixed transmitting and receiving antennas are at different sites and elevations. Also,

the fixed and mobile transmitting powers differ, and the radio noise or required carrier on the train may differ from that at the land receiver.

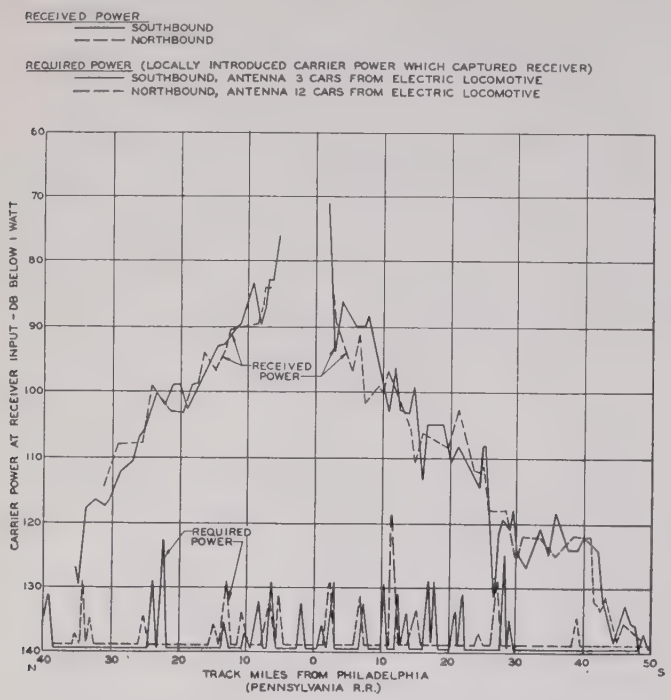
RECEIVED POWER
SOUTHBOUND
NORTHBOUND
REQUIRED POWER (LOCALLY INTRODUCED CARRIER POWER WHICH CAPTURED RECEIVER)
SOUTHBOUND, ANTENNA 3 CARS FROM ELECTRIC LOCOMOTIVE
NORTHBOUND, ANTENNA 12 CARS FROM ELECTRIC LOCOMOTIVE
CARRIER POWER AT RECEIVER INPUT - DB BELOW 1 WATT
RECEIVED POWER
REQUIRED POWER
TRACK MILES FROM PHILADELPHIA (PENNSYLVANIA R.R.)
N
S

Fig. 2—Field intensity versus distance.

The telephone speech tests showed generally commercial grades of transmission within fairly definite distances of the land stations, beyond which noise and "flutter" caused by fluctuations in received carrier intensity rapidly reduced the usefulness of the channel. These carrier fluctuations were observed on both routes, but were somewhat more pronounced on the B&O route, probably due to the more hilly character of the terrain traversed by this railroad, The results of the speech tests are indicated in Fig. 3, which shows the approximate coverage obtained on each railroad. On each road satisfactory two-way telephone conversations were obtained over about 70 per cent of the route mileage.

It will be noted from Fig. 3 that the rail distances from the approximate coverage boundary to the principal railroad station associated with each land transmitter vary considerably, from a minimum of about 7 miles on the Pennsylvania out of Washington to a maximum of over 50 miles on the B&O out of Jersey City. The explanation of the wide differences in range may be obtained from detailed consideration of the rail routes in relation to the location of the fixed radio transmitters and receivers and their respective antennas, particularly the transmitting antennas. The Newark and Philadelphia transmitting antennas are located on buildings about 400 feet above street level, having commanding views of the rural sections of the rail routes. The land transmitting antennas at Baltimore and Washington are much less favorably located with respect to these rail routes, since they were located to minimize mutual cochannel interference between the Baltimore and Washington urban mobile systems. In addition, the presence of hills and railroad cuts in these areas further reduces the range.

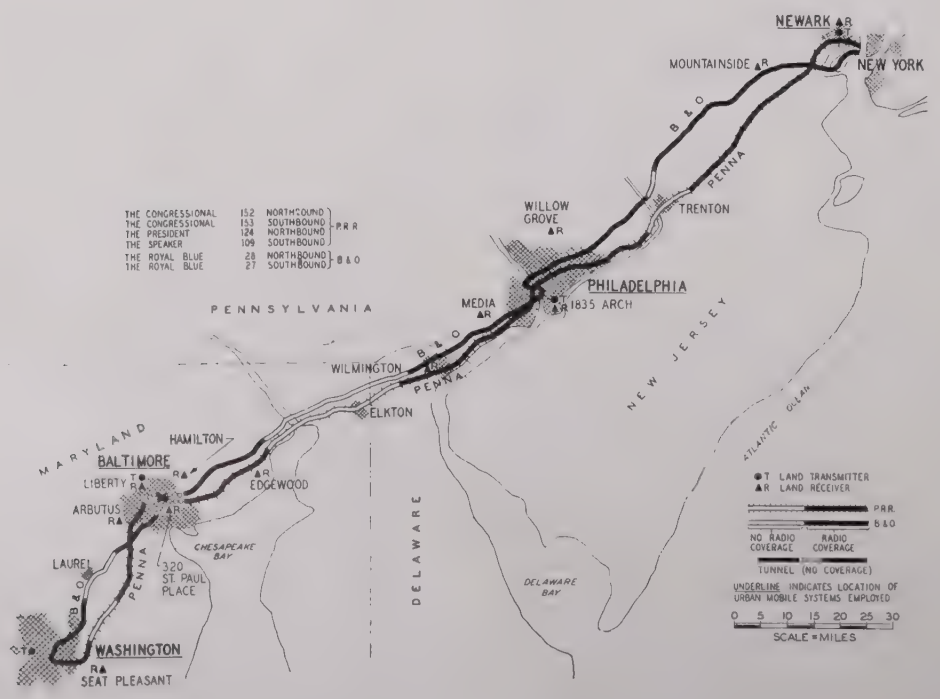

Fig. 3—Map showing coverage for B&O and Pennsylvania railroads.

In addition to the talking tests, dialing tests from the land stations to the train were made in a number of instances to insure that the train could be signaled when in talking range of a land station. It was found that the dialing range substantially equalled or exceeded the trainward talking range.

IV. Connection to Telephone Network

Facilities for combining the transmitting, receiving, and two-wire telephone and signaling paths are included in the control terminals as shown in Fig. 1. Each terminal is arranged for substantially automatic operation, but a licensed radio operator is available to supervise, test, and control the system.

In order to compensate for various line losses between the land telephone and the land transmitter, and also to some extent for variations in talking volume, the terminal includes a vogad[2] (voice-operated gain-adjusting device) in the transmitting branch. The function of the vogad is to provide substantially full modulation of the land transmitter on outgoing speech at all times. For selective calling[3] the outputs of the signaling oscillators are fed to the transmitter at the output of the vogad. The arrangement of the junction of paths in the terminal is shown in Fig. 4.

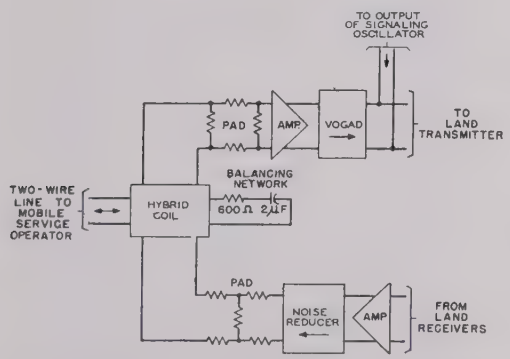

Fig. 4—Mobile system terminal for railroad trials.

The radio transmitter in a conventional mobile telephone installation is controlled by a "push-to-talk" switch operated by the subscriber. This arrangement prevents interference from the transmitter into the associated mobile receiver at times when reception from a distant land transmitter is desired. On calls between a mobile unit and a land telephone, the mobile subscriber listens to the land talker with the switch released, and the land subscriber listens to the mobile talker when it is operated. The junction of the 2-wire transmitting and receiving branches at the land terminal is normally arranged with a three-way combining pad, so that speech from the mobile talker is amplified by the vogad and radiated by the land transmitter as an indication to other mobile units that a call is in progress and the "party line" is therefore in a "busy" condition. The vogad gain changes back and forth from a value determined by the mobile talker's speech to another value which is generally higher as determined by speech arriving over the land line. During intervals when the "push-to-talk" switch in the mobile unit is released, the absence of carrier at the land receiver inputs results in muting their audio outputs by action of their "squelch" devices, so that the vogad gain adjustment and consequent modulation of the land transmitter is entirely by energy in the land portion of the connection.

In full duplex operation, a "push-to-talk" switch is not required, as the mobile transmitter and receiver are arranged to operate simultaneously, and the mobile listener is permitted to hear the output of the land receiver by reradiation from the land transmitter at all times. Moreover, since carrier from the mobile transmitter is applied to the land receivers continually, their outputs are not "squelched" and their "set noise" is transmitted through the combining pad, amplified by the vogad, and transmitted to the mobile listener. Such noise increases as the land talker's speech volume decreases, since the vogad gain increases to compensate for weak land speech.

To overcome this tendency, it is necessary to weaken the transmission between the land receiver and the vogad, relative to that between the land talker and the vogad. As indicated in Fig. 4, a three-winding transformer or hybrid coil is arranged so that energy in the receiving branch divides between the 2-wire line and a balancing network, only a small portion being transmitted to the vogad input. The effectiveness of this device depends on the degree of balance between the network and line impedances. In practice, a compromise balance allows enough reradiation for "busy" indications and for making calls between "push-to-talk" mobile units, and at the same time prevents any large reduction of range trainward by reradiated received noise.

By providing in the receiving branch of these terminals a device known as a "noise reducer,"[4] a further reduction of incoming noise is effected. This device operates to diminish the noise in the intervals between speech sounds. Thus transmission is benefited landward, and also trainward, whenever received noise reaching the vogad would otherwise be strong enough to be objectionable.

V. Description of Train Equipment

A train installation is illustrated schematically in Fig. 5. It includes two antennas, a coaxial filter, two radio transmitters, two radio receivers, a relay panel, a fuse panel, an attendant's control unit, two telephone sets, and suitable interunit cabling.

[2] S. B. Wright, S. Doba, and A. C. Dickieson "A vogad for radiotelephone circuits," Proc. I.R.E., vol. 27, pp. 254–257; April, 1939.

[3] C. N. Anderson and H. M. Pruden, "Coastal harbor radiotelephone system," Proc. I.R.E., vol. 27, pp. 245–253; April, 1939.

[4] C. C. Taylor, "Radiotelephone noise reduction by voice control at receiver," *Elec. Eng.*, vol. 56, pp. 971–974; August, 1937.

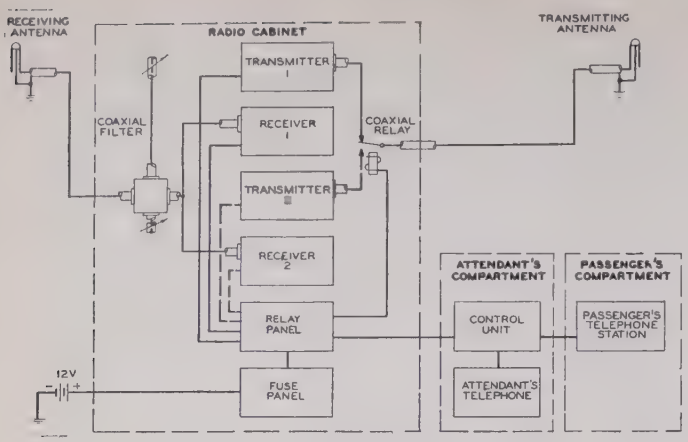

Fig. 5—Schematic diagram of train installation.

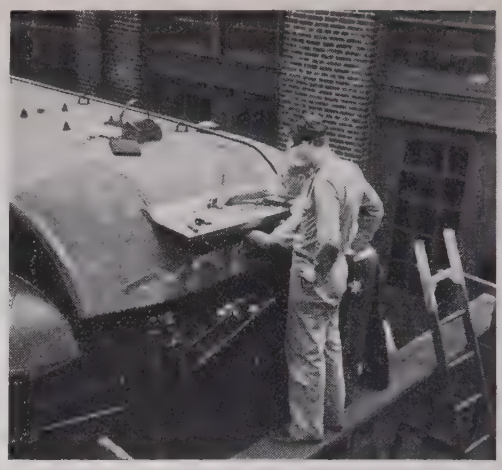

Fig. 6—Installing antenna on railroad car.

The two antennas are mounted at opposite ends of the car to provide for maximum separation between the transmitting and receiving circuits. These antennas are type 132-200 supplied by the American Phenolic Corporation, and are specially designed for railroad operation. They are strong enough to provide a handhold for train crewmen, and all external portions are solidly grounded to the car roof to minimize the hazard from accidental contact with high-voltage wires. Although each antenna is only $14\frac{1}{4}$ inches in height, tests have indicated that, from the radiation standpoint, they are substantially equivalent to a quarter-wave whip which, at the frequencies employed, would be in the order of 19 inches. This reduced height is essential to meet the railroad clearance requirements.

On the B&O trains the clearance permitted the mounting of the antennas on the center line of the flat portion of the roof of the cars. When so mounted the metal roof of the car provides the ground plane. About a 60-foot separation between antennas was obtained. On the Pennsylvania it was necessary to mount the antennas somewhat below the top of the roof. This was accomplished by installing a metal plate 36 by 18 inches on the rounded portion of each end of the roof of the car at the proper height, to provide the necessary clearance when the antenna was fastened to the plate. The antenna mounting arrangement is indicated in Fig. 6, which shows one of the antennas being installed. Although the antennas in this case project only $6\frac{3}{8}$ inches above the tops of the cars, no reduction in transmission efficiency was noted.

The antennas are connected to the radio sets by armored vinyl-covered polyethylene-insulated flexible coaxial lines 0.405 inch in diameter (RG/10U). These lines are carried for protection in $\frac{3}{4}$-inch iron conduit which is fastened to the roof of the car and terminated at the radio equipment. Parts of the lead-in conduits are visible in Fig. 6.

For simultaneous operation of the radio transmitter and receiver at the assigned radio frequencies, a loss of approximately 40 db at the nominal transmitter frequency was found necessary between the output of the mobile transmitter and the input to the mobile receiver. Since the loss in the air path between the two antennas was found to be 35 db without overhead wires, and 25 db under the Pennsylvania catenary, a filter for further suppressing the transmitter frequency is inserted between the receiving coaxial lead-in and the receiver input. This filter provides up to 40-db loss to the transmitter frequency, and only about 0.5-db loss to the receiver frequency.

The filter consists of two coaxial stubs connected to the transmission line between the receiving antenna and the receiver. One of the stubs is open-ended and approximately three-quarters of a wavelength long at the transmitter frequency f_2, i.e., $\frac{3}{4}\lambda_2$ where λ_2 is the wavelength corresponding to f_2. This stub provides a high loss to frequency f_2. It also provides a substantial loss to the receiver frequency f_1, which is about 3 per cent lower than the transmitter frequency f_2. In order to reduce this latter undesired loss, a second stub is connected to the transmission line at the same point as the first stub. This second stub is short-circuited at the end and proportioned so as to annul, at the frequency f_1, the reactance shunted across the transmission line at this frequency by the $\frac{3}{4}\lambda_2$ stub. This second stub is quite short, its length being approximately $\frac{3}{4}(\lambda_1-\lambda_2)$. For operation with either of two combinations of transmitters and receivers a compromise filter design was necessary. Thus, the frequencies f_1 and f_2 referred to above are approximately midway between the frequencies of the two receivers and the frequencies of the two transmitters, respectively. This compromise did not seriously affect the operation of the filter, however, since the two frequencies differ by only 240 kc in each case.

The lengths of both stubs are adjustable over a small range to obtain maximum efficiency. They are constructed from copper tubing, the outer conductor being $1\frac{5}{8}$ inches in diameter. External connections are made by means of standard uhf-type coaxial connectors at a junction box which is constructed as a part of the filter.

The radio equipment consists of two Western Electric 38C transmitters and two 38A receivers. Two transmitters and receivers are employed to provide for operation at either of two pairs of radio frequencies that may be available at the several land stations. These equipments are the same type as used in the automobiles served by urban mobile systems, and have been described elsewhere.[1] Each transmitter and receiver operates from a 12-volt source, as described below. The transmitter has a radio-frequency output of about 20 watts.

The train equipment is controlled locally by means of an attendant's control unit and associated relay panel. The former is located at the attendant's position and contains the necessary keys, buttons, and lamps for operating purposes. The relay panel is mounted with the radio equipment and contains the several relays required for turning the power on and off and changing the channels remotely from the attendant's control unit. All connections to the relay panel are made by means of plug-in connectors, so that the complete unit may be readily removed and replaced.

The control unit, a photograph of which is shown in Fig. 7, is used by the attendant on the train in setting up and receiving calls. It includes a lock-type power on-off

Fig. 7—Attendant's control unit.

switch, a channel-selecting key, and a switch which permits the attendant to communicate with the mobile service operator, switch the radio connection to the passenger's telephone, or talk directly with the passenger. There are also included several buttons and supervisory lamps. The lamp in the upper left corner lights when power is supplied to the equipment. The lamps marked Channel 1 and Channel 2 indicate incoming calls, and the lamp in the upper right corner shows the condition of the passenger's telephone set, i.e., whether the receiver is off or on the hook. The button on the lower left of the unit operates the transmitter, which is then locked on under control of the attendant's and passenger's telephone hook switches, both of which must be hung up to turn the transmitter off. The button on the lower right energizes a buzzer in the passenger's telephone set to signal the passenger when the attendant wishes the passenger to use the telephone.

Both the attendant and passengers' telephone sets are of a new wall-type construction recently standardized for Bell System use. For use on the train these sets are suitably modified for 4-wire operation. The attendant's telephone includes two small bells, one associated with each channel, for receiving incoming signals. The passenger's telephone is mounted, as shown in Fig. 8, in a small booth or compartment to reduce room noise and insure privacy. It contains the buzzer which is

Fig. 8—Telephone facilities installed on train.

operated by the attendant from the control unit to signal the passenger. Connections between the relay panel and the control unit, and between the control unit and the two telephones, are by standard multiconductor interior wiring cables.

Power for the radio sets and the miscellaneous equipment is obtained from a 12-volt storage battery which is supplied by the railroad. There are two such batteries on each train. One of these is arranged to be connected to the radio equipment while the other is being charged from the regular 32-volt car battery through suitable voltage-dropping, regulating, and protective equipment. The 12-volt supply is connected to the radio equipment through a power switch and fusetrons, both of which are mounted on the fuze panel.

The equipment, with the exception of the attendant's control unit, is mounted in a small compartment or cabinet. The several units are arranged vertically one above the other to conserve floor space. No shock mounting is provided for the radio sets. Fig. 9 shows the apparatus cabinet employed on one of the Pennsylvania trains. The upper two units are the Channel 1 radio transmitter and receiver, respectively. Below these are the corresponding units for Channel 2. The relay panel is mounted below the lower receiver and the fuze panel

Fig. 9—Radio equipment installed on train.

at the bottom immediately below the relay panel. The filter, which does not show in Fig. 9, occupies the vertical space in one corner of the cabinet. Except for the power leads, all interconnecting cabling is terminated in plug-in connectors to permit quick replacement of any unit. The compartment is ventilated by means of holes at the top and bottom which permit a continuous circulation of air. It is normally locked to prevent tampering with the equipment.

VI. Operation of System

Each car is assigned a telephone number with its associated code in the selective calling system, and the mobile service operators at the various locations are provided with information regarding the scheduled times of trains when the associated receivers having these codes would normally be able to receive calls. On a call to a train, the operator dials the designated number which rings a bell in the attendant's telephone set and operates a visual signal associated with the attendant's control unit. Upon receipt of these signals, the attendant picks up a handset and depresses a button on the control unit to start the train transmitter. The attendant next secures the name of the called passenger who (after being located) takes the call at the passenger's telephone. The attendant, after monitoring the connection to insure that the call is progressing satisfactorily, then hangs up. As soon as both telephones are restored to their cradles, the train transmitter is turned off and the system is ready for another call.

On a call originating on the train, the attendant selects the proper channel and monitors to find out if the channel is clear. If they are in range of a land system and no sounds are heard in the receiver, the attendant depresses the "transmitter-on" button which turns on the train carrier and signals the mobile-service operator. When this operator answers, the land-transmitter carrier is turned on, and the attendant may talk with the operator and ask for a number in the telephone network. When the call is ready, the attendant signals the passenger in the booth, who then takes the call. When the connection is satisfactorily made, the attendant's telephone is restored to its hook. When the call is ended, the attendant must obtain from the mobile service operator information as to the amount of the charges due and collect from the passenger.

VII. Summary of Results

The experience in providing experimental telephone service on railroad trains through a series of urban mobile radio telephone systems involves some special technical considerations. These are:

1. Antennas employed on railroad cars are somewhat more elaborate than the simple type of whip antenna used on automobiles.
2. Additional equipment in both the mobile unit and the control terminal of the urban mobile system is required to enable a passenger to use the telephone in the normal fashion, instead of operating a push-to-talk switch.
3. Special power-supply arrangements to convert the 32-volt railroad-car plants to the normal automobile-battery voltage are necessary.
4. A number of radio units at different locations are required to operate as a single system.
5. Due to the limitations of train speed, a limited time is available for making calls in a given area.
6. The repetitive character of the location of the mobile unit causes systems abnormalities to become more readily apparent than when calls are made from random locations at random times.
7. Servicing must be accomplished at terminals during the night and the short day lay-overs, while an automobile may be driven to a centralized repair shop.

VIII. Conclusions

While it is still too early to predict the extent of public railroad-train telephone service, the results of the first months of experimental operation have indicated general public acceptance of the service with generally favorable reactions from all concerned.

Acknowledgment

In conclusion, the authors wish to acknowledge the helpful co-operation of the communication and other personnel of the B&O and Pennsylvania Railroads for their assistance. Also, the co-operation of the telephone company personnel and of the writers' colleagues at the Bell Laboratories, who spent many hours on test and design work, is gratefully acknowledged.

Television Antenna and RF Distribution Systems for Apartment Houses*

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Summary—**The requirements of television antenna and rf distribution systems for apartment houses are discussed. Amplifiers and distribution networks are described which supply boosted signals of all available stations on their original carriers to all receivers in an apartment house.**

A LARGE PROPORTION of television receivers is located in city apartments where they are not easily supplied with satisfactory signals from all television stations of the area. Most commercial receivers require an input signal amplitude of the order of 1 millivolt, with interference, such as due to diathermy, at least 30 db weaker to be imperceptible. A type of interference, specific to television and particularly difficult to avoid, is due to signals from the desired station, reflected on distant structures such as tall buildings, bridges, gas tanks, and the like, and then arriving at the receiver delayed by 1 microsecond for each 1000 feet detour. Such additional signals produce additional images, called "ghosts," superimposed upon the main image but displaced to the right (with present U.S. television standards) by approximately 1/60 of the picture width for each 1000 feet detour.[1]

An indoor antenna seldom yields satisfactory signals from more than one or two stations, even if it is relocated and reoriented for each. The indoor field strength is found[2] to vary considerably from point to point and to change with movement of people, or objects such as elevators.

The installation of a roof antenna, preferably facing the transmitters, is usually necessary; yet, in general, this encounters several difficulties, as follows:

1. Landlords object to having their roofs obstructed by ubiquitous private antennas, even if properly installed.

2. There is seldom space enough on a roof to install antennas for more than a small minority of the receivers without excessive interaction.

3. Even with free choice, a single location is seldom found where an antenna will yield sufficiently strong signals, yet without appreciable "ghosts," from all stations of the area. Experience shows that for adequate television reception a number of separate antennas is needed, preferably each tuned, located, and oriented to receive a particular station; it is obvious that there is seldom space for more than one such set of antennas available on the roof of an apartment house.

It follows that the benefits of such a set of antennas should be shared by all receivers in the building, by means of a distribution system. The system here described is designed for this purpose. It serves to supply simultaneously all receivers of an apartment building, each with strong and clear signals of all television stations of the area, as if received by an individual ideal antenna. The signals are thus distributed on their original carrier frequencies and suitable for any type of commercial receiver without modification to either the receiver or the system. It will be found that the cost of such a system for each of at least twenty participants compares favorably with that of a simple individual antenna installation.

Over-All Requirements and General Layout

To supply from twenty to, perhaps, one hundred television receivers, with sufficient attenuation between any two of them to prevent interaction, will, in general, require the installation of preamplifiers. For ease of installation and maintenance, these should be centrally located. They should have enough gain to supply from even the weakest usable signal, of the order of 25 microvolts, the power required to supply to each receiver signals of the order of 1 millivolt. Since the channels now assigned to television broadcasting cover much of the frequency range from 44 to 216 Mc, no single wide-band amplifier for this frequency range is at present practicable. Thus, a separate amplifier is provided for each television station of the area, as much as seven amplifiers for the seven stations planned for large cities such as New York. Since FM broadcasting in the band from 88 to 106 Mc also benefits from such a system, it will often be desired to provide one more amplifier to cover the FM band, and to supply signals of the FM stations over the distribution system as well.

Since switching at the outlets should not be necessary, it is preferable to distribute all signals over a common cable, and indeed the relatively high costs of cable installation make it imperative to design the system so as to require the fewest, and shortest, cables possible. In the system here described all outlets are connected as taps on two branches of a single distribution cable.

The general layout of a typical installation is shown in Fig. 1, with an antenna and an amplifier each provided for the television channels 2, 4, and 5, as well as

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[1] A. B. DuMont and T. T. Goldsmith, Jr., "Television broadcast coverage," PROC. I.R.E., vol. 32, pp. 192–204; April, 1944.

[2] G. L. Beers, "Design of experimental television receivers," PROC. I.R.E., vol. 21, pp. 1692–1706, section on antennas and Fig. 4; December, 1933.

for the FM band. Their output is fed via a coaxial distribution cable to outlets for receivers marked *R*. The installation may be discussed under three headings: (1) the antennas; (2) the amplifiers; and (3) the distribution network.

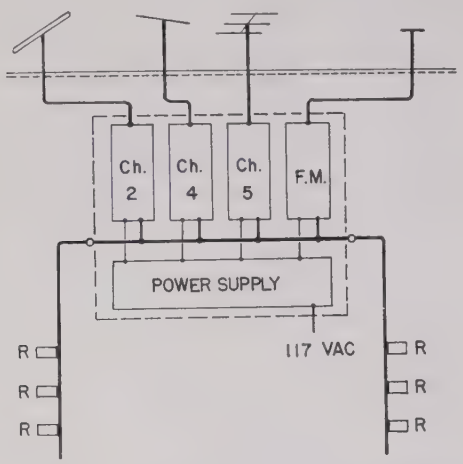

Fig. 1—A distribution system with antennas and amplifiers for three television channels and FM broadcasting.

Antennas

No detailed discussion of antennas will be necessary. As a rule, a separate antenna is provided for each television channel, located, tuned, and oriented for best reception of one station. Such an antenna may be a simple dipole, or a highly directive multielement array, if the need justifies such expenditure of space and material. Ample bandwidth and proper match to the cable are easily achieved for the 6-Mc frequency range of a single channel. No directivity is needed for the reception of FM broadcasting. All antennas are connected by matched coaxial cables to the central amplifiers.

Amplifiers

All amplifiers with their power supply and output network are housed in a single cabinet, preferably mounted near the roof inside the building. Fed from the 117-volt power line, they are left to run continuously, unattended except for occasional service checks. Each television amplifier must be capable of at least a 60-db gain, since it may operate with input signals as low as 25 microvolts and yet should supply approximately 30 millivolts to the output network, in order to allow for adequate attenuation in each outlet. Such high gain must be maintained stably over long periods, with very conservative amplitude and phase response over the whole 6-Mc channel, yet with considerable attenuation for all other channels. Such selectivity requires six or more tuned circuits, even where stronger input signals would permit use of low-gain amplifiers.

With the relatively poor selectivity of antennas and their sometimes precarious orientation, so as to dodge "ghost" signals, it is not uncommon that the signals of some other station at the input of an amplifier are stronger than the desired signals, yet appreciable amplification of the undesired signals leads to twofold trouble.

1. Amplification of a signal through other than its own amplifier will generally be subject to a different time delay, thus giving rise to multiple images similar to "ghosts."

2. The output power of each amplifier is mainly limited by the nonlinearity of its output tube characteristic. At satisfactory power output, even weak signals of other stations may produce perceptible crossmodulation at the grid of the output tube. Effects of slight crossmodulation are the granulated appearance of normally smooth background and the appearance of the spurious line synchronizing pulse as a dark vertical bar weaving across the image, somewhat like a windshield wiper.

For ease of servicing, each amplifier strip is built as a small plug-in unit. Fig. 2 shows such a unit, with two knife contacts at the input end, and six knife contacts at the other end for supply voltages and for the signal

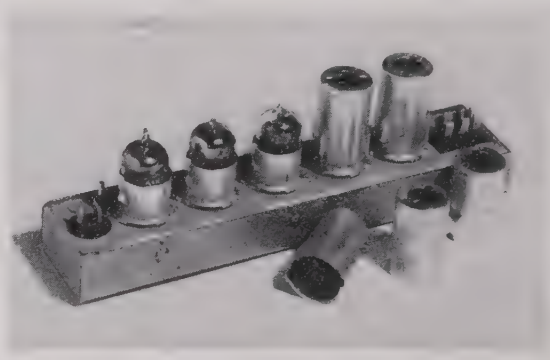

Fig. 2—Five-tube plug-in amplifier for a television channel.

output. Amplifiers may have five or seven miniature tubes, with all coupling circuits pretuned at the factory for one channel. Except for their tuning, amplifiers for all channels are alike. Their input circuit is shown in

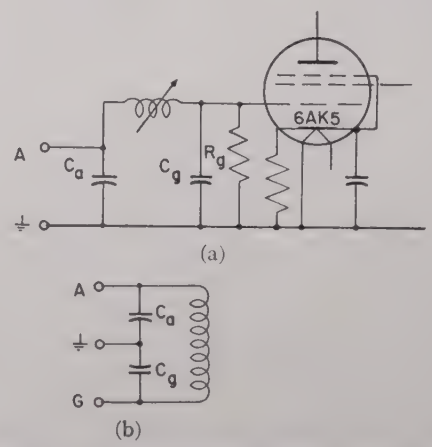

Fig. 3—(a) Amplifier input circuit, and (b) its equivalent network.

Fig. 3(a); with a tube input capacitance C_g of the order of 6 $\mu\mu$f a voltage step-up in excess of 4:1 is easily procured over very ample bandwidth. Thus the signal-to-

noise ratio of such an input stage is excellent in spite of the distribution noise originating in the pentode tube. The value of the antenna-coupling capacitor C_a is chosen so as to terminate the 52-ohm impedance of the antenna cable properly with the transformed grid loading impedance R_g, as is easily understood from the resonant-circuit diagram, Fig. 3(b). The relatively large value of the capacitance C_a allows convenient adjustments to absorb the incidental capacitances at the input connector. The tuning coil, as are all others, is space wound on a form of 0.156-inch diameter with 0.125-inch diameter bore for a 0.120-inch low-μ iron tuning core.

Except for the input and output stage, all amplifier stages are stagger-tuned at approximately ± 3.5 Mc. All tubes, with the possible exception of the output tube, are miniature pentodes of the type 6AK5, fed with screen-grid voltages of approximately 105 volts and anode voltages of approximately 95 volts. The control grid bias of the first two or three stages is adjustable for

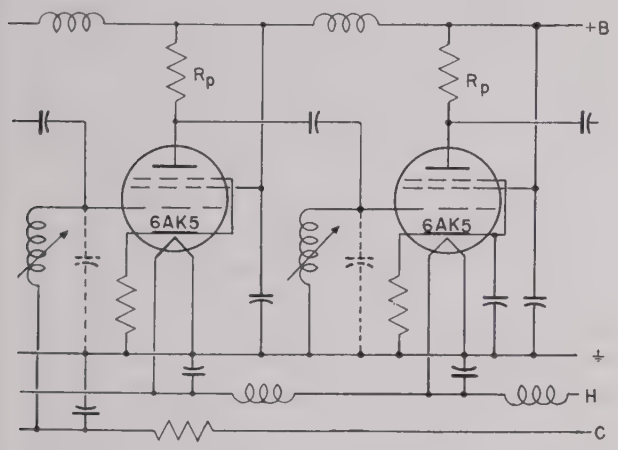

Fig. 4—Circuit diagram of typical amplifier stages; the first is gain-controlled.

gain control by a preset potentiometer in the common power-supply chassis. Degenerative resistors of the order of 30 ohms in the cathode lead of each gain-controlled tube serve to minimize change of input capacitance, and thus change of tuning, with change of gain. The first of the two stages in the circuit diagram of Fig. 4 shows the components of a gain-controlled stage, with only the tube and wiring capacitances used for tuning with the iron-core-tuned inductance, and with the plate resistor R_p chosen for proper circuit damping. There are low-Q chokes in one heater lead and in the anode supply lead and various by-pass capacitors, each of the order of 500 $\mu\mu$f. The very similar stage shown second in Fig. 4 is not gain-controlled, its control-grid bias being determined by the cathode-bias resistor.

In a high-gain wide-band amplifier that must operate stably at frequencies above 200 Mc, rf by-pass capacitors are conspicuous by their number. Small tubular silvered-ceramic capacitors have been used but their leads must be kept extremely short, so that their wiring to a miniature tube socket requires much care and the wiring of other components is obstructed. Moreover, the uncertainty of just where the wire leads make contact with the silver coating appears to result in uneven rf by-passing by seemingly identical capacitors. Fig. 5 is a photograph of two fixed-gain amplifier stages, each using a multiple by-pass capacitor developed for the purpose. The two triple by-pass capacitors may be recognized as the short white-painted cylinders surrounding the grounded center-shield pins of the seven-pin miniature tube sockets.

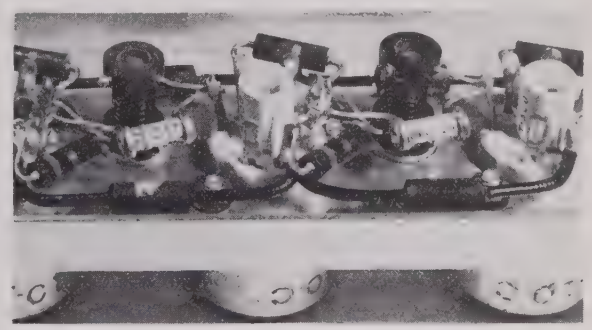

Fig. 5—Two amplifier stages with ceramic triple by-pass capacitors.

Each by-pass unit consists of a ceramic tube of high dielectric constant, $\frac{5}{16}$ inch long, i.e., as long as the lugs of the tube socket with an outside diameter of $\frac{1}{4}$ inch and a wall thickness of 0.025 inch. The whole inside surface of this tube is silvered to form the grounded electrode; it is connected, by a tinned-copper tape 0.045×0.025 inch around the bottom edge to the grounded heater lug No. 4. The outside of the ceramic cylinder carries three separate patches of silver, each connected by a radial lead of tinned copper tape to one of the three by-passed lugs Nos. 2, 3, and 6 of the tube socket. The advantages of

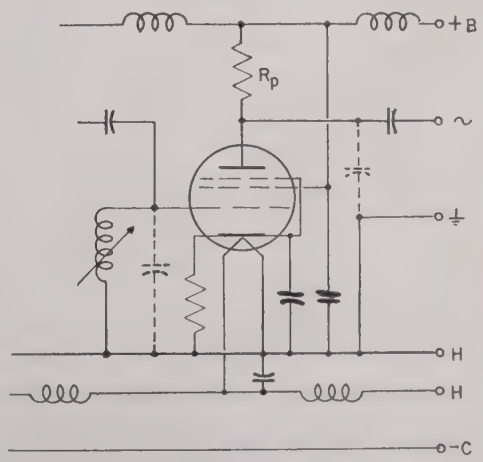

Fig. 6—Circuit diagram of amplifier output stage.

such units are that they offer the shortest possible leads; their points of connection are unambiguous; they do not obstruct other connections; they are placed so as to aid

in shielding between the grid and anode lug; and they may be wired into the tube socket before assembly on the chassis. With the dimensions given, allowing for insulating gaps around the outside patches, and assuming a dielectric constant of 12,000, as much as 3×5000 $\mu\mu$f may be had with such units, but much less is needed in this case.

The output stage, Fig. 6, resembles the amplifier stages, although but for larger output power a somewhat larger, otherwise similar, tube such as the type 6AN5[3] with 25 ma plate current may be used.

All the signals at the anodes of the output tubes are added by shunt-feeding them into a combining network. Since in this case the network presents a load impedance of 26 ohms, and one-half of its matching impedance of 52 ohms, the voltage gain of the output stages is actually less than unity. The plate resistors R_p, of the order of 1000 ohms, merely supply dc to the tubes with reasonably small drop in dc potential, yet without presenting an appreciable shunt load to the output network. Plate- and screen-current consumption of a seven-tube amplifier strip ranges from 40 to 75 ma according to the gain. Both the plate and the heater supply are fuze-protected in each strip.

Typical over-all amplitude-response curves are shown for television channel 2 in Fig. 7. The curves for 20- and for 40-db gain are moved up towards the curve for 65-db gain for easier comparison. It will be

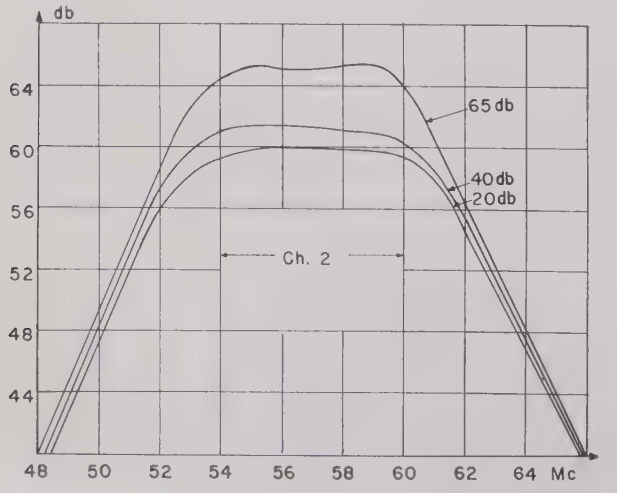

Fig. 7—Amplitude-response curves of a typical television amplifier, for over-all gains of 20, 40, and 65 db.

noted that the curves neither deform nor move appreciably with change of gain. Attenuation of about 30 db at the carrier frequency of the next television channels of the area is usually required. If the selectivity of the amplifiers does not suffice, small pretuned bandpass filters may be inserted in the input chassis, between the antenna cables and the connectors to the input of the amplifiers.

[3] Raytheon Manufacturing Company.

The amplifier covering the FM broadcast band resembles those for television. Since there is no control over the relative amplitudes of the received signals, the gain of this amplifier is held to a moderate value, a little above 40 db. The gain control for this amplifier is used only for checking. The amplitude and the phase response of this amplifier need not be as conservative as those of the television amplifiers, but a response characteristic such as that shown in Fig. 8 is easily accomplished.

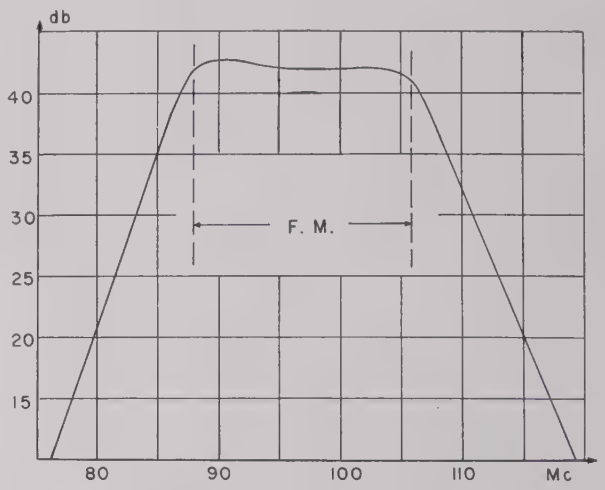

Fig. 8—Amplitude-response curve of a typical amplifier for the FM broadcast band.

Since the intensity of the direct signal from a television transmitter may be taken to be reasonably constant, automatic control of the amplifier gain may be dispensed with, provided that their present gains remain unaltered for long periods of operation. The types of amplifier tubes are chosen accordingly, and their dissipation kept far below rating. Heater, anode, and grid-bias voltage are supplied to groups of each four amplifier strips. They are regulated where necessary to protect both against variations in line voltage and against change of load if another strip of the group fails.

The change of gain with change of heater voltage due to variations in line voltage is too slight to merit attention. Changes due to change of load are held to negligible proportions by keeping the resistance of the heater winding on the power transformer to a low value.

The plate-voltage supply is regulated within close limits for ±10 per cent change in line voltage and for load currents ranging from 40 to 300 ma. Thus the adjustment of the gain of one amplifier does not affect the gain of the others, and the gain of an amplifier will remain unaffected even if every other one of the group of four should fail. The relatively large current at 110 volts is supplied by a bridge of four selenium rectifiers, as shown in Fig. 9. The regulator circuit comprises a low-impedance, high-current twin-triode 6AS7G as series element and a low-current, high-gain pentode 6SJ7 as shunt control tube. The anode current of the

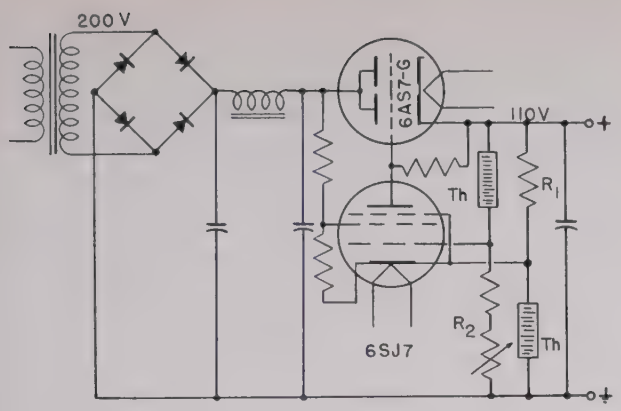

Fig. 9—Plate-supply regulator circuit controlled by a nonlinear bridge.

latter, of the order of 20 μa, is controlled by a nonlinear bridge comprising the two constant resistors R_1 and R_2 and the two voltage-sensitivity Thyrite resistors *Th*. Since the operation of this circuit has been discussed elsewhere[4] in detail, it may suffice to mention that the nonlinear bridge, consuming less than one milliampere, controls the output without difficulty at voltages even below 110 volts, with an effective power supply source impedance of a few ohms.

The four gain-control potentiometers require a stable supply of −4 volts, independent of line-voltage variations but not necessarily from a low-impedance source. This voltage is supplied by the simple regulated bias-supply circuit shown in Fig. 10. A separate transformer

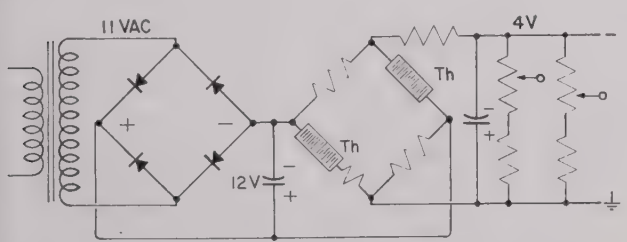

Fig. 10—Bias-supply circuit regulated by a nonlinear bridge.

winding and a selenium rectifier bridge supply 12 volts of poorly smoothed dc to a nonlinear bridge comprising two Thyrite units *Th* as the nonlinear elements. This bridge is degraded, as has been discussed elsewhere,[4] by insertion of a constant resistor in the arm of a nonlinear resistor, to such an extent that its originally reversing characteristic, shown as the broken line in Fig. 11, now barely drops and remains flat over an appreciable range of input voltage, as shown by the solid curve in Fig. 11. Not only does the output voltage stay constant within better than 1 per cent with ±10 per cent change of line voltage, but the instantaneous action of the bridge also contributes substantially to the smoothing of the residual ripple in its input voltage. Since no current is drawn from the bias-control potentiometers, they present a constant load to their source, and their output voltages will be quite independent of each other. Varying the bias voltage of three amplifier stages from −1.5 to −4 volts reduces the over-all gain from about 65 to about 0 db at an approximately linear rate of 26 db per volt.

[4] Heinz E. Kallmann, "Nonlinear circuit element applications," *Electronics*, vol. 19, pp. 130–136; August, 1946.

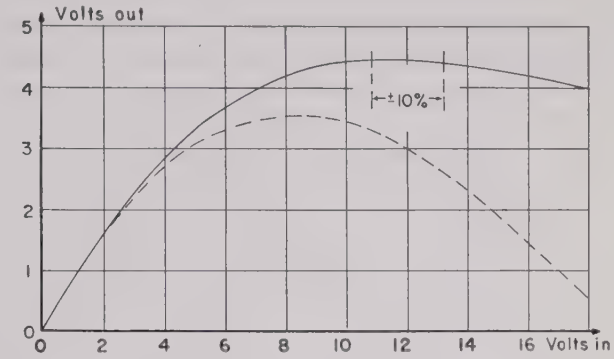

Fig. 11—Regulation characteristic of bias-supply circuit.

The amplifier-output combining network is also housed in the power-supply chassis. It is designed to satisfy the following conditions: (1) it must match the coaxial distribution cable, of, e.g., 52 ohms impedance, over the whole range of television carrier frequencies, up to at least 216 Mc; (2) it must combine the output of as much as eight separate amplifiers without mismatch at any point for the signals of any one of them; and (3) its transmission loss should be equal—and preferably small—for the output of all amplifiers. The network is derived from the ordinary low-pass filter, shown in Fig. 12(a), comprising series inductances *2L* and

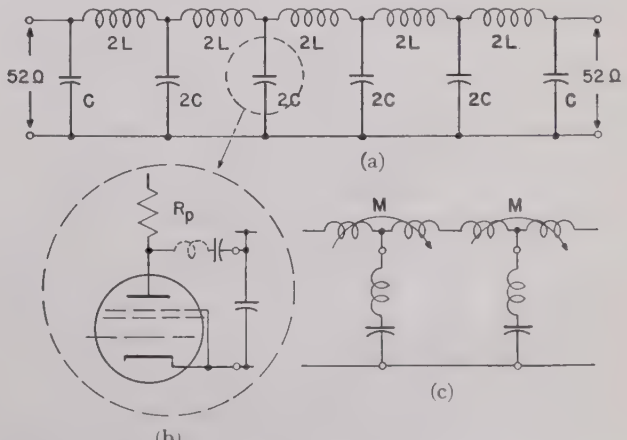

Fig. 12—(a) Amplifier-output combining network: basic low-pass filter; (b) tube output impedance replacing shunt capacitor; (c) mutual inductance compensating for lead inductance in amplifier output.

shunt capacitances *2C*, designed for a nominal impedance of 52 ohms so as to match coaxial cables connected to either end, and with a cutoff frequency well above 216 Mc, say 250 to 300 Mc. The shunt elements *2C* will

then be found to be each of the order of 20 $\mu\mu$f. For the greater part of these shunt capacitances $2C$ is substituted the output impedance of one of the amplifiers, as represented within the broken-line circle of Fig. 12(b), consisting, mainly, of the tube output capacitance, the connector capacitances, the tube plate impedance, and the plate resistance R_p, all in parallel. The total of these capacitances may amount to, perhaps, 12 $\mu\mu$f, and can thus be accommodated by corresponding reduction in the value of the network shunt capacitors. The plate impedance of the output pentode is so high that it may be ignored compared with the plate resistance R_p, which is of the order of 1000 ohms. Even this resistance, however, is so large compared with the network impedance of 52 ohms that its effect on the network cannot be detected. The attenuation through the network remains very small in spite of the slight leakage so introduced, and there is no perceptible mismatch due to the tube output impedance. The substitution would thus be quite simple, except for the fact that the inductance in the connector lead and in the tube leads, indicated as a broken-line coil in Fig. 12(b), is not to be ignored at frequencies of the order of 200 Mc. It is important, but not sufficient, to make these leads as short as possible. The inductance then left can be disposed of, however, by an application of well-known network theory. A coupling inductance M between the series inductances of a low-pass filter, such as shown in Fig. 12(c), is fully equivalent to an inductance inserted in series with the shunt capacitor. Moreover, by choice of the sense of the coupling M, the sign of the equivalent inductance in the shunt arm may be made positive or negative. If the coupling M is in such a sense as results from winding the series inductances on a common core, and in the same direction, then the sign of the equivalent inductance in the shunt arm is negative.[5] It follows, and is confirmed by observation, that by suitable couplings between the successive series inductances one can neutralize a certain amount of inductance in series with the shunt capacitances. Thus the output network can be made to satisfy all stipulated requirements. Since, seen from the tube output, both directions of the network appear to be connected in parallel, the load impedance for the tube is usually 26 ohms. To maintain a signal level of 30 millivolts rms in the network therefore requires an anode current of about 1.1 milliampere rms. The network ordinarily feeds one branch of distribution cable at each end. If a single branch only is needed, the other is replaced by a matched terminating resistor; a larger number of branches may be fed by connecting several cables, in parallel, to each end of a network of proportionately lower impedance.

A group of four amplifiers with their power supply, input, and output chassis is housed in a wall-mounted cabinet of about 5-inch depth, 14-inch width, and 23-inch height. Its total power consumption is about 13(watts. Where eight amplifiers are needed, two groups o four are mounted side by side, and their output com bining networks connected in series. By keeping the tw power supplies separate, a breakdown in one of then will disable only four of the amplifiers.

[5] Quantitative data can be found in Fig. 24 in Heinz E. Kallmann, "Equalized delay lines," PROC. I.R.E., vol. 34, pp. 646–657; September, 1946.

Distribution Network

Common to all practicable rf distribution systems i the distribution network of coaxial cables. This is, as a rule, much the costliest item of the system, even com pared with very elaborate antennas and amplifiers. Th costs of the network are mainly those of the installa tion; they rise not only with the length of the connec tions but also with the number of cables, if more tha one is needed to carry all signals; they also rise with th increased labor of installing heavier cables. The ampli fiers were thus designed to feed all signals over on rather thin cable. The network is then planned for short est possible total length of cable; a few vertical risers each feeding an outlet on each floor, have generally bee found to offer the most practical solution. For all excep long uninterrupted connections a thin coaxial cable wit low-loss dielectric has been found adequate. This is th type RG 58-U, with an impedance of 52 ohms. Its cente conductor is a copper wire of 0.022 inch diameter; it dielectric is solid polyethylene extruded to 0.116 inc diameter; its outer conductor is a single braid of tinne copper; its jacket of polyvinyl plastic has a maximun diameter of 0.205 inch. The attenuation per hundre feet averages, for this cable, 2.7 db at 40 mc, 5.2 db a 100 Mc, and 6.2 db at 200 Mc. For uninterrupted lon runs, these losses at the higher frequencies may becom appreciable. In such cases a thicker cable with lowe losses is used. This is the type RG 8-U, also of 52 ohm impedance. Its stranded center conductor consists o seven copper wires and has an over-all diameter o 0.086 inch; its dielectric is solid polyethylene extrude to a diameter of 0.285 inch; its outer conductor is a single braid of bare copper; and its polyvinyl jacket ha an over-all diameter of 0.415 inch maximum. The aver age attenuation for this cable, per one hundred feet, i 1.4 db at 50 Mc, 2.0 db at 100 Mc, and 3.1 db at 20(Mc.

In order not to preclude use of the distribution net work at even higher frequencies of possible future broad cast services, all frequency-range limiting elements suc as filter networks and transformers are rigorously ex cluded from it and the outlets. Except for attenuatio rising with frequency, its performance is entirely fre quency-independent and all connections can be matche for all frequencies up to, perhaps, 1000 Mc to an measurable degree of accuracy. In order to prevent an reaction from any receiver on the cable, as well as an interaction between any two receivers through th cable, a substantial attenuation is inserted at each out let. An outlet attenuation of not less than 30 db ha been found desirable; larger values would require main-

tenance of an unreasonably high power level in the cable. With signals of 1 millivolt fed to the individual receivers, a signal level of 30 db above 1 millivolt, i. e., about 30 millivolts, is maintained in the cables, adjusted to be approximately the same for all television stations so that no, or very little, automatic or manual control of the receiver gain is needed when switching to another station.

Each outlet contains essentially a resistive tee attenuator, as shown in Fig. 13(a). The cable Z_1-Z_1 is shunted by the relatively large resistance R_p through

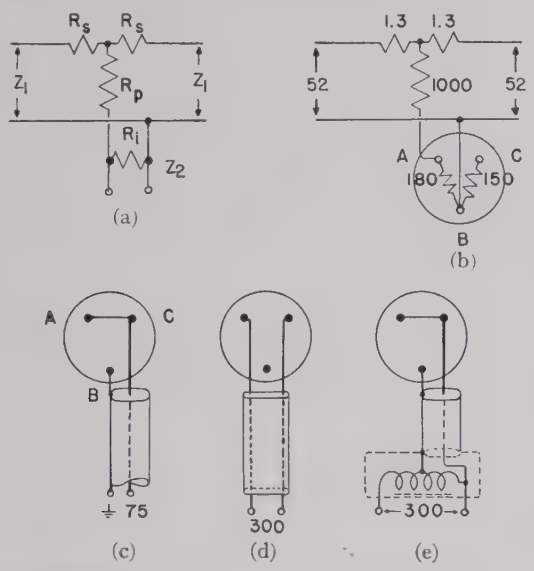

Fig. 13—Distribution network outlet: (a) basic outlet circuit; (b) outlet network for choice of two receiver-input impedances; (c) plug connections for 75-ohm receiver-input impedance; (d) plug connections for balanced 300-ohm receiver-input impedance; and (e) plug connections for 75:300-ohm receiver-input transformer.

which a small fraction of the signal power is fed to the sidebranch cable Z_2. The sidebranch source impedance is matched to the receiver antenna cable Z_2 by means of the resistor R_i, so that

$$Z_2 = \frac{R_i(R_p + Z_1/2 + R_s/2)}{R_i + R_p + Z_1/2 + R_s/2}$$

or, approximately,

$$R_i \approx R_p Z_2/(R_p - Z_2).$$

If the two small series resistors R_s are omitted, the attenuation along the cable, due to the presence of the outlet, is negligible, but each outlet then creates a slight mismatch and becomes the source of reflections with a voltage-standing-wave ratio VSWR $= 1 + Z_1/R_p$. For a value of $R_p = 1000$ ohms, VSWR ≈ 1.05. Such echo, particularly if no cumulative effect of many in-phase echoes is to be feared, may often be disregarded and the resistors R_s omitted. They are needed only for exact matching, their value being

$$R_s = \sqrt{Z_1^2 + R_p^2} - R_p.$$

For a shunt resistor $R_p = 1000$ ohms, the value for R_s is about 1.3 ohms, and the attenuation along the cable, due to the insertion of the matched outlet, is about 0.42 db, from

$$\text{voltage ratio } A = Z_1/R_p + \sqrt{1 + (Z_1/R_p)^2},$$

the price paid for rigorous matching.

In either case, whether the sidebranch cable Z_2 is left open, or is terminated with a receiver, or short-circuited, does not perceptibly affect the main cable. All parts of an outlet find place in a metal case of $1\frac{3}{4}$-inch diameter and about 1-inch depth, which serves as an electrical shield and also protects the cable ends and the outlet resistors against moisture. By judicious spacing of the solder lugs, the slight increase in the distributed inductance of the cable at its braidless ends may be made to compensate for the incidental shunt capacitances at the junction point with the resistor R_p, by completing a matched low-pass filter section of very high cutoff frequency.

The input circuits of presently manufactured television receivers are either designed to match a coaxial cable of 72 ohms impedance or a balanced line of 300 ohms impedance. The outlet circuits are designed to meet either requirement without need for any adjustment. As shown in Fig. 13(b), each outlet has three terminals A, B, and C to receive a three-prong plug. The dropping resistor R_p is connected to A, while the terminal B is grounded to the braid of the cable. A resistor of 180 ohms is connected from A to B and another resistor of 150 ohms from C to B. Receivers with either input impedance are matched simply by connecting their antenna cable to the proper prongs of the plug. For a receiver of about 75 ohms single-ended-input impedance, the center conductor of its coaxial cable is connected to both the prongs A and C in parallel, as shown in Fig. 13(c). The source impedance of the outlet is then that of 1027 ohms and 180 ohms and 150 ohms in parallel, i.e., approximately 75 ohms as required. The connections for the other group of receivers are shown in Fig. 13(d). The two conductors of the balanced 300-ohm line are connected to prongs A and C; if there is a grounded shield it may be tied to prong B. The source impedance of the outlet is then a balanced impedance of 300 ohms, with the 150-ohm resistor in series with the parallel combination of 1027 ohms and 180 ohms. Moreover, due to the coupling between the conductors of even a short balanced cable, the signal amplitudes at the two receiver input terminals will be found to be well balanced.

A serious disadvantage of the balanced 300-ohm input connection to a television receiver is that the unshielded line commonly used tends to pick up television signals directly to such an extent that they show as "ghosts," but to the left of the main image. In locations close to a transmitter, this direct pickup may be so strong that even a very large signal from the distribution system could not swamp it without overloading the input of the receiver. To keep the length of the an-

tenna connection short is helpful; but use of a shielded balanced cable of 300 ohms impedance is certain to minimize the unwanted pickup. At present such cables are, however, thick and quite unwieldy.

An alternative connection for 300-ohm balanced receiver input impedance is shown in Fig. 13(e), employing a small shielded rf transformer covering the whole television band. The primary of this transformer is connected to the outlet by a coaxial cable of 72 ohms impedance, as in Fig. 13(c) while its balanced secondary winding of 300 ohms impedance is directly connected to the receiver input terminals. The transformer supplies a somewhat larger signal to the receiver and also helps to suppress reradiation of the local-oscillator energy in receivers having a poorly balanced push-pull rf-amplifier stage.

With attenuation of about 30 db between the distribution cable and each receiver, the attenuation between any two sets is at least 60 db. As a consequence, no interaction, via the cable, has been observed between any two sets, each tuned to any desired program, even if a set radiates as much as 5 millivolts local-oscillator amplitude into its antenna outlet and at a frequency that would cause a beat pattern in the image of the next receiver. Due to this protection, almost all types of commercial receivers may be operated with the distribution system. Those that are objectionable would interfere as much with their neighbors if each had a separate antenna. Some types of receivers, lacking an rf amplifier stage, have been found to develop as much as 1 volt of local-oscillator amplitude at the antenna terminal.

Not a few types of receivers interfere with others by direct radiation coupling between their insufficiently shielded chassis, perceptible up to, perhaps, a distance of 30 feet between chassis without antenna connected to either set. Besides causing interference, such receivers are subject to direct signal pickup similar to that due to an unshielded antenna connection.

A Broad-Band High-Level Modulator*

R. J. ROCKWELL†, FELLOW, IRE

Summary—**A class-B modulator has been devised to assure a broad pass band having uniform gain with quite low distortion and noise level. Unique features include (1) a cathode follower, (2) no modulation transformer, and (3) broad-band feedback.**

THERE ARE MANY vexatious problems concerned with the design of amplitude-modulated transmitters capable of true high fidelity. Means have been found to obviate most of the objectionable features encountered, a generous application of negative feedback being especially helpful. Because the class-C final is effectively in series with the modulator output, it is desirable that over-all rectified feedback be returned to the modulator from the output of the class-C amplifier. Such an arrangement is particularly useful because it provides cancellation of disturbances arising within the class-C equipment, as well as the modulator.

In practice, the modulation transformer is so disruptive to stability that feedback must be rendered ineffectual well before reaching the upper frequency limit of the transformer pass band. This leaves uncorrected the higher-frequency distortion and noise originating within the class-C portion, with resultant diminution of fidelity.

Broad-Band Modulator

The obvious alternative to the conventional circuit of Fig. 1 is a system which eliminates the modulation transformer without sacrificing electrical efficiency Such a basic class-B modulator stage is portrayed in Fig. 2.

Here modulator tube M_1 operates into L_1 as a normal reactance-coupled amplifier. Since it is biased nearly to cutoff, it responds only to positive grid excursions

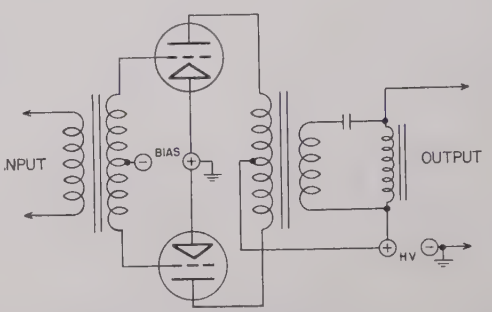

Fig. 1—Conventional output.

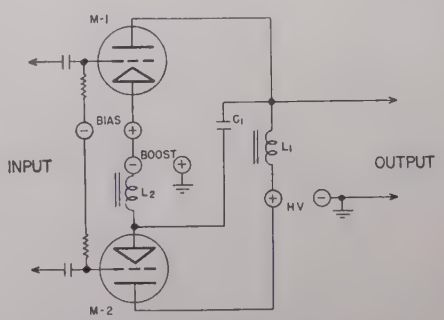

Fig. 2—Broad-band output.

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† Crosley Broadcasting Corporation, Cincinnati 2, Ohio.

which produce negative excursions of both its plate and the direct-coupled class-C load. Modulator tube M_2 is similarly biased near cutoff, and responsive only to positive excursions. However, it is connected as a cathode follower through a reactor L_2, across which develop positive excursions of its cathode. These positive excursions are capacitance-coupled through C_1 to L_1.

This enables negative and positive half-cycles to be alternately developed across a common output reactor L_1, and thereby modulate the class-C load. Thus is achieved an active electric network which transmits audio power from a class-B amplifier to a single-ended load without resort to a transformer.

Modulation Capability

To realize 100 per cent modulation, the class-C plate current must be varied between zero and twice its steady-stage value. To develop such an output potential without the step-up afforded by a transformer, the broad-band modulator must operate with higher plate voltage than the class-C final.

The necessary voltage differential may be obtained across a heavily by-passed resistor in series with the modulator output. However, it would be more efficient to connect an equivalent voltage supply between the modulator cathodes and ground. This cathode-boost supply would then be in cumulative series between the grid-bias and high-voltage supplies, as shown in Fig. 2.

Driver Stage

Upon examination of the driver circuit, Fig. 3, it will be noted that the grid of modulator M_1 is driven in a

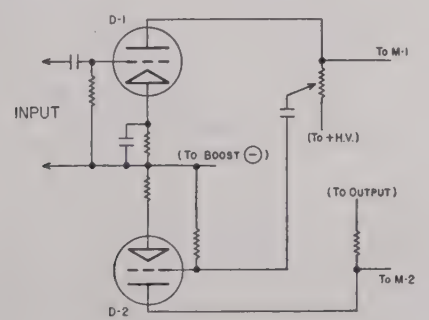

Fig. 3—Driver stage.

conventional manner from the plate of driver D_1, and modulator M_2 is similarly actuated from the plate of driver D_2. The grid of driver D_2 operates from the plate of driver D_1, in phase with the input to modulator M_1, to provide the opposite phase at the grid of modulator M_2.

Of particular interest is the plate supply for driver D_2, which is not obtained from a steady source, but from the full modulator output. This positive-feedback connection provides adequate drive voltage between ground and the grid of modulator M_2. Because M_2 is highly degenerative, it requires a positive grid-to-ground voltage equal to its cathode excursion, plus its usual grid excitation. Thus, at driver D_2, a negative grid excursion creates a positive plate movement which is transferred through modulator M_2, superposed upon the direct voltage at the output terminal, and simultaneously returned in phase with the load voltage developed by driver D_2. This novel arrangement offers both design simplicity and economy of operation. A circuit analysis of this cathode-follower and inverter-driver operation is appended.

Performance

A low-power modulator (see Fig. 4) employing the broad-band design was set up to experimentally test this circuit. Two type-AB-150 (845) tubes served as modula-

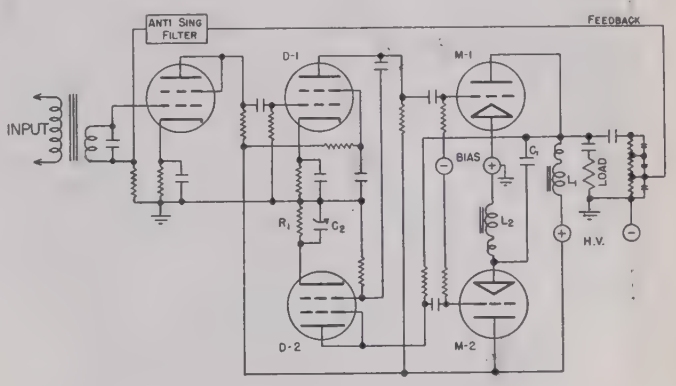

Fig. 4—Experimental circuit.

tors, and two 6L6 tubes as drivers. Air-core reactors were placed in series with, and on the tube side of, the iron-core reactors to avoid response degradation at the higher audio frequencies.

Note that the cathode-boost supply was omitted, because this experiment was concerned only with the quality of the modulator output signal. For convenience, the test modulator was operated into a resistive load. Feedback was returned from a resistance-capacitance network across the modulator output.

It has been shown[1] that, to be properly effective, an inverse-feedback loop must be under accurate control as to frequency response, phase shift, and rate of attenuation for a range extending several octaves beyond the desired pass band. Such feedback control was utilized to enhance the pass range of this modulator.

Magnitude and Phase Adjustment

With all circuit elements properly chosen, the cathode resistor R_1 varied the gain of driver D_2 to balance the alternate half-cycles appearing at the modulator output. Certain photographs of oscilloscope traces illustrate, by various adjustments of R_1, oppositely unbalanced conditions compared to correct balance. Abnormally high grid bias was used to emphasize the junction point. Feedback was entirely inoperative for all

[1] F. E. Terman, "Radio Engineers Handbook," McGraw-Hill Book Company, New York, N. Y., pp. 218–226; 1943.

examples of Fig. 5(a). These same three adjustments, but with some 35 db of feedback fully operative to 100 kc, were repeated in Fig. 5(b). It will be seen that application of feedback gave clean waveform, despite both considerable misadjustment of R_1 and exaggerated bias on the modulator grids.

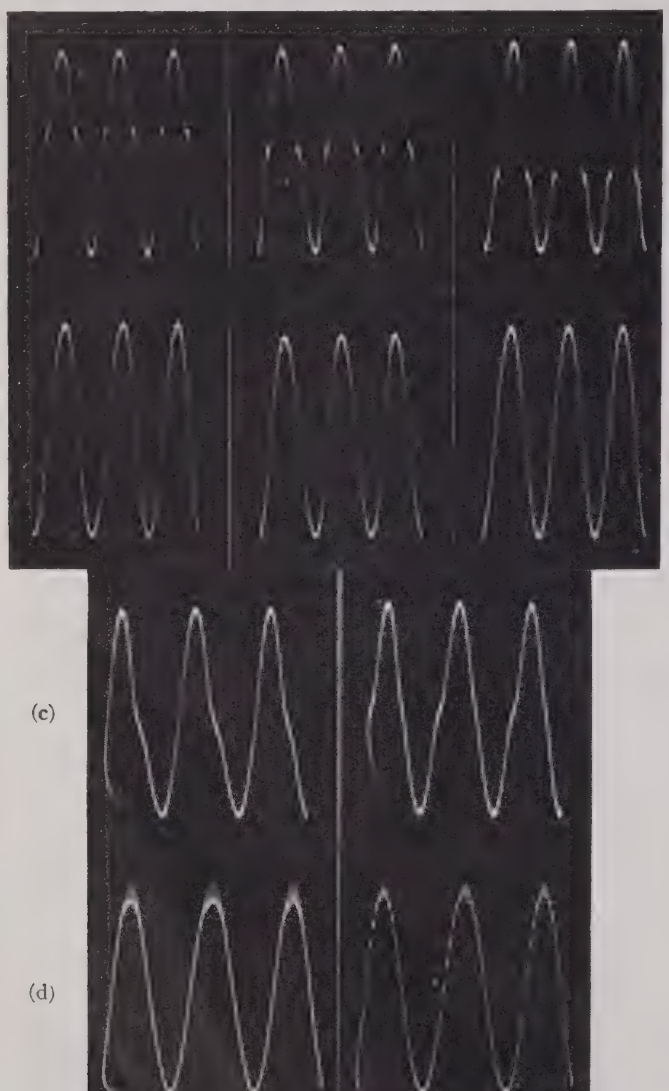

Fig. 5—(a) Balance without feedback.
(b) Balance with feedback.
(c) Phase without feedback.
(d) "Sing" due to improper constants.

The adjustable capacitance C_2 shunted across R_1 enabled slight phase correction of the voltages delivered to the common load by opposite modulator sides. Without feedback, the results of two differently improper adjustments of C_2 are portrayed in the oscilloscope traces of Fig. 5(c). Application of feedback obviated these phase variations and, because of similarity to Fig. 5(b), no illustrations were made.

Certain improper filter constants were deliberately inserted in the feedback loop. Two separate results were the "sing" frequencies, Fig. 5(d), superimposed upon fundamental wave of 20 kc. The necessity of prope feedback constants was thus confirmed.

Measurements

The several measurement results at various frequencies are depicited in Fig. 6. The frequency response without feedback followed line *BB*, whose drooping end

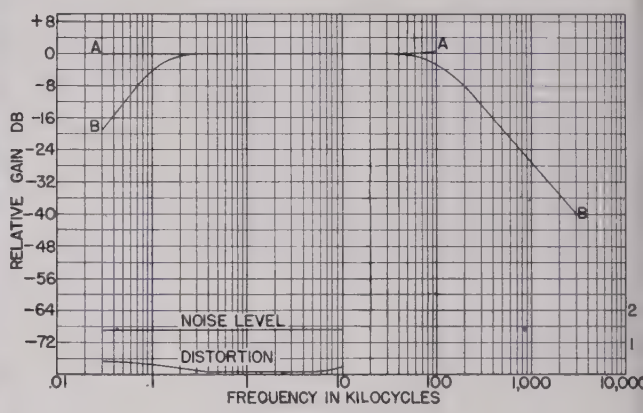

Fig. 6—Response curves.

were of correct attenuation slope, as previously mentioned. The over-all gain without feedback became zero at approximately 2500 kc. Line *AA* was obtained after the application of 35 db of negative feedback. With feedback, uniform gain was obtained to 50 kc, and the increased slightly to 100 kc where measurement ended.

Distortion varies somewhat, but remained below $\frac{1}{2}$ 1 per cent throughout the measured region. The noise level remained constant at 69 db below full output. Neither distortion nor noise-level measurements extended above 10 kc, that being the upper frequency limit of the available measuring equipment.

Conclusion

By comparing the conventional modulator of Fig. to the band-band circuit of Fig. 2, it will be seen that the number and cost of components are roughly equal the additions being offset by the items deleted. The broad-band design utilizes a cathode follower, eliminates the modulation transformer, and allows full use of over-all feedback. The prime advantage of this improved modulator lies in its ability to deliver intelligence onto a carrier envelope with very high fidelity throughout a wide range of frequencies.

Acknowledgment

The author wishes to acknowledge the assistance of Howard Lepple, of the WLW Engineering Laboratory who set up the test and measurement circuits for the project and prepared the appended circuit analysis.

[3] I.e., $1-\mu B=\mu$ from H. W. Bode, "Network Analysis and Feed-Back Amplifier Design," D. Van Nostrand Company, Inc., New York, N. Y., p. 32; 1945.

and of P. A. Young, of the WLW Propagation Department, who aided in the arrangement of material in this paper.

APPENDIX

Circuit Analysis

This explanation of the broad-band circuit presents, first, the modulator operation; second, the phase-inverter–driver analysis; and, finally the procedure for selection of a driver tube and associated circuit constants.

Modulator Operation

Fig. 7 is the diagram of a proposed broad-band modulator. In this circuit both tubes M_1 and M_2 are biased

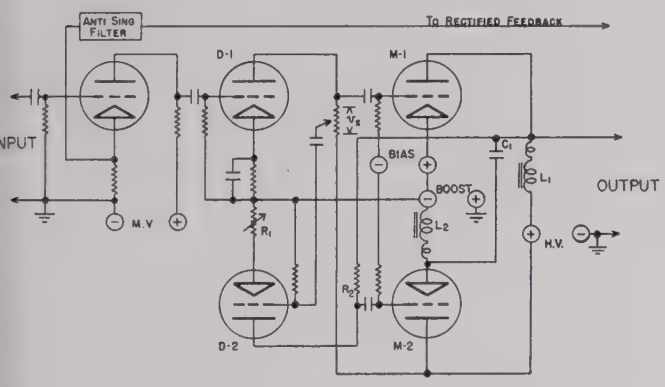

Fig. 7—Proposed broad-band modulator.

nearly to cutoff for class-B operation. This renders them responsive to only positive grid excitation. Alternate operation is obtained by connecting the input signal directly to tube M_1, but through a phase-inverter stage to tube M_2. The modulator stage operates as follows:

During the positive half cycle of input voltage v_s, tube M_1 and reactor L_1 act to decrease the potential between the output terminals. During the negative half-cycle of v_s, tube M_2 and reactor L_2 act to increase the potential between the output terminals. The modulator output voltage therefore varies as a full sinusoidal wave.

It will be noted that the plate voltage available to the driver tube is also varying sinusoidally, since it is obtained from the high-potential output terminal. Furthermore, neglecting the voltage-divider effect of the driver-load and modulator-grid resistors, this same alternating voltage is simultaneously applied to the grid of tube M_2. Therefore, the potentials of the grid and cathode of tube M_2 rise and fall together, except for the voltage drop across resistor R_2 occasioned by tube D_2 plate current. The voltage drop across R_2 must vary sinusoidally and be of equal magnitude to v_s for both M_1 and M_2 to receive identical excitation. It is therefore evident that the inverter-driver stage must have unity amplification. This discussion of the action of tube M_2 during the negative half-cycle of v_s is summarized by the voltage relation depicted in Fig. 8.

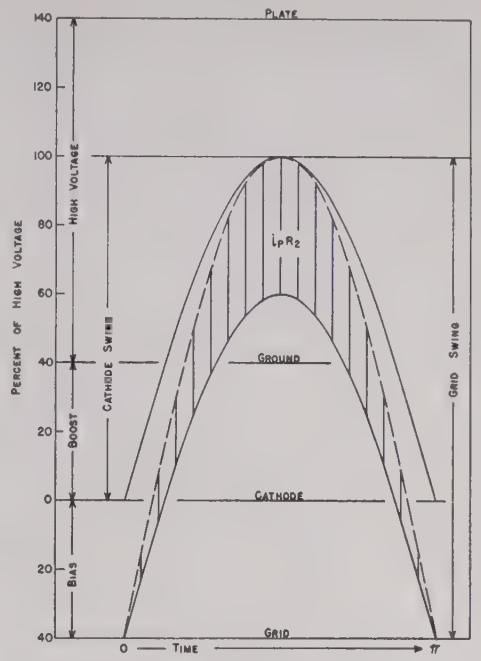

Fig. 8—Cathode-follower operation.

Inverter-Driver Analysis

Symbols used in the analysis include:

A = voltage amplification of modulator stage
I_b = direct current in the plate circuit of D_2
$i_p = I_p \sin \omega t$ = alternating current in the plate circuit
k = a fraction not exceeding unity
μ = amplification factor of tube D_2
R_p = alternating-current plate resistance
R_1 = cathode resistor
R_2 = load resistor
V_b = direct high voltage
v_g = grid-to-cathode signal voltage
v_0 = alternating voltage developed across load R_2
$v_s = V_s \sin \omega t$ = input signal voltage.

The action of inverter-driver D_2 during the negative half-cycle of v_s may be analyzed from the circuit shown in Fig. 9. The voltage supply for the plate is

$$V_b + Av_0.$$

The signal voltage exciting the grid is

$$v_g = kv_s - i_pR_1. \tag{1}$$

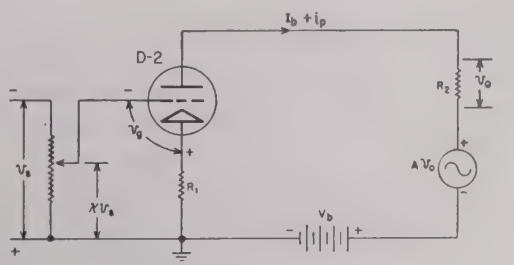

Fig. 9—Inverter-driver circuit.

From the equivalent constant-voltage generator circuit of Fig. 10,

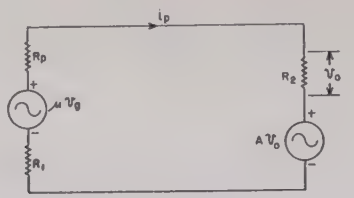

Fig. 10—Equivalent plate circuit.

$$\mu v_g - A v_0 = i_p(R_1 + R_2 + R_p).$$

Substituting for v_g

$$\mu k v_s - A v_0 = i_p[(\mu + 1)R_1 + R_2 + R_p] \qquad (2)$$

or

$$i_p = \frac{\mu k v_s - A v_0}{(\mu + 1)R_1 + R_2 + R_p}. \qquad (3)$$

Specifying the amplification as unity (i.e., $|v_0| = |v_s|$)

$$\left|\frac{v_0}{v_s}\right| = 1 = \left|\frac{i_p R_2}{v_s}\right| = \frac{(\mu k - A)R_2}{(\mu + 1)R_1 + R_2 + R_p}. \qquad (4)$$

From (4) it is found that

$$R_1 = R_2\frac{\mu k - A - 1}{\mu + 1} - \frac{R_p}{\mu + 1}. \qquad (5)$$

For R_1 to be a positive resistance requires that

$$R_2\frac{\mu k - A - 1}{\mu + 1} > \frac{R_p}{\mu + 1}$$

or

$$R_2 \geqq \frac{R_p}{\mu k - A - 1}, \qquad (6)$$

where equality holds only for $R_1 = 0$.

Now a value of R_2 may be chosen and (5) solved for R_1.

If the total alternating and direct current in the plate circuit is considered,

$$I_b + i_p = \frac{V_b + \mu k v_s - A v_0}{(\mu + 1)R_1 + R_2 + R_p}.$$

Since $v_s = V_s \sin \omega t$, and the amplification has been specified $|v_0/v_s| = 1$, substituting for v_s yields

$$I_b + i_p = \frac{V_b}{(\mu + 1)R_1 + R_2 + R_p} + \frac{(\mu k - A)V_s \sin \omega t}{(\mu + 1)R_1 + R_2 + R_p}. \qquad (7$$

An ideal tube would operate until the total plate current became zero at $\omega t = 270°$, at which moment

$$V_b = (\mu k - A)V_s.$$

However, practical operation of an existing tube requires the plate current not to reach zero during the negative (i.e., $180° < \omega t < 360°$) half-cycle of the input signal v_s. Such a limitation is imposed when

$$V_b > (\mu k - A)V_s. \qquad (8$$

or

$$k < \frac{V_b + A V_s}{\mu V_s}. \qquad (9$$

Selection of Constants

Knowing the amplification factor of the inverter driver tube, the voltage amplification realized in the modulator stage, and the potentials of both the direct plate supply and the alternating input signal, there remain three pertinent unknowns, R_1, R_2, and k. Of these k has an upper limit indicated by the inequality (9). The value of R_2 must comply with a lower limit as expressed by (6). Having selected useful values of k and R_2 the remaining unknown, R_1, may be determined from (5).

Simplified Automatic Stabilization of a Frequency-Modulated Oscillator*

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Summary—A frequency-modulated exciter which incorporates a quartz-crystal discriminator for center-frequency stabilization of the modulated oscillator is described. The unit was designed as the exciter for a frequency-modulated broadcast transmitter.

The frequency-stabilizing circuits are the unique portion and are described in detail. Circuit simplicity is attained by operating the discriminator at the modulated-oscillator frequency, thus eliminating heterodyning and mixing circuits. A special bridge circuit, operating on the modulator bias, maintains the average frequency at the midpoint of the discriminator. Performance and stability of a completed unit are briefly discussed.

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ALTHOUGH FM broadcast transmitters are in many ways less involved than their counterpart, the amplitude-modulated transmitter, the

.re by no means as simple as the industry would like. The simplification that has been made possible by the elimination of high-level modulators or low-efficiency class-B stages has been considerably offset by the additional circuits and equipment required to insure operation at the specified frequency.

The problem of measuring and controlling the center or average frequency of a frequency-modulated oscillator has had numerous solutions. The required precision of frequency stability has been established by the Federal Communications Commission, whose standards state that the center or average frequency shall not deviate more than 2000 cps from the assigned exact value. Since assigned frequencies for frequency modulation broadcasting are from 88 to 108 Mc, this is a tolerance of approximately ± 20 cps per megacycle.

Because this rigid tolerance must be maintained in the presence of modulation, and since no system of determining the center of a frequency-modulated carrier is without error, the inherent stability in the absence of modulation must be considerably better than the expected over-all performance. Let us assume that a reasonable design goal in the absence of modulation is one-fourth of the maximum tolerance, with a similar additional tolerance being allowed for the center-frequency measurement during modulation. This means that a system which is capable of maintaining the frequency within 5 parts per million in the absence of modulation, and 10 parts per million in the presence of modulation, will be acceptable. This additional tolerance in the presence of modulation allows for measurement error of approximately $\frac{1}{2}$ of 1 per cent of the maximum frequency swing.

The problem, then, is to obtain this accuracy in the simplest and most reliable manner possible.

In its simplest form, a frequency-control circuit consists of just two parts: a frequency-measuring device and a frequency-correcting device.

For accurate frequency measurements, a comparison of a sample of the unknown frequency voltage to a voltage of known frequency is usually made. Where more error can be tolerated, a much simpler procedure is to measure the effect of voltage at the unknown frequency when induced into a closed circuit consisting of inductance and capacitance in series, as in the conventional wavemeter. A wavemeter or circuit consisting of physical inductance and capacitance cannot be depended upon for the accurate determination of frequency, because of its inherent physical instability and high loss factor. However, the equivalent capacitance-inductance network of a piezoelectric crystal, being many times more free of these limitations, is extremely useful for accurate frequency determination.

A network utilizing the inductance and capacitance of one or more quartz crystals as the frequency standard is perhaps the simplest practical solution to the measuring problem. Although the problem is made difficult by the fact that measurements must be made over a band of frequencies, the bandwidth is fortunately quite small, as it is in the order of 0.1 per cent of the frequency. For bandwidths of this order it is entirely practical to devise frequency-selective networks of sufficient linearity and stability by using piezoelectric crystals for the reactive portions. A frequency-discriminating network utilizing two piezoelectric crystal networks with direct and inverse impedance versus frequency characteristics, respectively, and adjusted for the required modulated oscillator frequency, is incorporated in the FM exciter described herein.

The equipment, which is designated as the correcting device, has a rather special duty to perform, since it must translate any off-frequency indications of the discriminating network into an electromotive force or mechanical operation which will effect and maintain a correction to the required frequency. It is not sufficient that a simple correcting force proportional to the frequency error be applied, as this force would decrease toward zero as correction was effected. The result would be a reduction in the magnitude of the frequency error, but some error would necessarily remain in order to produce the correcting force.

A suitable correcting device must have the ability to store and maintain the necessary force or operation after the initial motivating force has vanished. The quantity of energy so stored must be maintained at all times at that amount which will cause the frequency error to be zero. A frequency-correcting device should then consist of a storage unit and a suitable circuit for converting frequency-error indications from the frequency standard into changes in the quantity of energy stored.

One such storage device is a capacitor in which the terminal voltage produced by its stored energy can be utilized to vary the equivalent resistance or reactance of a suitable electron tube which, in turn, is a portion of the frequency-determining circuit of the oscillator.

A suitable circuit for controlling the charge on the capacitors must be capable of increasing, decreasing, or retaining undisturbed the capacitor charge without itself being affected by the charge already existing. It must be energized by the output voltage of the frequency-discriminating network in such a manner that a frequency error will cause an appropriate change in the capacitor charge, which change must continue until the error vanishes.

A simple circuit for accomplishing this task was developed by utilizing two pentode tubes in a balanced bridge arrangement with capacitors connected as the bridge diagonal. Thus, when the two plate currents are identical, no current flows in the capacitor branch. The plate currents are controlled by and solely dependent on the output voltages of the crystal discriminator networks, and are therefore identical only when the discriminator halves are delivering identical output voltages to the control tubes. These tubes are operated at such electrode potentials that their plate resistances are

almost infinite, thus making their plate currents practically independent of plate voltage. This latter feature provides the necessary independence between plate voltage and capacitor charge, and makes certain that changes in capacitor charge will be directly proportional to plate-current unbalance.

Voltage applied to the discriminator at any frequency other than that which produces identical output voltage from the halves will cause an appropriate change in the capacitor charge, which, in turn, causes the frequency to change in the proper direction to bring about balance.

The Circuit

Fig. 1 is a block diagram of the complete frequency-modulation generator, showing the relationship of the various sections. It can be seen that this generator consists of a source of rf voltage (oscillator) whose frequency is to be determined and maintained at a prescribed value; a pair of frequency-selective networks

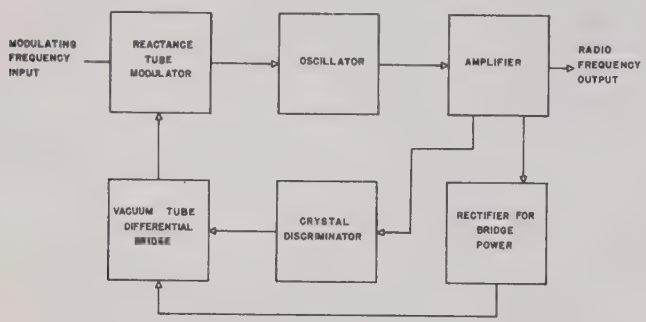

Fig. 1—Block diagram of a frequency-modulated exciter.

(crystal discriminator) whose output voltages are directly and inversely proportional, respectively, to the applied frequency, but are of equal magnitude when the frequency is at the prescribed value; an electron-tube differential bridge to compare the output voltages of the networks and convert any difference into dc voltages; a reactance-tube modulator acting on the oscillator to convert modulation-frequency voltages or control-circuit voltages into corresponding frequency variations; and a source of dc power for operating the bridge. (Fig. 8 shows a more complete schematic diagram.)

Although the oscillator and modulator are designed in accordance with accepted practice for reactance-modulated oscillators, it should be pointed out that circuit and control simplicity were the guiding factors in the specific design. Both the oscillator and modulator are push-pull dual-beam tetrodes. Although push-pull modulators have the important advantage of better sensitivity and linearity than single-ended types, push-pull oscillators have no outstanding electrical advantages over single-ended designs, except that it is easier to maintain good circuit symmetry and simple mechanical design through their use. By utilizing resistance-capacitance phase shifting between the modulator plates and grids, and direct coupling between the oscillator and modulator, a simple, reliable, and easily adjusted frequency-modulated oscillator containing only one resonant circuit was developed. Isolation of the oscillator from the effects of load variations in the external circuits or the internal frequency-control circuit is provided by a third dual-beam tetrode whose sections independently drive these loads. The tetrode section providing the useful output is operated as a tripler, and thus further isolates the oscillator from external circuit loading.

The Discriminator

The crystal discriminator, shown in Fig. 2, has been described as a pair of frequency-selective networks with direct and inverse frequency characteristics. It consists of two simple tee networks with resistive series

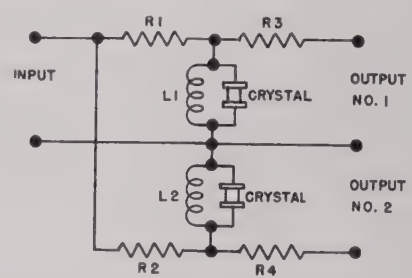

Fig. 2—Schematic diagram of the frequency discriminator.

arms and inductance-shunted piezoelectric-crystal shunt arms.[1,2] The network inputs are operated in parallel from a common source to insure identical voltages. The output terminals are connected individually to a bridge-type differential voltmeter which will be described later.

The operation of this crystal network is most easily described by referring to the approximate circuit of a quartz crystal (see Fig. 3). Here a piezoelectric crystal is shown as the equivalent of a combination of two

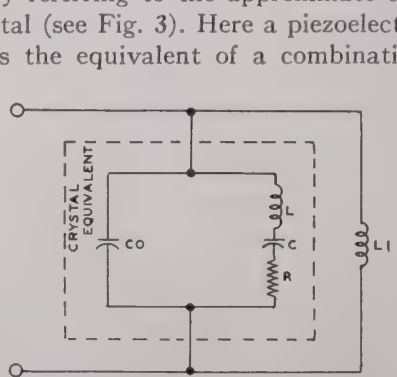

Fig. 3—Piezoelectric crystal circuit with a shunting inductance.

capacitors C and C_0, with an inductance L and a resistance R. The circuit arrangement is such that both resonance and antiresonance will occur when excitation of the proper frequency is applied across the terminals. Resonance of the inductance L and the capacitance C

[1] W. P. Mason, "Electromechanical Tranducers and Wave Filters," D. Van Nostrand Co., Inc., New York, N. Y., 1942, chap. 8.
[2] W. P. Mason, "Electromechanical wave filters employing quartz crystals as elements," *Bell Sys. Tech. Jour.*, vol. 13, p. 405; July 1934.

causes the impedance between terminals to become simply the value of the resistance R. At a slightly higher frequency, a second condition of resonance, which is more properly called antiresonance, causes the impedance between terminals to become a very-high-value resistive impedance. This occurs when the reactance of the series LC branch is inductive and equal to the reactance of the shunt capacitance C_0. These two resonance conditions normally occur at very closely adjacent frequencies, and are the high and low extremes of a continuous impedance-versus-frequency curve. The dotted curve of Fig. 4 illustrates the variation of the impedance versus frequency for such a crystal.[1,2]

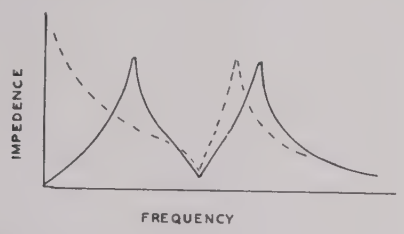

Fig. 4—Illustrative plot of impedance versus frequency for the circuit of Fig. 3. The dotted curve illustrates the impedance variation of the crystal alone.

It will be noted that, in the region between resonance and antiresonance, a reasonable degree of linearity of impedance versus frequency exists, so that it might be possible to utilize this range for a frequency discriminator whose impedance increases with frequency. Two serious drawbacks are evident, however. The first is that the two resonance points are not sufficiently separated in frequency to accommodate the required frequency swing. The second and most apparent is that the degree of linearity is not sufficient to convert FM into AM without serious distortion, and a resulting shift of the ac axis of the derived amplitude-modulated wave. This is particularly serious in view of an earlier assumption that the maximum tolerable shift of this axis was 0.5 per cent of the maximum frequency swing. It will be noted also that no equivalent inverse impedance-versus-frequency curve is present.

Since the antiresonant frequency is controlled by the values of the shunt capacitance C_0, it is possible by changing the value of this shunt admittance to vary the frequency. An inductance in shunt with the capacitive admittance of C_0, as shown in Fig. 3, will effectively reduce this shunt admittance and cause the antiresonant frequency to move to a higher value, thus further separating it from the unaffected resonant frequency. The addition of this shunt has another equally important effect in that it produces a second condition of antiresonance which is lower in frequency than the resonant point.

The solid curve of Fig. 4 shows this second antiresonant point, and reveals also that we now have a negative impedance-versus-frequency slope which approximates the same shape as the positive slope on the high-frequency side of resonance. This shunt inductance then provides a means of setting the slope of the curve from resonance to either of the antiresonant points over a limited range. Neither curve, however, appears to be linear within the required 0.5 per cent, so that additional factors must be introduced. By utilizing two identical shunted crystals, it is possible, by proper adjustment of the shunting inductance, to make the negative slope from antiresonance to resonance of one the image of the positive slope from resonance to antiresonance of the other. It is not practical, however, to make the opposite slopes identical at the same time. This is no drawback, as the series-resonant frequencies of the two crystals can be separated a slight amount without serious detriment to the inverse image characteristics of the intervening slopes. By spacing the resonance frequencies of the two crystals symmetrically about the desired operating frequency, and separated about one and one-half to two times the maximum modulation swing, a very acceptable curve can be obtained. Fig. 5(a) is a double plot of the impedance variation versus frequency for two crystals so spaced and adjusted.

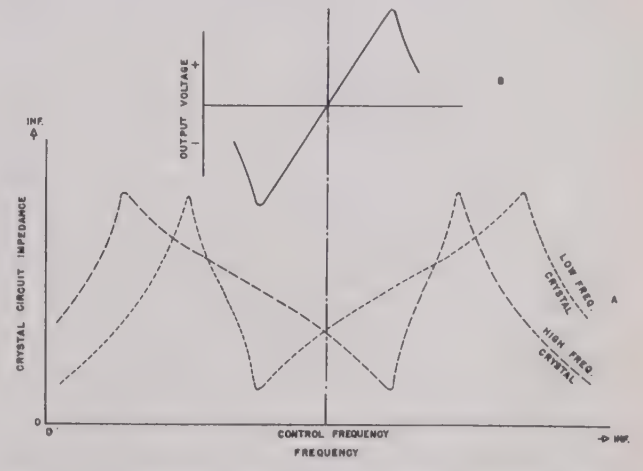

Fig. 5—(a) Illustrating the individual impedance-versus-frequency curves which can be produced by properly shunted piezoelectric crystals with slightly different resonant frequencies. (b) Illustrating the frequency discriminator which results from utilizing the impedance difference of the two curves.

The two slopes which intersect at the operating frequency in Fig. 5(a), namely, the positive slope from resonance of the low-frequency crystal and the negative slope to resonance of the high-frequency crystal, are the useful curves, because everything else is outside the operating range. It will be noted that, individually, these slopes are not very linear; however, they do have identical impedances at the desired operating frequency. A study of the impedance difference between the two slopes reveals that symmetry about the operating frequency is readily possible, but nothing can be concluded about the linearity of such impedance difference versus frequency without precise information on the curve shapes. If we were concerned only with sinusoidal frequency modulation, symmetry about the operating point would be sufficient to guarantee that the ac axis of a derived amplitude-modulated signal would cor-

respond to the zero axis of the original frequency-modulated signal. Since modulation is to be speech and music, which is not sinusoidal, linearity of conversion is very important.

A rigorous mathematical analysis of the curve shapes is extremely cumbersome, and requires a number of assumptions to permit solution at all. Such an analysis shows that it is not practically possible to obtain a strictly linear slope through the operating point. To analyze the degree of nonlinearity in terms of the ac-axis shift it would produce is even more impractical, since it involves a number of measurements on the crystal characteristics followed by a point-by-point calculation to derive the curve. Such an analysis was attempted, but was found to be practically useless in the face of the large number of variables which had to be considered. Since an axis shift of approximately 0.5 per cent of the maximum frequency swing can readily be tolerated, it is evident that some degree of nonlinearity is entirely practical. Consideration of program content and correction time constants indicates that even greater nonlinearity is acceptable for practical purposes.

In the face of all these variable and compromisable factors, an experimental determination of required linearity is all that is practical. This was done with the aid of an oscilloscope connected to the differential electron-tube bridge while the oscillator was sinusoidally modulated in frequency. It was relatively easy to adjust the shunt-inductance values to produce the discriminator curve shown in Fig. 5(b). The resulting symmetry was as nearly perfect as the eye could judge, when the discriminator curve was displayed on the face of a 5-inch oscilloscope with a superimposed transparent rectangular-co-ordinate screen. The degree of nonlinearity which was present following this adjustment was found to be insignificant during subsequent stability tests with both sinusoidal and speech and music modulation.

Although no rigorous design procedure can be set down at this time for the design and adjustment of this discriminator, a few general rules which have developed from experience and a consideration of the basic fundamentals can be stated:

First, it is preferable to use two crystals that have been cut from the same slab at the same time and processed as nearly identically as possible.

Second, resonant frequencies should be carefully adjusted during the initial calibration for symmetrical spacing above and below the desired operating frequency. These resonant frequencies should be removed from the operating frequency by approximately 150 to 200 per cent of the maximum frequency swing. The minimum spacing produces maximum discriminator linearity, but reduces the safety factor for overmodulation. In the case of FM broadcasting, this is between 0.15 and 0.2 per cent of the crystal frequency.

Third, the adjustment of the shunt impedance should be such that the lower antiresonant frequency of the high-frequency crystal network and the higher antiresonant frequency of the low-frequency crystal network fall just beyond the limit of the last important sideband component. For FM broadcasting with the oscillator in the 3.25- to 4.0-Mc region, this is at least 30 kc each side of the operating frequency.

Fourth, discriminator characteristics must be investigated at least to the limits of the above-mentioned sidebands, and any crystal discarded which, through spurious resonances or other defects, causes irregularities in the otherwise smooth curve.

Fifth, the resistances associated with this network as shown in Fig. 2 should be noninductive, stable, and as high in value as considerations of available source voltage and required differential bridge input voltage will permit.

The Capacitor-Charging Circuit

The electron-tube differential bridge, Fig. 7, is of special interest because of its ability to remember. This was previously pointed out as a necessary feature for any correcting circuit that is to produce full correction, and not merely reduce the amount of error. The essential element in this bridge which gives it memory is the capacitor branch across which is developed at all times the exact voltage required to maintain the modulated oscillator at the prescribed frequency. To produce this result, the capacitor-charging circuit must be capable of changing the charge in direct proportion to any frequency error, but leave undisturbed any existing charge when there is no error. This calls for either an infinite-impedance charging and discharging circuit or some type of balanced circuit whose balance is unaffected by charges on the capacitor. Since the previously described discriminator produces balanced output voltages when operation is at the prescribed frequency, a balanced charging circuit is indicated. A suitable charging circuit, then, would be one which does not affect the existing charge when the two dis

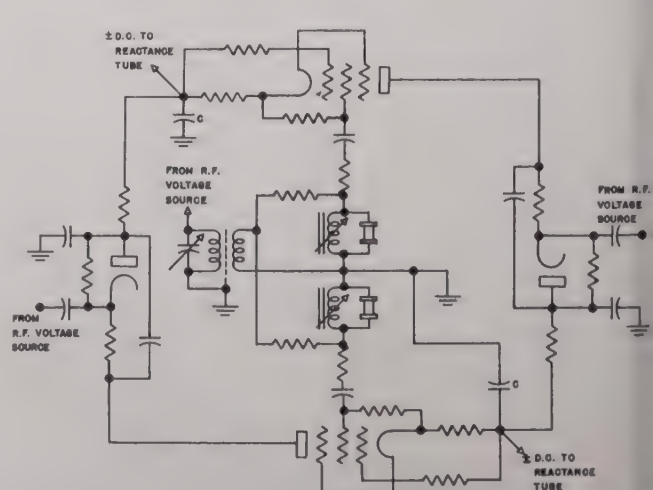

Fig. 6—Schematic diagram of the frequency-control portion of the frequency-modulated exciter. The rf voltage source is common to the three inputs shown.

criminator output voltages are balanced, but produces a change in the charge at any time the discriminator output is out of balance.

A consideration of the effect of the capacitor terminal voltage on any conceivable two-tube charging and discharging circuit utilizing electron tubes reveals that the plate currents of such tubes must be absolutely independent of plate-to-cathode voltage over at least a limited voltage range, if plate currents are to be independent of capacitor charge. This requires essentially

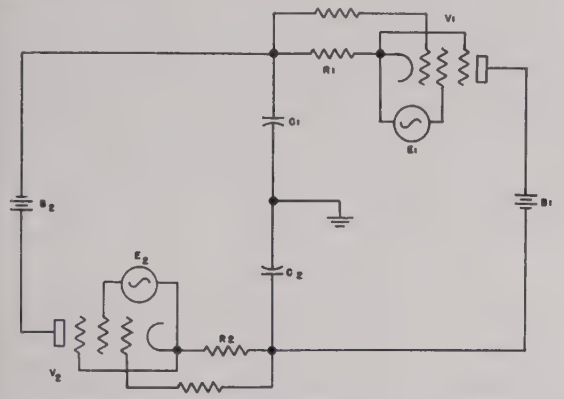

Fig. 7—A simplified schematic diagram of the differential bridge circuit which maintains the proper charge in the control capacitors C_1 and C_2.

infinite plate resistance. A pentode tube operating with a plate voltage that is greater than its screen voltage and at very low plate current will meet this requirement to a practical degree. The circuit shown in Figs. 6 and 7 was developed from these considerations.

Fig. 7 is a simplification of Fig. 6, and includes only those components which constitute the differential bridge. In this figure the capacitors C_1 and C_2 in series constitute the capacitor arm. The ground point in the center makes possible the subsequent utilization of the capacitor voltages by a push-pull modulator, but is not essential to a consideration of the bridge circuit itself.

It will be noted that this is a closed loop for dc with the two tubes acting in series, thus making it essential that the tubes pass identical direct currents. Any difference in current flow through the tubes constitutes a charging current to the capacitors, and therefore must be of short duration, for, when allowed to continue, the voltage developed by the charge increases into balance by sufficiently increasing the cathode-anode voltage of one tube while decreasing the voltage across the other. It is essential, therefore, that for at least the range of usable capacitor voltage the plate currents must be absolutely independent of plate voltages. Battery voltages B_1 and B_2 help make this possible in two ways: First, by making this voltage high with respect to the maximum capacitor voltage, the plate-to-cathode voltage variations are held to a small per cent of the total; and, second, by maintaining the plate voltage considerably greater than the maximum screen voltage, maximum plate resistance is assured.

An additional factor which influences the plate resistance of a pentode is its plate current, with maximum resistance occurring at minimum plate current. It is absolutely essential, therefore, to use the minimum plate current consistent with reliable operation. In the circuit of Fig. 7, R_1 and R_2 are very high value (1 megohm) cathode resistors which produce essentially cutoff bias. A last, but very important, factor influencing plate resistance is the screen voltage, with infinite plate resistance resulting from zero applied screen voltage.

It was found by experiment that a higher average plate resistance could be maintained by using the screen as the control electrode, and utilizing the grid only as a biasing electrode. The grid must not be bypassed, as that seriously reduces the plate current which will flow during positive excursions of the screen driving voltage. This effect is apparently due to the internal capacitance of the tube from screen to grid which provides coupling from the screen to the grid. The net result of this method of excitation is that plate current flows in short pulses at the peak of each rf cycle, and is therefore at cutoff most of the time. This results in a plate-current pulse whose value is determined almost entirely by the peak value of the applied screen voltage, and is little affected by the plate-to-cathode voltage.

A few troublesome, but not too serious, limitations are imposed by practical circuit considerations. These limitations are leakage and stray charging circuits. Since, for absolute control, the electron-tube bridge circuit should be the only path through which the capacitors may be charged or discharged, any other paths must be eliminated or reduced to a negligible amount. In considering the possible undesired charging paths, any circuit connected to the capacitor terminals must be viewed with suspicion as a possible path to ground. The circuit for transforming the capacitor voltage into a modulator bias voltage must have infinite dc input resistance. A dual triode (6SN7), operating as a cathode follower at very low plate current and with reduced heater voltage, was found to be entirely practical for this circuit.

The cathodes of the bridge tubes are a potential source of trouble, as leakage to or emission from the heater, and thence to ground or to the opposite tube, is possible. It was found that emission from heater to cathode was the most serious source of trouble, but this was effectively eliminated by operating the heaters at reduced voltage. Naturally, wiring and component insulation must be the best possible. A second dual-triode cathode follower was added to provide a monitoring meter, but the same precautions were followed as on the cathode follower for modulator bias, so that no trouble was experienced.

Careful shielding of the entire bridge and discriminator from stray rf fields is essential, since it is easy for such fields to produce additional excitation of the bridge tubes, or to be rectified and act as a charging current for the capacitors.

It will be noted that, where Fig. 7 shows series batteries in the dc loop, Fig. 6 shows diodes and an rf voltage source. Since the power requirement for this bridge is so very low, it was practical to derive the dc voltage for the bridge from the same source of rf that furnishes excitation to the discriminator. This minimizes the circuit complexity and maintains the maximum freedom from leakage.

Example of Operation

If it is assumed that capacitors C_1 and C_2 (see Fig. 7) are short-circuited, and the oscillator frequency is adjusted to the value which produces identical currents in the two bridge tubes, there will be no current flowing in the capacitor-shorting conductor. Removing this shorting conductor will, therefore, leave the capacitors in an uncharged condition with zero voltage between terminals. Now, assuming that the oscillator frequency changes to a higher value, the output voltages from the two halves of the crystal discriminator will no longer be equal. Since the frequency has moved closer to the series resonance of the high-frequency crystal, that half of the discriminator containing this crystal will have a lower shunt-arm impedance, and therefore lower output voltage. The output voltage from that discriminator half containing the low-frequency crystal, however, will be high, since its action is the inverse of the other.

These output changes then cause a plate-current increase in one control tube, and a decrease in the other control tube. Since the difference in plate currents of these tubes is a charging current to the capacitors, they must simultaneously develop a potential difference between terminals. Since, also, the terminals of these capacitors are connected to the grids of a reactance-tube modulator, the voltages developed across them will cause the frequency to change to a higher or lower value. In this example, the voltages are such as to cause the frequency to be lowered. It will be observed, then, that, as the capacitor is charging, the oscillator frequency is being lowered in proportion to the charge. As the frequency is lowered, however, it approaches and must finally return to the value which produces identical control-tube currents. At this frequency, then, the charging current will cease, and the capacitors will remain with whatever charge and terminal voltage were required to return the oscillator to its original frequency.

Now, if the oscillator resonant circuits change back to their original values, the actual frequency will be too low because of the effect of the charge on the capacitors. The control circuit will, however, discharge the capacitors, charging them with the opposite polarity, if necessary, until the frequency is returned to the balancing value.

Hunting or overshooting during correction can be completely eliminated in this circuit. Hunting is caused by a delay between the discerning of a need for correction and the subsequent correction. By maintaining the time constant between the charging capacitors C_1, C_2 and the modulator grids at a very low value, the oscillator frequency can be made to follow capacitor voltage changes with negligible delay. The necessary time constant that filters out the modulation component and prevents rapid frequency correction is provided by the high impedance of the bridge circuit which limits the charging current to the relatively large capacitors. A further deterrent to hunting is the asymptotic reduction to zero of the charging current as frequency correction is affected.

The fact that this oscillator may be modulated in frequency by an ac modulating signal during the above outlined operation does not alter the considerations, in all currents and voltages referred to are considered as average values. Linear impedance versus frequency characteristics of the crystal discriminator over the range of frequencies included by the ac modulation is, of course, a necessary prerequisite to this conclusion.

Performance

Several complete units utilizing the circuits described in this paper have been in operation for several months. Although operating data have not been sufficiently analyzed for the drawing of final conclusions, a few performance remarks are in order.

One performance check is the ability of the unit to maintain frequency in the face of serious oscillator-frequency drift. Fig. 9 is a plot of the measured frequency

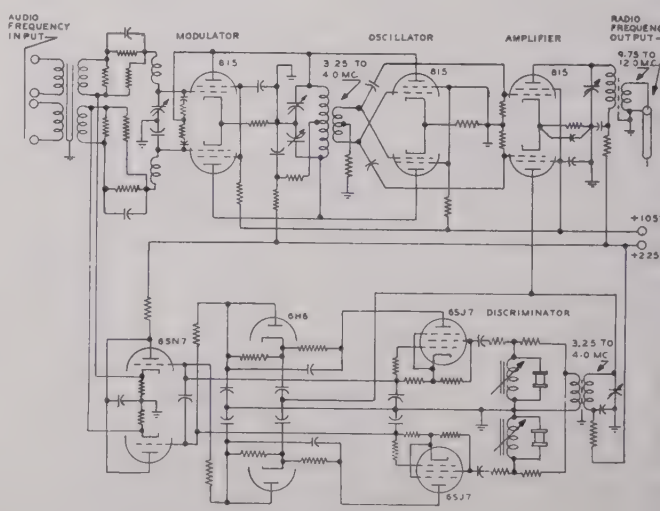

Fig. 8—A simplified circuit diagram of the complete frequency-modulated exciter.

error during automatic control versus the natural frequency of the oscillator when operated without control. It was obtained by manually detuning the oscillator tank circuit with the unit under control and measuring the operating frequency, then short-circuiting the control capacitors and again measuring the oscillator frequency. It will be noted that, with the oscillator tank circuit detuned as much as 30 kc, which is practically

the limit of the modulator, correction was maintained well within the ±76-cps FCC tolerance. It will be noted further that, if a stability factor of ±0.1 per cent is assumed as practical for a self-excited oscillator, this circuit will correct any variations within this tolerance to within 4 cps of exact frequency.

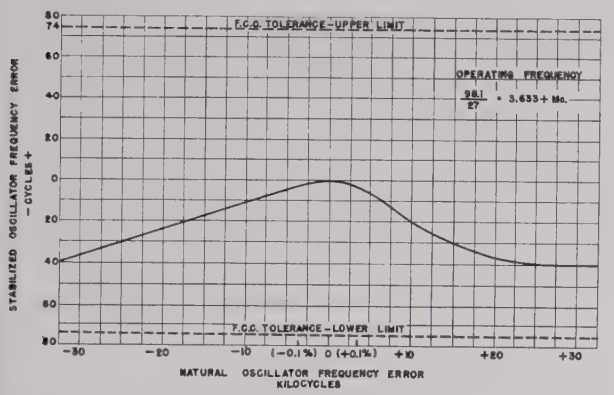

Fig. 9—Measured frequency-control ability with changes in oscillator natural frequency.

A second important basic measurement is the frequency stability under various degrees of variable-frequency sine-wave modulation. It can be shown that, if there are spurious responses or lack of symmetry in the discriminator circuit beyond the limit of frequency swing, but inside the limit of the highest-frequency sidebands, this control circuit will respond as if there were a center-frequency shift at certain modulation frequencies. The modulation frequency most likely to produce trouble is that which produces the first sideband pair at the series-resonant frequencies of the crystals. Frequencies above this value may cause center-frequency shift of a lesser degree if the discriminator return slopes are not sufficiently symmetrical. Measurements of this type on the completed unit showed a maximum of 1000 cps (at 100 Mc) deviation with continuous 100 per cent modulation at the most troublesome frequency. Since this is a condition of modulation which can never be experienced with actual program modulation and can only be repeated by actual 100 per cent sine-wave modulation at one particular frequency, it is not considered a serious fault of the equipment.

The effect of program modulation of all types has been carefully observed, and to date there has been no observable correlation between program modulation and center-frequency drift. As observed on a General Electric FM broadcast frequency monitor, there is an ambiguity of about 400 cps with a maximum recorded indication of about 1100 cps error. These are 100-Mc measurements where the maximum tolerance is ±2000 cps.

Conclusion

That a relatively simple, straightforward frequency-modulated exciter for broadcast use utilizing directly the reactances of piezoelectric crystals as a frequency standard is practical has been proved by the unit around which this paper was written. The simplicity of the circuit lends itself to a compact, yet readily accessible, mechanical design which is hardly larger than the standard-broadcast-frequency crystal oscillators in use today. Because of its compactness, it is easy to make dual installations for a frequency-modulated transmitter the better to insure uninterrupted service. The design illustrated with this paper is intended for dual installation, either in the transmitter or in a standard equipment rack along with the frequency monitor. Two units mount side by side and together occupy the space of one $10\frac{1}{2}$-inch panel-height unit. One of these exciters has been in continuous operation for over a year as the exciter for WLWA of the Crosley Broadcasting Corporation.

Although this application of the crystal discriminator is for transmitter frequency control, its usefulness is not confined to that alone, as it could be very easily adapted for use as a direct-reading center-frequency monitor or a direct-reading monitor for broadcast or short-wave use.

Acknowledgment

The author gratefully acknowledges the help and co-operation of the following men in particular, as well as many others not mentioned: R. J. Rockwell, vice-president in charge of engineering; C. R. Pfitzer, junior engineer; C. C. Bopp, development engineer; and C. A. Robson, development engineer, all of the Crosley Broadcasting Corporation.

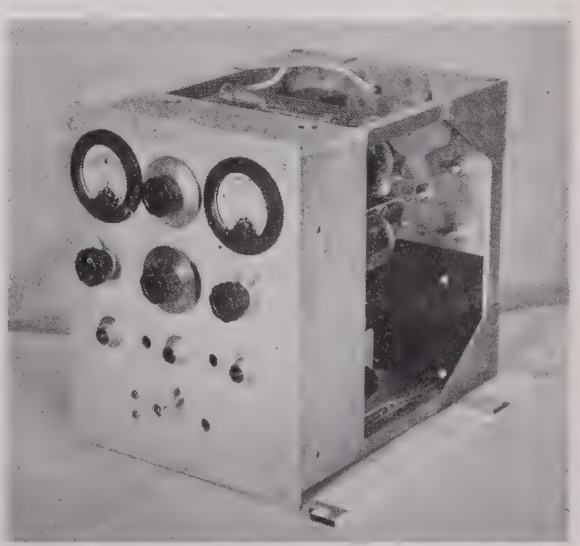

Fig. 10—A frequency-modulated exciter incorporating circuits described in the text.

The Cathode-Coupled Clipper Circuit*

L. A. GOLDMUNTZ†, STUDENT, IRE, AND HERBERT L. KRAUSS†, MEMBER, IRE

Summary—**The cathode-coupled amplifier circuit is used as a clipper with some advantages over the conventional pentode and diode types. An analytical solution is obtained for proper operating conditions to give the desired size of output pulse, allowable range of input voltages, bias values for symmetrical clipping, and the like. Regenerative feedback is shown to give considerable improvement in the clipping action, and design methods for obtaining optimum clipper performance are outlined.**

Introduction

CLIPPING AND LIMITING circuits[1] have found widespread use in frequency-modulation, radar, and pulse transmission circuits. The cathode-coupled clipper combines the desirable features of both the diode and overdriven-amplifier clippers. The cathode-coupled circuit does not depend on grid-current flow for clipping action, it allows large grid swings on the input stage without the flow of grid current, it has the high input impedance of a cathode follower, and it provides gain and produces a symmetrical[2] or unsymmetrical output pulse, as desired. With regeneration, the rise time of the output is limited only by the interelectrode capacitances.

Symbols and Definitions

μ = amplification factor of the tube at the quiescent operating point
μ' = amplification factor of the tube at plate-current cutoff
r_p = dynamic plate resistance of the tube
g_m = grid-plate transconductance
K = a tube "linearity factor" defined in the text
R_k = cathode resistor
R_g = grid resistor
R_b = plate load resistor
C_c = coupling capacitor
E_{bb} = plate supply voltage
e_b = instantaneous plate-cathode potential
i_b = instantaneous plate current
E_s = effective value of signal voltage
e_{co} = grid-cathode potential for plate-current cutoff
E_{cc} = grid-bias voltage
e_+ = instantaneous grid-to-ground potential on tube 1 which cuts off tube 2
e_- = instantaneous grid-to-ground potential on tube 1 which cuts off tube 1
E_{sm} = peak amplitude of the input signal
E_0 = peak amplitude of the output pulse
e_{max} = grid-to-ground potential on tube 1 which makes the instantaneous grid-cathode potential zero
e_{k+} = voltage drop across the cathode resistor when tube 2 is just cut off
e_{k-} = voltage drop across the cathode resistor when tube 1 is just cut off
e_c = instantaneous grid to cathode potential drop
E_{fb} = peak amplitude of feedback signal.

The Cathode-Coupled Clipper

The cathode-coupled clipper utilizes the cathode-coupled amplifier circuit[3,4] as shown in Fig. 1. The circuit acts as a linear amplifier for small input signals, but large signals drive the tubes alternately to cutoff, thus producing a clipped output voltage. On the positive half-cycle of a sufficiently large input signal, the cathode potential rises to cut off tube 2. Once tube 2 has been cut off, a further increase in the input-grid potential has no effect on the output voltage. Similarly, on the negative half-cycle tube 1 is driven to cutoff because the plate current of tube 2 tends to hold the cathode potential at a high level, and once tube 1 is cut off, the signal voltage has no effect on the output. The clipping action begins at potentials essentially determined by the plate-supply and tube parameters. The grid biases determine the operating points of the stage, and, therefore, the amplitude and shape of the pulse output. Positive biases are used on the grids to increase the size of the output

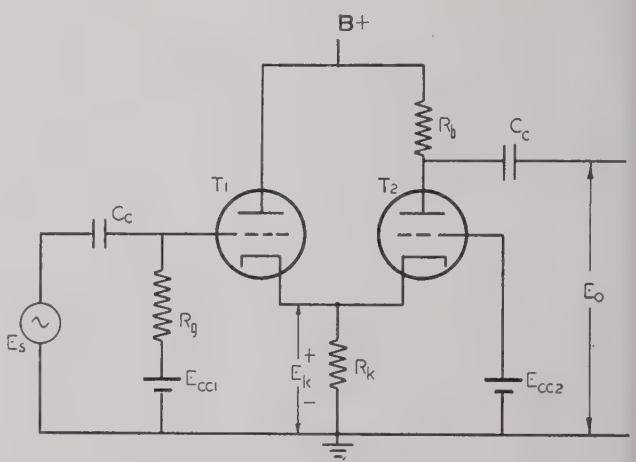

Fig. 1—Cathode-coupled clipper.

* Decimal classification: R139.2×R363.2. Original manuscript received by the Institute, December 18, 1947; revised manuscript received, March 22, 1948.

† Yale University, New Haven, Conn.

[1] N. W. Mather, "Clipping and clamping circuits," *Electronics*, vol. 20, pp. 111–113; July, 1947.

[2] The term "symmetrical pulse" means that the clipper produces a square-wave output.

[3] K. A. Pullen, "The cathode-coupled amplifier," PROC. I.R.E., vol. 34, pp. 402–405; June, 1946.

[4] Murray G. Crosby, "A two-terminal oscillator," *Electronics*, vol. 19, pp. 136–137; May, 1946.

pulse. By proper adjustment of these biases, symmetry of clipping becomes independent of the relative sizes of R_k and R_b.

Graphical Analysis

A graphical analysis of the cathode-coupled clipper circuit may be used to determine the values of the grid biases and the minimum input signal necessary to obtain a pulse of given amplitude and duration. This analysis is possible because, when the input signal is large enough to cause clipping of both positive and negative signal peaks, each tube operates independently during its conduction period, except for the switch-over time.

Therefore, it is possible to draw two load lines corresponding to the two tubes and their respective circuit impedances on a set of plate characteristics. The operating point of tube 2 is determined by the current flow necessary to produce the desired output pulse E_0. From this operating point the instantaneous grid to ground potential (e_-) on tube 1, which cuts off tube 1, may be determined. The clipping potential e_+ on the grid of tube 1, that just causes tube 2 to cut off can be found by a graphical trial and error process.[5] Referring to Figs. 1 and 2, if symmetrical clipping is desired, the bias voltage E_{cc_1} should be adjusted to the average of the two clipping potentials

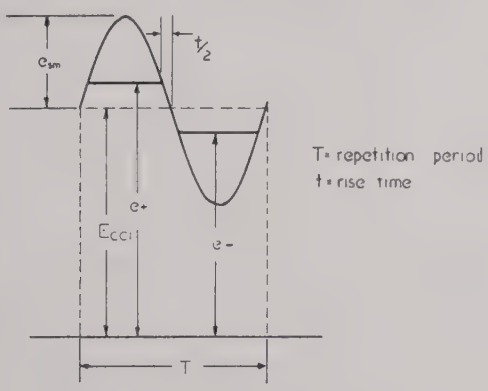

Fig. 2—Definition of rise time (t/T).

$$E_{cc_1} = \frac{e_+ + e_-}{2}. \tag{1}$$

Then the peak amplitude of a sinusoidal signal which just causes clipping to occur is

$$E_{sm} = e_+ - E_{cc_1}. \tag{2}$$

Signals of at least ten times this amplitude would be necessary to obtain reasonably square output pulses. The per cent rise time of the pulse may be defined (see Fig. 2) as the ratio of the rise time to the repetition period. Under the assumptions that the rise time is small compared to the period, and that the interelectrode capacitances are negligible, the *per cent* rise time is given approximately by

$$\text{per cent rise time} = \frac{(e_+ - e_-)100}{2\pi E_{sm}}. \tag{3}$$

Analytical Solution

The analytical solution, given in full in Appendix A, employs two tube parameters that are not ordinarily used: K, a tube "linearity" factor, and μ', the ratio of plate voltage to grid voltage at plate-current cutoff. The factor μ' is a constant over a wide range of plate voltages and can be determined experimentally, or from the plate characteristics.

The derivation in Appendix A yields the exact expressions for the two clipping potentials (e_+ and e_-) and the amplitude of the output pulse (E_0) in terms of circuit and tube parameters. If the circuit and tube parameters are such that the assumptions $\mu \gg 1$, $\mu' \gg 1$, $K \ll E_{bb}$, and $(1+\mu)R_k \gg r_p + R_b$ are valid, the exact expressions for e_+, e_-, and E_0 can be approximated by

$$e_+ \cong E_{cc_2} + E_{bb}\frac{(\mu - \mu')}{\mu\mu'} \tag{4}$$

$$e_- \cong E_{cc_2} - E_{bb}\frac{(\mu - \mu')}{\mu\mu'} \tag{5}$$

$$E_0 \cong \frac{R_b}{R_K}\left[\frac{E_{bb}}{\mu} + E_{cc_2}\right]. \tag{6}$$

Thus, for symmetrical clipping, the values of E_{cc_1} and E_{cc_2} are approximately equal. If a more accurate adjustment for symmetry is required, the exact solution may be used, or the bias may be adjusted in the laboratory.

Input Ratio of the Clipper

An important figure of merit of this clipper is the "input ratio," which is a measure of the ratio of (1) the maximum grid-to-ground signal that may be applied to tube 1 without causing grid current to flow, to (2) the minimum grid-to-ground signal at which clipping just begins. The input ratio is expressed as

$$\text{Input Ratio } (IR) = \frac{e_{\max} - E_{cc_1}}{e_+ - E_{cc_1}} = \frac{E_{sm}}{e_+ - E_{cc_1}} \tag{7}$$

where $e_{\max}$ is the highest positive grid-ground potential that can be applied to tube 1 without causing grid current to flow. Since grid current is undesirable, the peak signal (E_{sm}) which may be applied to tube 1 is usually limited to $e_{\max} - E_{cc_1}$. Since the rise time of the output pulse, as already shown, is approximately an inverse function of the amplitude of the applied sinusoid, the largest possible input ratio is desired in order to obtain the best possible square-wave output.

[5] Research Report 1, Contract W36-039 SC 33649, Dunham Laboratory, Yale University, New Haven, Conn.

The potential e_{max} used in (7) is given approximately in terms of the circuit parameters by

$$e_{max} \cong \frac{E_{bb}R_k}{R_k + r_p}. \tag{8}$$

Since $E_{cc_1} \cong E_{cc_2}$ for symmetrical clipping, by substituting (4), (5), (6) and (8) in equation (7), the input ratio becomes

$$IR \cong \frac{\mu\mu'}{E_{bb}(\mu - \mu')}\left[\frac{E_{bb}R_k}{R_k + r_p} + \frac{E_{bb}}{\mu} - \frac{E_0R_k}{R_b}\right]. \tag{9}$$

Thus, to obtain minimum rise time, the maximum allowable input signal ($E_{sm} = e_{max} - E_{cc_1}$) should be used, and the circuit should be adjusted for the maximum input ratio.

Use of Regenerative Feedback

If regenerative feedback is used in the cathode-coupled clipper, the expression for the input ratio becomes (see Appendix B)

$$IR = \left[\frac{R_kE_{bb}}{R_k + r_p} - \frac{R_kE_0}{R_b} + \frac{E_{bb}}{\mu}\right] \Big/ \left[\frac{E_{bb}(\mu - \mu')}{\mu\mu'} - \frac{E_{fb}}{2}\right] \tag{13}$$

In this equation the input ratio may be considered to be a function of just one variable, R_k, if the pulse output E_0 and load resistor R_b are fixed. (The size of R_b is generally fixed by the upper frequency limit, and E_0 is determined by the requirements of the next stage.) A plot of the input ratio versus R_k for various values of E_0/R_b from (9) is shown in Fig. 3 for a 6SN7 tube. For low values of E_0/R_b, the value of R_k that yields a maximum input ratio is not critical, but for high values of E_0/R_b the size of R_k must be adjusted carefully.

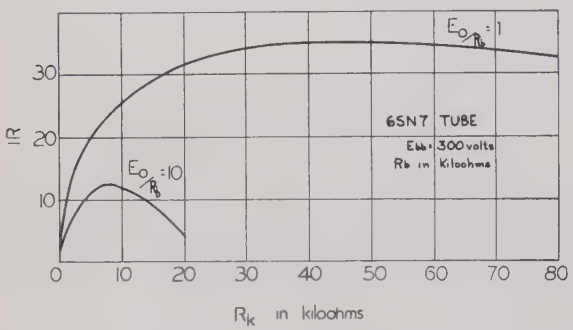

Fig. 3—Input ratio or ratio of overloading signal to minimum clipping signal as a function of cathode resistance for a 6SN7 tube.

For a given pulse output the values of R_k and E_{cc_2} that provide a maximum input ratio are

$$R_k = \sqrt{\frac{E_{bb}R_br_p}{E_0}} - r_p \tag{10}$$

$$E_{cc_2} = \sqrt{\frac{E_{bb}E_0r_p}{R_b}} - \frac{E_{bb}}{\mu} - \frac{E_0r_p}{R_b}. \tag{11}$$

In some applications the value of E_{cc_2} indicated by (11) may cause grid current to flow in tube 2. The maximum input ratio then cannot be realized.

The rise time of the pulse, if interelectrode capacitances may be neglected, can be expressed in terms of the input ratio, by using expressions (3) and (7):

$$\text{per cent rise time} = \frac{100}{\pi(IR)}. \tag{12}$$

where E_{fb} is the peak amplitude of the feedback voltage.

This expression for input ratio with regenerative feedback, when compared to expression (9) for the input ratio without regenerative feedback, shows that feedback increases the input ratio. This fact indicates that the circuit may be made to slip with smaller input signals or that the time rise of the output may be decreased for the same input signal, if the rise time is not limited by the interelectrode capacitances.

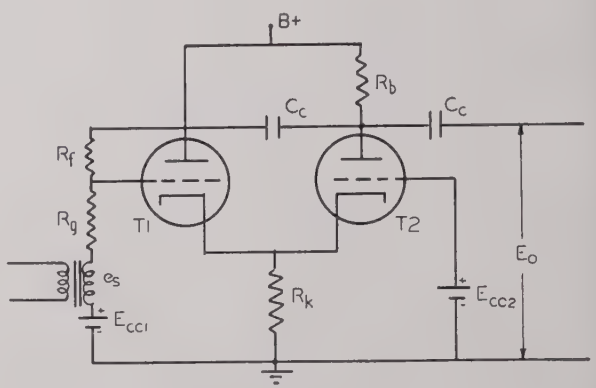

Fig. 4—Basic regenerative cathode-coupled clipper.

The simplest variation of the basic clipper circuit which involves regenerative feedback is shown in Fig. 4. In this circuit the feedback voltage is

$$E_{fb} = E_0 \times \frac{R_g}{R_g + R_f}. \tag{14}$$

The disadvantage of this circuit is that signal voltage is fed into the output circuit through the feedback path, and also that, if transformer coupling cannot be used, a high-impedance driving source is needed so that the impedance presented to the feedback voltage is not appreciably altered.

The circuit of Fig. 5 is usually more desirable, since the input signal cannot be fed into the output through the feedback path. The load resistor of the input tube

R_{b1} is made only large enough to supply the necessary feedback voltage. The reduction in input ratio due to R_{b1} in the plate circuit of tube 1 is more than offset by

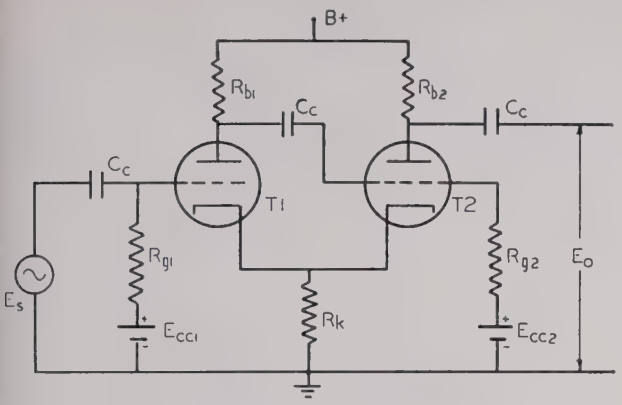

Fig. 5—Preferred regenerative clipper.

the increase in input ratio due to regeneration. The time constant C_cR_{g2} in the circuit of Fig. 5 should be large compared to the period of the lowest frequency to be clipped, if a flat-topped output is desired.

If the feedback voltage E_{fb} should be increased so as to make the denominator of (13) zero, the feedback voltage would approximately equal the difference between the clipping potentials. (See (4) and (5).)

$$E_{fb} = 2\frac{E_{bb}(\mu - \mu')}{\mu\mu'} = e_+ - e_-. \tag{15}$$

This means that not only would the input ratio become infinite, yielding a rise time limited only by the interelectrode capacitances, but also that the clipper would either multivibrate or trigger, depending on the elements of the feedback path. The circuit triggers or multivibrates when the feedback voltage equals the difference between clipping potentials, because the nonlinear loop gain $(E_{fb}/e_+ - e_-)$ is then greater than unity. When the nonlinear loop gain is greater than unity, the circuit is capable of driving itself so that either one or the other tube is cut off.

The function of the input signal under these conditions is to synchronize the resulting free-running multivibrator or trigger circuit to input-signal frequency. If the nonlinear loop gain is very much greater than unity, the synchronizing signal must be large. Experimentally, it has been found that more rectangular output voltages are obtained for a given input signal when the clipper is operated with a gain slightly greater than unity. If it is undesirable to have the circuit multivibrate or be in a triggered state when the input signal is removed, the nonlinear and linear loop gain must be less than unity.

The frequency of the free-running multivibrator shown in Fig. 5 may be shown by standard transient analysis[5] to be

$$f = \frac{1}{R_{g2}C_c \ln\left[\dfrac{(E_0 - \beta + \alpha)(E_0 - \beta - \alpha)}{(\beta - \alpha)(\beta + \alpha)}\right]} \tag{16}$$

where

$$\alpha = E_{cc_1} - E_{cc_2}$$

$$\beta = \frac{E_{bb}(\mu - \mu')}{\mu\mu'}.$$

This frequency should be lower than the lowest frequency to be clipped, so that the triggering from one state to another is always initiated by the input signal.

With this type of operation, R_k and E_{cc2} should be chosen on the basis of frequency and output-pulse requirements, and not necessarily in accordance with (10) and (11), which hold only when the gain is less than unity.

Tubes best suited for this circuit are high-g_m duo-triodes which are linear, i.e., whose "cutoff" factor, $(\mu - \mu')/\mu\mu'$, is low. Table I is a list of "cutoff" factors for various tubes. The g_m should be high so that a large output pulse may be obtained with a small load resistor, and the cutoff factor should be low so that the input ratio can be high and the difference between clipping potentials low.

TABLE I

Tube	$\frac{\mu - \mu'}{\mu\mu'}$	g_m
6J4	0.0142	12,000
6J6	0.0206	5,300
6SN7	0.0214	2,600
6SL7	0.00494	1,600

Acknowledgment

The investigation of this clipper circuit was made in connection with a research project in pulse-modulation methods sponsored by the United States Signal Corps. The authors wish to thank P. F. Ordung, of Yale University, for suggesting this type of clipper circuit, and for his helpful criticism during the investigation.

Upon completion of this work, the writers learned that a similar circuit had been developed simultaneously at the Bell Telephone Laboratories.

Appendix A

In the circuit of Fig. 1, the potential e_+, on the grid of tube 1, necessary to cut off tube 2, may be expressed in terms of a summation of voltages in the grid-cathode circuit of tube 1:

$$e_+ = e_{k+} + e_{c1}. \tag{17}$$

The voltage drop e_{k+} across the cathode resistor, necessary to cut off tube 2, may be expressed as

$$e_{k+} = i_{b1}R_k = E_{cc_2} - e_{co_2} \tag{18}$$

where e_{co_2}, the cut off bias of tube 2, is given by

$$e_{co_2} = \frac{-(E_{bb} - e_k)}{\mu'}. \tag{19}$$

The total instantaneous plate current flowing in a given tube, with a given plate supply and bias voltage, may be expressed as

$$i_{b1} = g_m e_{c1} + \frac{e_b}{r_p} - \frac{K}{r_p}$$

$$= g_m e_{c1} + \frac{E_{bb} - i_{b1}R_k}{r_p} - \frac{K}{r_p} \tag{20}$$

in which K is a linearity factor determined from the linear portion of the static characteristics. For a 6SN7, $K=12$; for a 6J6, $K=9$.

Equations (17), (18), (19), and (20) may be combined to obtain

$$e_+ = \frac{\mu' E_{cc_2} + E_{bb}}{(1+\mu')} - \frac{E_{bb} - K - (R_k + r_p)\left[\dfrac{\mu' E_{cc_2} + E_{bb}}{(1+\mu')R_k}\right]}{\mu} \tag{21}$$

for the potential e_+, on the grid of tube 1, necessary to cut off tube 2.

By a similar analysis, e_-, the potential on the grid of tube 1, necessary to cut off tube 1, may be found.

$$e_- = \frac{R_k(1+\mu')\left[\dfrac{\mu E_{cc_2} + E_{bb} - K}{r_p + R_b + (1+\mu)R_k}\right] - E_{bb}}{\mu'}. \tag{22}$$

The amplitude of the pulse output may be determined in terms of the current that flows in tube 2, when only tube 2 conducts. An equation similar to (20) may be written for tube 2, giving

$$i_{b2} = g_m e_{c2} + \frac{E_{bb} - i_{b2}(R_k + R_b)}{r_p} - \frac{K}{r_p} \tag{23}$$

where

$$e_{c2} = -e_{k-} + E_{cc_2} = -i_{b2}R_k + E_{cc_2}. \tag{24}$$

Then E_0, the output pulse, is given by

$$E_0 = R_b\left[\frac{E_{bb} + \mu E_{cc2} - K}{r_p + (1+\mu)R_k + R_b}\right]. \tag{25}$$

Appendix B

The effect of regenerative feedback can be shown by assuming a feedback voltage E_{fb} in series with the grid of tube 2, as shown in Fig. 6. Tube 2 is cut off when the following equation is satisfied:

$$e_{k+} = -e_{co_2} - \frac{E_{fb}}{2} + E_{cc} \tag{26}$$

where

$$e_{co_2} = -\left[\frac{E_{bb} - e_{k+}}{\mu}\right] \tag{27}$$

and where E_{fb} is the peak-to-peak feedback voltage applied to the grid of tube 2.

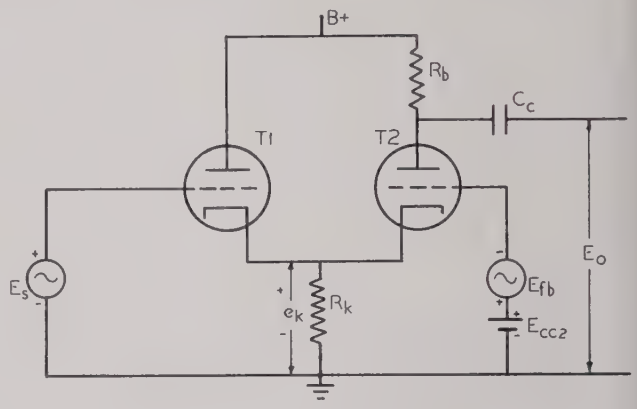

Fig. 6—Circuit used in analysis of cathode-coupled clipper.

Combining (26) and (27),

$$e_{k+} = \frac{E_{bb} + \mu'(-E_{fb/2} + E_{cc_2})}{(1+\mu')}. \tag{28}$$

Since tube 2 is cut off,

$$e_{k+} = i_{b1}R_k \tag{29}$$

and by combining (28) with (29), equation (30), which is the current in tube 1 at the instant tube 2 is cut off, may be obtained:

$$i_{b1} = \frac{E_{bb} + \mu'(-E_{fb/2} + E_{cc_2})}{R_k(1+\mu')}. \tag{30}$$

Summing the voltages in the grid-cathode circuit of tube 1 when tube 2 is just cut off by the potential e_+ on the grid of tube 1,

$$e_+ = e_{k+} + e_{c1}. \tag{31}$$

The value of e_{c1} may be found from (20) in Appendix A, so that, by combination of (20), (28), and (30), the positive potential that just cuts off tube 2 may be written:

$$e_+ \cong E_{cc_2} - \frac{E_{fb}}{2} + E_{bb}\frac{(\mu - \mu')}{\mu\mu'} \tag{32}$$

if

$$\mu \gg 1, \qquad \mu' \gg 1, \qquad E_{bb} \gg K$$

and

$$(1 + \mu)R_k \gg r_p.$$

Tube 1 is cut off when the following summation of voltages in the grid-cathode circuit of tube 1 is satisfied:

$$e_{k-} + e_{co_1} = e_- \tag{33}$$

where

$$e_{co_1} = -\left[\frac{E_{bb} - e_{k-}}{\mu'}\right]. \tag{34}$$

Solving (33) and (34) gives

$$e_{k-} = \frac{E_{bb} + \mu' e_-}{1 + \mu'}. \tag{35}$$

The drop across the cathode resistor e_{k-} when tube 1 is cut off is due to the current flowing in tube 2, so that

$$e_{k-} = i_{b2}R_k. \tag{36}$$

By reference to (23) and (24) of Appendix A, the constant current in tube 2, while tube 1 is cut off, may be expressed as

$$i_{b2} = \frac{E_{bb} + \mu(E_{cc_2} + E_{fb/2}) - K}{r_p + (1 + \mu)R_k + R_b}. \tag{37}$$

By a combination of (35), (36), and (37), the potential on the grid of tube 1 that just cuts off tube 1 is

$$e_- \cong -\frac{E_{bb}(\mu - \mu')}{\mu\mu'} + E_{cc_2} + \frac{E_{fb}}{2} \tag{38}$$

if

$$\mu' \gg 1, \quad \mu \gg 1, \quad (1 + \mu)R_k \gg r_p + R_b, \quad E_{bb} \gg K.$$

To obtain the difference between clipping potentials, subtract (38) from (32).

$$e_+ - e_- = 2\frac{E_{bb}(\mu - \mu')}{\mu\mu'} - E_{fb}. \tag{39}$$

By reference to expression (39) above and to expressions (4), (5), (6), and (7) in the text, the expression for the input ratio may be obtained

$$IR = \frac{\dfrac{R_k E_{bb}}{R_k + r_p} - \dfrac{R_k E_0}{R_b} + \dfrac{E_{bb}}{\mu}}{\dfrac{E_{bb}(\mu - \mu')}{\mu\mu'} - \dfrac{E_{fb}}{2}}. \tag{40}$$

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Lawrence Goldmuntz (S'47) was born on June 10, 1922, in New York, N. Y. He received the B.E. degree in electrical engineering from Yale University in 1947, and is now a graduate student in electrical engineering at that University.

L. Goldmuntz

During the war Mr. Goldmuntz was engaged in sonic deceptive warfare in this country and in the Mediterranean theater. He studied electrical engineering at the university of Michigan under the ASTP for nine months. He is a member of Tau Beta Pi.

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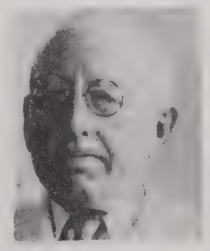
James L. Hollis (S'37–A'40–M'44–SM'46) was born near Omaha, Neb., in 1915, but received his formal education in Kansas, graduating from Kansas State College with the B.S. in electrical engineering in 1938.

After graduation, he spent one year with First National Television, Inc., of Kansas City, leaving there in early 1939 to join the broadcast engineering staff of the Crosley Corporation in Cincinnati, Ohio. While associated with Crosely, he was engaged in the design, development, and installation of the experimental television equipment; a broadcast frequency satellite station; the OWI "Voice of America" transmitters and plant near Bethany, Ohio; and an FM transmitter and installation. He was advanced to the position of chief design engineer during the "Voice of America" project.

In June, 1947, Mr. Hollis joined the engineering staff of the Collins Radio Company, where he is now active as a design engineer in the broadcast transmitters section. He is a registered professional engineer in the State of Ohio, a member of the National Society of Professional Engineers, member of Kappa Eta Kappa, member of the Cedar Rapids Engineer Club, 1948 program chairman of the Cedar Rapids Section of the IRE, and an active radio amateur.

James L. Hollis

Stanford Caldwell Hooper (F'28–A'–33 F'46) was born on August 16, 1884, in Colton, Calif. His early education was received in the public schools of San Bernardino, and he worked as a relief telegraph operator during his summer vacations. He was graduated from the United States Naval Academy in 1905, and instructed in electricity, physics, and chemistry at the Naval Academy from 1910 to 1911. Later he served for two years as the first Fleet Radio Officer, resuming that post again from 1923 to 1925, and he was in charge of the Radio Division of the Navy Department for eleven years. He was Director of Naval Communication from 1928–1934. Admiral Hooper has been a leader in developing the field of wireless radio communications in the Navy by carrying out pioneer tests, establishing a chain of land stations for communication between fleet and land, and serving as technical advisor and head of numerous boards and committees dealing with communications. He suggested the office of Fleet Radio Officer as necessary to the new



S. C. Hooper

Contributors to Waves and Electrons Section

radio communications, and served in this post two years. In the first World War he was awarded the Navy Cross for distinguished service as commanding officer of the U. S. S. *Fairfax*. He received an honorary LL.D degree from Drury College in February, 1948.

Admiral Hooper was retired in 1945, after forty years of service, with civilian, military, and foreign awards and medals for "outstanding contributions to the radio art, particularly in building up the wireless communications system of the United States Navy from the stature of an engineering experiment to a major military arm for control, detection and communication."

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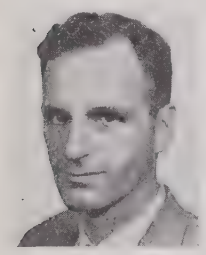
Heinz E. Kallmann (A'38–M'41—'SM'43) was born on March 10, 1904, in Berlin, Germany. He received the Ph.D. degree in physics from the University in Goettingen in 1929. From 1929 to 1934, Dr. Kallmann was a research engineer in the laboratories of the C. Lorenz A. G. in Berlin, concerned with the development of rf test equipment, and of ultra-short-wave receivers for communications and for wide-band television. He was also in charge of television development.



H. E. Kallmann

From 1934 to 1939, Dr. Kallmann was a television engineer with Electric and Musical Industries, Ltd., Hayes, England, doing research and advanced circuit development. Since 1939, He has been a consulting engineer in New York, N. Y., except for the period from 1943 to 1945, when he joined the Radiation Laboratory of the Massachusetts Institute of Technology, where he had charge of the circuit section of the test equipment group, and was a member of the fundamental microwave research group.

Dr. Kallmann is a member of the American Physical Society; an associate member of the Society of Motion Picture Engineers, and a member of its committee on television projection practice; and a member of the RMA committee on uhf television systems and some of its subcommittees.

Newton Monk was born in Stoughton, Mass., on December 5, 1897. After an interruption in his college work of two years when he served in the United States Army Signal Corps during World War I, he was graduated from Harvard College in 1920 with the degree of A.B., and in 1922 he received the degree of B.S. in communication engineering from the Harvard Engineering School. Immediately upon graduation, he joined the department of development and research of the American Telephone and Telegraph Company, where he worked on interference suppression and carrier transmission problems. In 1934 he transferred to the Bell Telephone Laboratories with the merger of the development and research department and that organization. At the Laboratories he continued his work on carrier transmission, and pioneered in the application of carrier systems to railroad communication lines.

Newton Monk

During World War II Mr. Monk was active in the development of voice-frequency and carrier equipment for the Signal Corps. Subsequently he has been concerned with the development of radio communication for railroads and airplanes.

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For a photograph and biography of Herbert L. Krauss, see the January, 1948, issue of the Proceedings of the I.R.E.

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R. J. Rockwell (A'25–M'31–SM'43–F'45) was born on February 5, 1904, in Omaha, Neb., and received the B.E. degree in electrical engineering from the Iowa State College in 1927. He became interested in radio prior to World War I, and had the first broadcast station, 9VE, west of the Mississippi River. This station began a regular daily broadcast for the Omaha Weather Bureau in 1920, and possibly was the first regularly scheduled broadcast in this country. He became chief engineer of radio station WOW, in Omaha, at its beginning, and after graduation from Iowa State College, he joined the test department of the General Electric Company, in Schenectady, N. Y. In 1928 he designed and installed radio stations KTHS, in Hot Springs, and KLRA, in Little Rock, Ark. In early 1940 he designed and constructed radio station WLWO, the first of the high-power international shortwave stations, and he completed the design and installation of the Bethany transmitting plant containing three 200-kilowatt shortwave transmitters in 1944.

R. J. Rockwell

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S. B. Wright (A'36–SM'45), was born on November 5, 1897, at Baltimore, Md. and was graduated from Cornell University with the degree of M.E. in electrical engineering in 1919. He joined the American Telephone and Telegraph Company and began making field studies of transmission over various types of long-distance telephone circuits, later becoming concerned with development of voice-operated devices for controlling two-way operation of transoceanic-type radio-telephone circuits. After transferring to Bell Telephone Laboratories in 1934, he was assigned to work on emergency radio and carrier systems. During the war he acted in an advisory capacity to the U. S. Navy on several confidential radio communication projects. In 1946 he acted as project engineer in the development of rural party-line telephone service by radio installations operating on subscribers' premises. His present work is concerned with applications of mobile telephone systems.

S. B. Wright

Mr. Wright is a member of the American Institute of Electrical Engineers and of the Honorary engineering societies Tau Beta Pi and Eta Kappa Nu.

Abstracts and References

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ACOUSTICS AND AUDIO FREQUENCIES

016:534 2118

References to Contemporary Papers on Acoustics—A. Taber Jones. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 201–213; March, 1948.) Titles of 334 papers, including 134 published in Gemany, mostly in the years 1939 to 1943. For earlier parts, see 1531 of July and back references.

534.138 2119

Theoretical and Observed Absorption of Sound in Suspensions—R. J. Urick. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 225; March, 1948.) Summary only.

534.2 2120

Absorption and Scattering for Impedance Boundary Conditions on Spheres and Circular Cylinders—M. Lax and H. Feshbach. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 108–124; March, 1948.)

534.2 2121

Compressional Wave Front Propagation through a Simple Vortex—R. B. Lindsay. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 89–94; March, 1948.)

534.21 2122

The Propagation of an Acoustic Wave along a Boundary—I. Rudnick. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 149; March, 1948.) Correction to 3387 of 1947.

534.213:534.241 2123

Examples of Propagation of Underwater Sound by Bottom Reflection—R. W. Young. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 224; March, 1948.) Summary only.

534.213:534.321.9 2124

The Attenuation of Ultrasonic Sound Waves in Water—R. W. Leonard. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 224; March, 1948.) Summary only. Measurements of attenuation in distilled water in the range 50 to 450 kc are in good agreement with those of Fox and Rock (2456 of 1941) in the range above 10 Mc. Measurements on filtered sea water from 50 to 180 kc give values for the attenuation about 10 times those for distilled water.

534.213:534.321.9 2125

Absorption of Sound in Fresh Water and in the Sea—L. N. Liebermann. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 223–224; March, 1948.) Summary only. At 500 kc and 1000 kc, the absorption coefficient in fresh water is proportional to (frequency)^2. Measurements in the sea at 120 kc gave a value more than 5 times the fresh-water upper limit. Measurements at 500 kc and 1000 kc indicate that in sea water the absorption is not proportional to (frequency)^2. At 1000 kc the absorption is only slightly higher than in fresh water.

534.32:786.6 2126

Some Measurements of the End-Corrections and Acoustic Spectra of Cylindrical Open Organ Pipes—F. Ingerslev and W. Frobenius. (*Trans. Dan. Acad. Tech. Sci.*, no. 1, pp. 7–44; 1947.) A formula is derived for the physical length of an open pipe. This can be used for both cylindrical and conical pipes; the accuracy is reduced in the latter case. Results are given showing the effect of (a) scale ratio, (b) height and width of the mouth, (c) air consumption, and (d) nicks in the tongue, on the harmonic content of the emitted sound.

534.321.9:526.956.5 2127

Progress of Echo Sounding—(*Electrician*, vol. 140, pp. 339–340; January, 1948.) Modern echo-sounding gear uses magnetostriction oscillators operating on frequencies between 10 and 40 kc. For very great depths, quartz oscillators are used. Similar apparatus is used for transmission and reception and continuous automatic records of depth are obtained to an accuracy within about 1 per cent. Some details are given of various types of equipment developed by the Admiralty.

534.321.9:534.22 2128

A Study of Ultrasonic Velocity and Absorption in Liquid Mixtures—C. J. Burton. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 186–199; March, 1948.) Measurements at 5 to 25 Mc in mixtures of water with alcohols, glycols, and glycol ethers.

534.422:534.321.9 2129

High Frequency Whistles; Edge Tones and Resonance—W. L. Nyborg and H. K. Schilling. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 224; March, 1948.) Summary only.

534.44 2130

The Analysis of Acoustic Signals by Means of the Self-Consistent Spectrum—J. J. Markham. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 95–98; March, 1948.)

534.61 2131

Measurement of Directivity Factor—C. J. Burbank. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 223; March, 1948.) Summary only. A mechanical mounting permits rotation of the equipment about two mutually perpendicular axes. Electronic apparatus integrates the power.

534.64 2132

A Method for Measuring Acoustic Source Impedance—J. E. White. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 221–222; March, 1948.) Summary only.

534.64 2133

Acoustic Impedance from Motional Impedance Diagrams—R. D. Fay and J. E. White (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 98–107; March, 1948.) When an acoustic load is coupled to an electromagnetic transducer by means of an air column, there is a critical length at which an increment in acoustic load produces a maximum increment of electrical impedance. A method is described which makes use of this critical coupling to obtain optimum precision in determining acoustic impedance. The characteristic impedance of the air column is directly measured.

534.64 2134

Acoustic Impedance Measurement of Very Porous Screen—C. M. Harris. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 222; March, 1948.) Summary only. Equations are derived which express the impedance of the screen in terms of the acoustic characteristics of a small rectangular chamber or closed cylinder. Results for very porous cloth are given.

534.7:611.85 2135

Construction Material of the Sound Conduction System of the Human Ear—H. G. Kobrak. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 125–130; March, 1948.)

534.75 2136

The Atonal Interval—I. Pollack. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 146–149; March, 1948.) The difference between the thresholds of audibility and tonality for sinusoidal signals, the atonal interval, ranged from about 2.5 db at 500 cps to about 6.5 db at 4000 cps.

534.75 2137

The Perception of Short Bursts of Noise G. A. Miller. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 160–170; March, 1948.)

534.75 2138
The Perception of Repeated Bursts of Noise—G. A. Miller and W. G. Taylor. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 171–182; March, 1948.)

534.756+534.78 2139
New Possibilities in Speech Transmission—D. Gabor. (*Jour. IEE* (London), part III, vol. 95, p. 39; January, 1948.) Discussion on 1228 of June.

534.78 2140
The Influence of Interaural Phase Relations upon the Masking of Speech by White Noise—J. C. R. Licklider. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 150–159; March, 1948.)

534.835 2141
Attenuation of Sound by Light-Weight Multiple Structures—L. L. Beranek. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 222; March, 1948.) Summary only. The full paper presents theory and charts for the calculation of attenuation in typical aircraft and office partitions.

534.839:623.827 2142
A Sheet of Air Bubbles as an Acoustic Screen for Underwater Noise—D. P. Loye and W. F. Arndt. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 143–145; March, 1948.) Measurements of noise from submarines in dock could be made satisfactorily by use of a bubble screen across the open end of the dock. This effectively shut out external noise.

534.843 2143
The Acoustical Performance of Rooms—H. W. Rudmose. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 225; March, 1948.) Summary only. Report of tests, in studios and rooms of average size, with equipment giving af signals varying continuously in frequency and in bandwidth. The upper frequency limit was 15 kc and bandwidth varied from 15 to 150 cps. The minimum bandwidth for a smooth response for a polycylindrical (plywood) studio is very much less than that required for a rectangular studio with conventional acoustic treatment. Knowledge of the minimum bandwidth for smooth response may help to determine the diffusion of a room.

534.845 2144
Sound Absorption by Porous Materials: Part 1—J. v. d. Eijk, C. W. Kosten, and W. Kok. (*Appl. Sci. Res.*, vol. B1, no. 1, pp. 50–62; 1947.) Maximum absorption is obtained when there is a small airgap between the structural wall or ceiling and the absorbing material. A new interferometer is described and measurements of the effect of varying the airgap are given and discussed. Very small airgaps have a marked effect on the frequency at which maximum absorption occurs.

534.845.1 2145
A New Method for Measuring Sound Absorption—C. W. Kosten. (*Appl. Sci. Res.*, vol. B1, no. 1, pp. 35–49; 1947.) The relation between the electric impedance of an electrodynamic loudspeaker and the acoustic impedance in front of the loudspeaker is given and discussed in detail. The consequences of the flexibility of the cone are studied. Simple formulas and graphs are given, connecting the electrical behavior and the absorption coefficient corresponding to the load on the loudspeaker. The method seems to permit absorption measurements at low frequencies (50 to 500 cps).

534.85 2146
Audio Noise Reduction System—H. F. Olson. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 222; March, 1948.) Summary only. A threshold type for sound reproducing systems. A nonlinear element allows the useful signal to pass but discriminates against the noise.

534.851 2147
Dynamic Noise Suppression Circuits—J. D. Goodell. (*Radio News*, vol. 39, pp. 46–48, 162; January, 1948.) For similar paper see 1233 of June. See also 932 of May (Scott.)

534.851:621.395.813 2148
Wow Meter for Turntable Testing—G. L. Sansbury and E. W. Pappenfus. (*Electronics*, vol. 21, pp. 108–111; March, 1948.) A toothed wheel, centered on the turntable and rotating between two pickup coils connected for maximum output, generates a tone. The output is fed through a peak clipper to a discriminator and rectifier. Instantaneous turntable speed variation is read directly on a calibrated meter.

534.86 2149
Does Distortion Matter? (*Wireless World*, vol. 54, pp. 97–98; March, 1948.) Report on an IEE discussion on the subject "To what extent does distortion matter in the transmission of speech and music." Points raised include the public reaction to an increased high-frequency response, the most desirable attenuation versus frequency characteristic and the effect of rooms on reproduction.

534.862:621.317.755 2150
Cathode-Ray-Oscillograph Images of Noise-Reduction Envelopes—B. H. Denney. (*Jour. Soc. Mot. Pic. Eng.*, vol. 50, pp. 37–49; January, 1948.) A method of presentation for use in sound-on-film work. An electronic switch generates repetitive pulses of af signals and a second switch picks up these signals and the resulting noise-reduction-bias envelope, recording both in suitable positions on the oscillograph.

534.87 2151
The Magnetrostrictive Radial Vibrator—L. W. Camp. (*Jour. Acous. Soc. Amer.*, vol. 20, p. 225; March, 1948.) Summary only. Description of apparatus for underwater signalling. A plastic cast for the toroidal windings improves the efficiency.

621.395.623.7:534.842 2152
A Proposed Loudness-Efficiency Rating for Loudspeakers and the Determination of System Power Requirements for Enclosures—H. F. Hopkins and N. R. Stryker. (Proc. I.R.E., vol. 36, pp. 315–335; March, 1948.) "Experimental and computed data relating to the loudness contribution of various ranges of the frequency spectra of speech and music are correlated with the corresponding energy distribution. A relatively simple measurement of sound pressure and a knowledge of certain acoustic radiation phenomena are applied to this correlation to form the basis of a method for predicting the loudness established by loudspeakers in enclosures. A loudness-efficiency rating for loudspeakers is suggested, and its application to sound-system engineering problems is described."

621.395.625 2153
The Development of Sound Recording and Reproduction—E. T. Fisk. (*Jour. Roy. Soc. A.* vol. 96, pp. 105–117; January 16, 1948. Discussion, pp. 118–120.) A historical review, with discussion of the design, performance, and scope of some modern film, disk, and magnetic tape and wire recording systems.

621.395.625.3 2154
Build Your Own Magnetic Tape Recorder—L. B. Hust. (*Radio News*, vol. 39, pp. 39–42, 167; February, 1948.) Complete specifications for the construction of a recording and playback unit with a playing time of 30 minutes.

621.395.625.3 2155
Basic Amplifier for a Wire Recorder—L. S. Hicks. (*Radio News*, vol. 39, pp. 44–45, 169; February, 1948.)

621.395.92 2156
Models 65 and 66 Hearing Aids—J. R. Power. (*Bell Lab. Rec.*, vol. 26, pp. 30–33; January, 1948.) Two compact, 3-stage instruments with built-in crystal microphones and weighing 6 and 8 oz respectively. Model 65 is suitable for persons with a small hearing loss. Model 66 is designed to provide maximum quality and output for anyone with correctable hearing. Additional amplification may be obtained with external batteries and the output characteristic may be modified by the use of alternative receivers.

621.395.92 2157
A Cavity Pressure Method for Measuring the Gain of Hearing Aids—E. L. R. Corliss and G. S. Cook. (*Jour. Acous. Soc. Amer.*, vol. 20, pp. 131–136; March, 1948.) The sound pressure generated by the hearing-aid receiver is measured by placing the receiver on a "artificial ear" coupler of volume 2 cm³. Free-field and pressure techniques are compared. The pressure and free-field methods give results of comparable accuracy; from a practical standpoint the pressure method has decided advantages.

621.396.645.371.029.3 2158
Negative Feedback Compression—J. T. Goode. (*Radio News*, vol. 39, pp. 118, 139; January, 1948.) Experimental results for a simple circuit.

531+534]:621.392 2159
Electromechanical and Electroacoustical Analogies and Their Use in Computations and Diagrams of Oscillating Systems [Book Review]—B. Gehlshøj. G. E. C. Gad, Copenhagen, 142 pp., 12 Kroner. (*Wireless Eng.*, vol. 25, p. 88; March, 1948.) Based upon work for a higher degree at the Royal Technical College of Denmark. "The treatment throughout is very clear and the book can be strongly recommended."

621.395.625.3 2160
Further Studies in Magnetophones and Tapes [Book Notice]—F.I.A.T. Final Report No. 923. H. M. Stationery Office, London, May 13, 1947, 131 pp., 17s. 6d. A report of the investigation of German recorders, Models K7 and K4, and K7 as modified for broadcasting. Discussion of principles of operation, circuit diagrams, response curves, and practical operating and maintenance. Recording playback, and erasing by an 80-kc tone are provided in a single unit. Details of the manufacture of three types of tape are included. See also 2320 of 1947 and back references.

ANTENNAS AND TRANSMISSION LINES

621.315.212:621.3.09 2161
The Propagation of Signals in a Coaxial Cable—L. A. Zhekulin. (*Radiotekhnika* (Moscow) vol. 3, pp. 22–35; January and February, 1948.) In Russian.

621.315.212.011.2 2162
Study of the Impedance Irregularities of Coaxial Cables by Oscillographic Observation of Pulse Echoes: Part 1—P. Herreng and J. Ville. (*Câbles and Trans.* (Paris), vol. 2, pp. 111–130; April, 1948. With English summary. A theoretical treatment. The exact nature of the observed errors is discussed and theoretical considerations are developed which enable systematic errors to be eliminated and errors due either to irregularities of construction or to impedance unbalance to be correctly interpreted.

621.392.029.64 2163
Magic-Tee Waveguide Junction—G. Saxon and C. W. Miller. (*Wireless Eng.*, vol. 25, pp. 138–147; May, 1948.) A mathematical analysis of the junction leads to an alternative explanation of the properties described by previous writers, and explains some additional properties. Performance data are included for a range of frequencies, and a modified construction is described whereby the useful

frequency range may be extended. Applications of the junction include an impedance bridge, the measurement of SWR, transmission round awkward corners, phase-shifting, frequency discrimination, and frequency changing.

621.392.029.64 2164

Radiating Slits in Circular Waveguides—Ya. N. Feld. (*Radiotekhnika*, (Moscow), vol. 2, pp. 42–54; May and June, 1947. In Russian, with English summary.) Equations are derived for the field within a waveguide with a longitudinal slit. Formulas are obtained for the voltage across such a slit and for the reflection coefficient in the waveguide due to the slit. If a metal partition is fixed beyond the slit, it is possible to obtain a system of traveling waves in the waveguide.

621.392.029.64 2165

Waveguide Hybrids—W. A. Tyrrell. (*Bell Lab. Rec.*, vol. 26, pp. 24–29; January, 1948.) Discussion of a 4-arm junction. If a suitable wave front is passed into either of one pair of arms with high isolation from one another, then the output will be equally divided between the second pair. Impedance mismatch is eliminated by the use of rod and post tuning elements, the input arms being balanced with respect to each other provided that the output arms are symmetrically terminated.

This type of junction has been used successfully with a balanced converter, the incoming rf signal being applied to one of the input arms and the local oscillator connected to the other. The if signal is equally divided between the output arms and is passed to two rectifier systems.

621.392.029.64:621.3.09 2166

Propagation Characteristics in a Coaxial Structure with Two Dielectrics—A. Baños, Jr., H. Gruen, and D. S. Saxon. (*Phys. Rev.*, vol. 73, p. 531; March 1, 1948.) Summary of Amer. Phys. Soc. paper. The propagation characteristics are expressed in terms of the ratio of the two dielectric constants, the ratio of the radii of the dielectrics, and the frequency. The possibility of adjusting the phase velocity by varying these parameters suggests application to linear accelerators.

621.392.43 2167

The Matching of High-Frequency Transmission Lines using a Frequency-Variation Method—A. S. Edmondson. (*Proc. Phys. Soc.*, vol. 59, pp. 982–989; November, 1947.) The variation with frequency in the sending-end voltage of a transmission line many wavelengths long is reproduced on the screen of a cathode-ray oscilloscope and from the curve it is possible to see whether the standing waves along the line are large or small in amplitude, and how they vary with frequency. A description is given of an apparatus applying the new method to ultra-high frequencies and used in connection with the matching of antennas and filters to a coaxial transmission line. The method has also been used for demonstrating several important transmission-line properties and in this connection is useful for educational purposes.

621.396.67 2168

Note on Carson's Theory of Reciprocity—J. Maillard. (*Onde Élec.*, vol. 28, pp. 77–81; February, 1948.) Discussion of the generalized form of the reciprocity theorem shows that the directivity and the bandwidth of an antenna are the same for transmission and reception, that the transmitter impedance corresponding to maximum radiation is the same as that of the receiver picking up maximum power, and that for two antennas, the ratio of the maximum powers transmitted and picked up is independent of the sense of the transmission.

621.396.67 2169

The Comb Antenna—R. Grimm. (Proc. I.R.E., vol. 36, pp. 359–362; March, 1948.) The reception of vertically polarized mf waves by a comb antenna array is analyzed. The formulas derived can be applied to any array in which the elements are arranged in line and in which the current ratios and phase relations are known. Close coupling of the elements greatly increases the directivity and signal level by adjusting the line velocity to an optimum value. This type of antenna is particularly suitable for the reception of loran signals.

621.396.67 2170

A Note on an Omni-Directional Array of Stacked V-Antennas—T. P. Pepper. (*Canad. Jour. Res.*, vol. 26, sec. A, p. 22; January, 1948.) Five V-antennas, each consisting of two mutually perpendicular end-fed $\lambda\backslash 2$ dipoles, with $\lambda\backslash 2$ spacing, were fed from a double coaxial transmission line. An approximately circular radiation diagram was obtained. Matching problems were very much simpler than for stacked-loop antennas.

621.396.67 2171

Cathode-Follower TV-Antenna System—E. G. Hills. (*Communications*, vol. 28, pp. 22, 32; January, 1948). When the operating frequency changes, factors tending to reduce the efficiency of an array are: (a) change of electrical length of the feeder lines; (b) change in element feed-point impedances when isolated from other elements; (c) change in mutual impedances between radiators; (d) change in number of wavelengths in the spacing between the elements. (a) and (d) can be remedied together by making the effective path of the signal the same for all elements. (b) has been remedied in many broadband antennas and also in the cathode-follower antenna. (c) can be minimized by reducing the currents in the various elements to a very low value by terminating the receiving element with the high input impedance of a cathode follower. Typical cathode-follower arrays are described. Signal-to-noise ratios and antenna gain are discussed.

Such arrays can only be used for reception and the gain is relatively low. There is the added disadvantage that a source of power is required to operate the tubes.

621.396.67:517.512.2 2172

Fourier Transforms in Aerial Theory: Part 5—Fourier Approximation Curves—Ramsay. (*See* 2270.)

621.396.671 2173

Calculation of the Input Impedance of a Special Antenna—C. J. Bouwkamp. (*Philips Res. Rep.*, vol. 2, pp. 228–240, June, 1947.) The antenna consists of a vertical $\lambda/4$ end-fed wire, with either two equal horizontal wires pointing in opposite directions from the feed point, or four equal horizontal wires pointing in perpendicular directions. Sinusoidal current distribution is assumed. The radiation resistance is about 20Ω in each case.

621.396.671.4 2174

An Approximate Calculation of the Mutual Impedance of Aerials—G. T. Markov. (*Radiotekhnika* (Moscow), vol. 3, pp. 36–39; January and February, 1948. In Russian.)

621.396.677 2175

A Wave Channel with Radiators of an Improved Shape—S. I. Tetel'baum. (*Zh. Tekh. Fiz.*, vol. 17, pp. 1181–1186, October, 1947. In Russian.) An usw directive receiving system consisting of a number of elementary dipoles is discussed theoretically and methods are indicated for determining their optimum arrangement. The operation of the whole link comprising the transmitting and receiving antennas is also considered.

621.396.677 2176

Metallic Delay Lenses—W. E. Kock. (*Bell Sys. Tech. Jour.*, vol. 27, pp. 58–82; January, 1948.) Focusing of microwaves is obtained by phase-velocity reduction, so that the lens is shaped like its optical counterpart. The reduced velocity is caused by the presence of conducting elements which behave like the molecular dipoles of a nonpolar dielectric. If these elements are small compared with λ, the refractive index is constant over a wide frequency range.

Various experimental models using spherical and disk lattices, strip arrays, and sprayed sheets are described, and design and performance details are given for a strip type lens suitable for microwave repeater systems. Directional properties and impedance match are stated to be superior to those of parabolic reflectors. Theoretical calculations of the expected dielectric constant are in fairly good agreement with experiment for values less than 1.5. For earlier work see 1013 of 1947.

CIRCUITS AND CIRCUIT ELEMENTS

621.314.2 2177

A Note on a Parallel-Tuned Transformer Design—V. C. Rideout. (*Bell Sys. Tech. Jour.*, vol. 27, pp. 96–108, January, 1948.) Simple design formulas are obtained for a slightly overcoupled case. Theoretical performance data are included for: (a) a "matched" transformer with resistance loading on each side; (b) the same transformer with the output load removed; (c) a transformer with loading on one side only, designed to have the same characteristics as the "matched" transformer.

621.314.26:621.385.38 2178

Thyratron Frequency Changers—O. E. Bowlus and P. T. Nims. (*Electronics*, vol. 21, pp. 126–130, March, 1948.) Designed to permit parallel operation of 3-phase aircraft alternators driven at unequal and varying speeds.

621.314.3† 2179

Magnetic Amplifier Characteristics — Neutral Type—A. S. FitzGerald. (*Jour. Frank. Inst.*, vol. 244, pp. 415–439, December, 1947.) A description of the performance characteristics of amplifiers designed to actuate directly a moving armature relay. The minimum operating amplifier input lies in the range 6 to 0.1 μw.

Several graphs are given showing the gain and operating time characteristics for various amplifiers operating on 60 or 720 cps, or a combination of both. See also 960 of May, 1285 of June, and 1567 of July.

621.316.726.078.3:621.396.615.142.2:538.569.4 2180

Frequency Stabilization of Microwave Oscillators by Spectrum Lines—W. V. Smith, J. L. G. de Quevedo, R. L. Carter, and W. S. Bennett. (*Jour. Appl. Phys.*, vol. 18, pp. 1112–1115, December, 1947.) The method of stabilization is a development from that in which a cavity, tuned to the required frequency, controls the differential dc output from two crystal rectifiers in a waveguide bridge circuit, this output being applied to the oscillator to correct its frequency. In the present method, the cavity is replaced by a short-circuited waveguide filled with a gas (e.g., ammonia) at low pressure. The differential output from the crystals is zero when the frequency is that giving maximum absorption; otherwise an error voltage is obtained. Frequencies just below 24,000 Mc can be maintained constant to about 1 Mc by use of one or other of three ammonia absorption lines. See also 1690 of 1947 (Pound).

621.316.728 2181

Designing Saturable-Core Reactors for Specific Uses—C. Helber. (*Electronic Ind.*, vol. 1, pp. 4–5, 13, December, 1947.) The reactors were required for ac voltage regulators for inverters used in aircraft, but the simplified design has more general application. A simple graphical method is given for predicting the control characteristics.

621.317.729 2182

A Note on Frequency Transformations for

Use with the Electrolytic Tank—W. H. Huggins. (Proc. I.R.E., vol. 36, pp. 421–424, March, 1948.) Two frequency transformations are described which transform the circular tank described in 3651 of 1945 (Hansen and Lundstrom) into either a rectangular or an elliptical tank. The rectangular tank is suitable for simulating highly damped circuits with response extending over many octaves in frequency. The elliptical tank is suitable for representing band-pass circuits having characteristics symmetrical about a center frequency.

621.318.42 2183

Minimum-Cost Chokes—T. H. Oddie and J. L. Salpeter. (*Philips Res. Rep.*, vol. 2, pp. 281–312, August, 1947.) A method enabling the most economical dimensions, for given electrical specifications, to be found for ac chokes. Equations and tabulated data yielding the best solutions are given for rectangular types of choke, with and without limitations on the stacking height. Application of the method to chokes carrying dc with superimposed ac is indicated briefly.

621.318.572 2184

Decade-Ring Scaling Circuit—L. Seren. (*Rev. Sci. Instr.*, vol. 18, pp. 654–659, September, 1947.) A counting circuit is described which has a resolving time of 1.9 μs for any two, and 3.7 μs for any three consecutive pulses. A triode and milliammeter are used for indication, instead of a cr tube. See also 2496 and 2497 of 1946 (Regener).

621.318.572 2185

A Scale-of-16 Counting Circuit—P. Bassi and A. Loria. (*Elettronica*, vol. 2, pp. 347–349, November, 1947.) A circuit based on that of DeVault (*Rev. Sci. Instr.*, vol. 14, p. 23; 1943) and comprising (a) input multivibrator with 6N7 double triode, (b) 4-stage scaling circuit with type 79 double triodes, and (c) counting circuit with WE13 triode pentode.

621.318.572:518.5 2186

High-Speed *n*-Scale Counters—T. K. Sharpless. (*Electronics*, vol. 21, pp. 122–125; March, 1948.) Basic flip-flop and scale-of-two circuits are reviewed and the technique is extended to the design of ring and chain counters of higher scale. A cathode-pulsed *n*-scale circuit is described which requires no complex indicating schemes, eliminates counting errors and operates at speeds up to 180,000 pulses per second.

621.392 2187

A Contribution to Linear Network Analysis—D. K. C. MacDonald. (*Phil. Mag.*, vol. 37, pp. 778–789; November, 1946.) Analytical methods involving the application of sinusoidal, "step" or Heaviside, and pulse stimuli are reviewed. An alternative exponential type of stimulus is described, and the resulting response, or "exponential admittance," is compared with those obtained by other methods. The exponential admittances are obtained for the fundamental circuit elements and some of their simple combinations.

621.392 2188

A Table of Intermodulation Products—C. A. A. Wass. (*Jour. IEE* (London), part III, vol. 95, pp. 31–39; January, 1948.) "An expression is derived from which, by substitution of appropriate numerical values, an equation can be obtained for any of the intermodulation products which can be generated by the simultaneous transmission of any number of sinusoidal waves through a device with an output/input characteristic like $V=a_1v+a_2v^2+a_3v^3+\ldots$. This expression is used as a basis for classification of products, and a table of representative products is drawn up. Information is given about the numbers and relative importance of products of different kinds."

621.392:512.831 2189

Matrix Methods in the Solution of Ladder Networks—R. E. Vowels. (*Jour. IEE*, (London), part III, vol. 95, pp. 40–50; January, 1948.) Matrix theory is reviewed and useful results and transformations are summarized. The solution of circuit equations by "diagonalizing" the impedance matrix is developed, and applied to the solution of difference equations of ladder networks regarded as *n* similar T-sections in tandem.

621.392.4 2190

On the Synthesis of Bipoles by a Recurrence Method—R. Leroy. (*Câbles and Trans.*, (Paris), vol. 2, pp. 101–110; April, 1948.) The impedance function should be a positive real function of $p=j\omega$. Brune's solution of the problem (1932 abstracts, *Wireless Eng.*, p. 280) involves the absolute minimum referred to the zero of the real part of the impedance on the frequency axis; the method depends upon the sign of the reactive part. The zeros of the even part of the impedance, whatever their position in the p-plane, are discussed more generally. Brune's results are obtained without having to distinguish particular cases. By using zeros not situated on the frequency axis, recurrence methods are derived which only involve reactive elements.

621.392.5 2191

On the Theory of Quadripoles. Canonical Impedances and the Bisection Theorem—L. Bouthillon. (*Ann. Radioélec.*, vol. 3, pp. 3–20; January, 1948.) The series and derived impedances of lattice equivalents are frequently used to define symmetrical quadripoles. The theory here given introduces these impedances under the name of canonical impedances, without particular reference to the lattice quadripole, which is only one type of the quadripoles equivalent to a given quadripole. A new demonstration of Cauer's theorem (1932 abstracts, *Wireless Eng.*, p. 537) is given and the idea of canonical impedances is extended to nonsymmetrical quadripoles. The filtering characteristics of the dipole are shown to be the same, in terms of canonical impedances, for both symmetrical and nonsymmetrical quadripoles. It may thus be possible to develop a general filter theory for both these classes. A new demonstration is given of Bartlett's bisection theorem and this is extended to antisymmetrical linkages. Particular cases are discussed and the theorem is applied (a) to the design of quadripoles whose canonical impedances are given; a number of known types are at once found; (b) to the study of quartz plates with four electrodes; the characteristic impedances are calculated for various quadripoles which can be derived from such arrangements.

621.394/.397].645.3:621.314.25 2192

An Analysis of Three Self-Balancing Phase Inverters—A. A. Rizkin. (*Radiotekhnika*, (Moscow), vol. 3, p. 79; January and February, 1948 In Russian.) Comment on 1195 of 1946 (Wheeler). Rizkin claims that the circuit there described was patented by him on May 28, 1940.

621.396.611 2193

An Inductance-Capacitance Oscillator of Unusual Frequency Stability—J. K. Clapp. (Proc. I.R.E., vol. 36, pp. 356–358; March, 1948.) The circuit is similar to the Colpitts, but an *LC* series circuit replaces the inductor. If connections to the tube are kept short, there is practically no tendency toward spurious oscillation. Stability depends on the frequency and reactances used. Frequency changes of less than 1 part in 10^6 have been observed for changes in supply voltages of ± 15 per cent.

621.396.611.21 2194

Variable-Frequency Crystal Oscillators—H. Stanesby and P. W. Fryer. (*Jour. IEE*, (London), part IIIA, vol. 94, no. 12, pp. 368–378; 1947.) The frequency of a crystal oscillator can be varied very slightly by means of a variable capacitor. The range of variation may be substantially increased by associating inductance with the crystal plate. It is estimated that the range of variation, for a 3-Mc oscillator using an AT-cut plate, could be increased fivefold with added inductance while preserving far better frequency stability than that of a tuned-circuit oscillator. Experiments confirmed that a range of 7.7 kc (2560 parts in 10^6) could be obtained for one 3-Mc plate by using inductance. Circuits are described whereby the frequency can be made a linear function of the controlling reactance. The possibility of linear variation by means of a reactance tube is considered briefly. Applications are discussed.

621.396.611.21 2195

Crystal Oscillators and Their Application to Radio Transmitter Control—J. L. Creighton, H. B. Law, and R. J. Turner. (*Jour. IEE*, (London), part IIIA, vol. 94, no. 12, pp. 331–344; 1947.) Discussion of the mechanism of operation of high-grade crystal oscillators, with particular reference to frequency stability. The crystal unit is described with special regard to the temperature performance obtainable from the various cuts used to cover a frequency range on fundamental modes of 4 to 20,000 kc. The drive circuit is analyzed in order to establish stability criteria. Examples of practical designs are given.

621.396.611.4 2196

On a Cavity Resonator of High Quality for the Fundamental Frequency—K. F. Niessen. (*Appl. Sci. Res.*, vol. B1, no. 1, pp. 18–34; 1947.) Q is evaluated for a cavity resonator whose cross section is a parallelogram which can be divided into two isosceles right-angled triangles. Such a resonator can be used in a triode oscillator to prevent the occurrence of unwanted lf modes.

621.396.611.4 2197

Coupling Cavity Resonators through a Small Aperture—V. V. Vladimirski. (*Zh. Tekh. Fiz.*, vol. 17, pp. 1277–1282; November, 1947. In Russian.) The resonators have slightly different resonant frequencies. A system of equations (18) is derived from which the frequencies of the oscillations in each resonator and also the ratio of their amplitudes can be determined.

621.396.611.4:621.3.09 2198

The Propagation of Radio Waves along a Chain of Cylindrical Cavity Resonators—V. V. Vladimirski. (*Zh. Tekh. Fiz.*, vol. 17, pp. 1269–1276; November, 1947. In Russian.) Formulas (39) and (40) are derived for determining the phase and group velocities of an em wave, the resonators being coupled through small apertures.

621.396.611.4.029.63 2199

Simplified Oscillators for 2300 Mc—A. R. Koch. (*QST*, vol. 32, pp. 11–14; February, 1948.) Detailed construction of a $3\lambda/4$ cavity resonator with output coupling loop, and of a $\lambda/2$ resonator with a capacitance-probe output connection.

621.396.615 2200

The Theory of an Oscillator coupled to a Long Feeder, with Applications to Experimental Results for the Magnetron—C. Domb. (*Proc. Phys. Soc.*, vol. 59, pp. 958–972; November, 1947.) Two cases are considered: (a) the static case when the feeder is short enough for the time of travel of the wave to and fro along it to be small compared with the time of rise of the oscillation. The frequency and amplitude of a steady solution are first obtained; the initial rise of oscillations and the effect of antenna coupling circuits are deduced. The theory is compared with recent experimental frequency versus power curves. (b) the dynamic case when the feeder is long enough for the propagation time to be of importance. The steady solutions are shown to be the same as in case (a) and approximations are deduced which

are useful when the AM is small and the initial FM, before the oscillator settles down, is of prime importance.

621.396.615 2201

Oscillator with External Limitation of Amplitude—N. F. Vollerner. (*Radiotekhnika* (Moscow), vol. 2, pp. 34–41; May and June, 1947. In Russian, with English summary.) General description, with method of plotting the voltage wave form for continuous oscillations when operating on the linear portion of the tube characteristic with a linear limiter.

621.396.615 2202

Tests of the 6L6 as Oscillator, Frequency Multiplier, and Amplifier—L. Liot. (*Télév. Franç.*, Supplement *Électronique*, pp. 16–18, 20; March, 1948.) The differences between the 6L6 (metal tube), 6L6G (glass tube), and 6L6GX (glass tube with steatite base) are outlined and circuits are given, using 6L6G, for three types of crystal oscillator, simple frequency multiplier and push-push doubler, and a class-C amplifier.

621.396.615.12 2203

Single-Valve A.F. Oscillator—K. C. Johnson. (*Wireless World*, vol. 54, pp. 82–84; March, 1948.) A simple *RC* circuit comprises a modified parallel-T phase-shift network and a double-triode amplifier which automatically limits amplitude of oscillation. Frequency coverage 35 to 16,000 cps in two ranges. Output 3 v/rms, constant within 10 per cent.

621.396.615.17 2204

Producing High-Frequency Pulses—R. D. Carman. (*Wireless Eng.*, vol. 25, p. 164; May, 1948.) The method involves the use of a class-C amplifying tube which passes current only at the positive peaks of the excursions of the grid voltage. These peaks appear as pulses in the output. A pulse recurrence frequency of 5 Mc has thus been obtained. Still higher frequencies should be possible with a suitably designed load impedance.

621.396.619 2205

"Lagrangean" Formulae for the Direct Calculation of Harmonic Output and of Intermodulation Products—A. Bloch. (*Phil. Mag.*, vol. 37, pp. 694–700; October, 1946.) The use of interpolation formulas to determine the anode current of a tube when the grid voltage varies sinusoidally. For earlier work see 2163 and 2852 of 1946; see also 3872 of 1940 (Espley).

621.396.645 2206

Cathode-Follower Gate Circuit—J. Kurshan. (*Rev. Sci. Instr.*, vol. 18, pp. 647–649; September, 1947.) A double triode is operated as a cathode follower, the signal being applied to one grid and the gating pulse to the other. Leakage is low and practically independent of pulse amplitude. Possible applications suggested include coincidence counters and the separation of synchronizing signals in television reception.

621.396.645 2207

Some Designs and Applications for Packaged Amplifiers using Subminiature Tubes—B. Chance, J. N. Thurston, and P. L. Richman. (*Rev. Sci. Instr.*, vol. 18, pp. 610–616; September, 1947.) Design and construction details are given for both dc and ac amplifiers. Applications to control and measurement circuits are considered, and ways in which the use of such amplifiers can save time in the construction and repair of electronic apparatus are discussed.

621.396.645 2208

Utility Amplifier Unit—T. A. Patterson, Jr. (*Radio News*, vol. 39, pp. 42–43; January, 1948.) Circuit and component values for a 3-stage high-fidelity audio amplifier.

621.396.645 2209

Some Characteristics of a Delay-Line Coupled Wide-Band Pulse Amplifier—H. G. Rudenberg. (*Phys. Rev.*, vol. 73, p. 543; March 1, 1948.) Summary of Amer. Phys. Soc. paper. A number of tubes may be used in parallel, the control grids being coupled to the nodes of one artificial delay line and the anodes similarly coupled to a second identical line. A stage gain of 2.7 permits optimum use of tubes. A bandwidth of over 40 Mc has been obtained with a single stage using six tubes, each having a 30-pF capacitance shunting the electrodes.

621.396.645.012.2 2210

Circle Diagrams for Cathode Followers—J. M. Diamond. (Proc. I.R.E., vol. 36, pp. 416–420; March, 1948.) Universal circle diagrams are developed which determine the gain, input admittance, and output impedance. The variables are transconductance and the components of the cathode load. The Colpitts oscillator is considered as a cathode follower, and is analyzed both algebraically and with the aid of circle diagrams.

621.396.645.36 2211

Push-Pull Input Circuits: Part 3—Phase Reversers—W. T. Cocking. (*Wireless World*, vol. 54, pp. 85–87; March, 1948.) A comprehensive account of two typical phase-reversal circuits. Expressions are derived for the unbalance in amplitude and phase-shift of the output voltages, for each circuit. Phase unbalance up to 12 per cent in the af range is an undesirable feature in both cases. Parts 1 and 2, 1325 of June. Parts 4 and 5, 2212 below.

621.396.645.36 2212

Push-Pull Input Circuits: Parts 4 and 5—W. T. Cocking. (*Wireless World*, vol. 54, pp. 126–130 and 183–186; April and May, 1948.)

Part 4. The basic circuit of the anode-follower phase reverser is discussed and variants are considered. Formulas are derived for the unbalance at high, medium, and low frequencies. The advantages of this type of circuit are outlined.

Part 5. In cathode coupling, the input signal is applied to the grid of one triode and a second triode is driven from the first by means of a common cathode resistor. The output signals from the two anodes are out of phase and can be made nearly equal in magnitude. Unbalance effects are again analyzed. Part 3, 2211 above. Parts 1 and 2, 1325 of June.

621.396.645.371 2213

Multi-Channel Radio-Frequency Amplifiers—R. F. J. Jarvis and R. A. Brockbank. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 389–397; 1947.) Discussion of war-time developments in wide-band feedback amplifier design. With existing tubes, network performance at frequencies up to 50 Mc is approaching the theoretical limit. A representative selection of amplifiers developed to meet urgent specific demands is described; these range from a highly stable (60-db feedback) 100-mw repeater for speech channels on a coaxial cable, to a transmitter amplifier capable of feeding 12 simultaneous 40-w telegraph signals, in the frequency band 3 to 8 Mc or 8 to 15 Mc, direct into a rhombic antenna. The importance of Bode's analysis (4231 of 1940) of feedback amplifier networks is stressed.

621.396.645.371 2214

A Response-Compensated 6L6 Amplifier—(*Radio and Electronics, Wellington, N.Z.*, vol. 2, pp. 8–12, 56; January 1, 1948.) A 5-stage amplifier incorporating two 6L6 tubes in class-A push-pull provides an output of 18.5 w with a peak input of 0.87 v. Over-all negative feedback gives a frequency response flat within ±1 db from 20 to 10,000 cps. Independent high- and low-frequency boost is provided.

621.396.662.3 2215

A New Crystal Channel Filter—E. S. Willis. (*Bell Lab. Rec.*, vol. 26, pp. 13–17; January, 1948.) Details of the construction and performance of an improved single-lattice filter of economical design for use with broad-band carrier systems. Glass-sealed crystal units are used in a bridge type of circuit.

621.396.662.3.029.64 2216

A Non-Reflecting Branching Filter for Microwaves—W. D. Lewis and L. C. Tillotson. (*Bell Sys. Tech. Jour.*, vol. 27, pp. 83–95; January, 1948.) The branching filter is required in microwave repeater stations to pass each frequency band into an appropriate waveguide channel. High selectivity is unnecessary, but low loss and good impedance match are essential. The filter described diverts each unwanted frequency band into a resistive termination. This is achieved by an assembly of two waveguide "hybrids" and two reflection filters. N such assemblies constitute an N-channel branching network.

The mechanical and electrical characteristics of a satisfactory 5-channel system are described. The over-all SWR is less than 0.6 db and insertion loss less than 1.0 db. Discrimination against other channels is at least 20 db.

621.396.69+621.317.7+621.38 2217

Physical Society's Exhibition—(*Wireless Eng.*, vol. 25, pp. 157–162; May, 1948; *Wireless World*, vol. 54, pp. 174–179; May, 1948.) Short descriptions of some of the exhibits.

621.396.69:06.064 2218

Progress in Components. Review of the R.C.M.F. [Radio Component Manufacturers' Federation] Exhibition—(*Wireless World*, vol. 54, pp. 131–137; April, 1948.) A survey of productions in the main categories, with a list of exhibitors and their addresses. For other accounts see *Electronic Eng.* (London), vol. 20, pp. 114–117; April, 1948 and 1896 of August.

621.396.822 2219

Statistical Properties of a Sine Wave plus Random Noise—S. O. Rice. (*Bell Sys. Tech. Jour.*, vol. 27, pp. 109–157; January, 1948.) An extension of part of an earlier paper (440, 2168, and 2169 of 1945). The current may be written $I = R\cos(qt + \theta)$. The probability density of I and the time derivative θ' of the phase angle θ, the crossings of I, R, θ, and θ' and the correlation function and power spectrum of θ' are investigated.

"It is believed that some of the material presented here may find a use in the study of the effect of noise in frequency-modulation systems."

621.396.822 2220

Noise in Resistances and Electron Streams—J. R. Pierce. (*Bell Sys. Tech. Jour.*, vol. 27, pp. 158–174; January, 1948.) The following aspects of noise are discussed in terms of simple physical pictures, using nonrigorous mathematics: (a) Johnson noise in resistors; (b) shot noise, for which an expression is derived by a new method; (c) noise produced by bunching in electron multipliers; (d) reduction of noise in diodes due to space charge; (e) noise in triodes and pentodes.

621.314 2221

Electronic Transformers and Circuits [Book Review]—R. Lee. John Wiley and Sons, New York, N.Y., 1947, 282 pp., $4.50. (Proc. I.R.E., vol. 36, p. 383; March, 1948.) "A detailed, meticulous exposition of the design of transformers for electronic circuits."

621.392+621.385 2222

Electronic Circuits and Tubes [Book Review]—War Training Staff of the Cruft Laboratory, Harvard University. McGraw Hill Publishing Co., London, England, 948 pp., 45s. (*Wireless World*, vol. 54, pp. 151–152; April, 1948.) "A number of different authors are responsible for different chapters. . . . Some chapters are extremely detailed and very thorough, others are much more elementary and of a rather superficial character."

GENERAL PHYSICS

530.145 2223

The Theory of Radiation Damping—J. Hamilton. (*Proc. Phys. Soc.*, vol. 59, pp. 917–940; November, 1947.) An attempt to get a better understanding of the quantum theory of radiation damping developed by Weisskopf and Wigner, Heitler and Peng, and Wilson. By considering the emission from an atom enclosed in a cubical box with perfectly reflecting walls, and the scattering of a photon or a meson by electrons or nucleons inside a similar box, the emission and scattering problems are solved for the discrete energy level case, subject to certain limitations. The addition of some physically unimportant states to the equations of the emission problem reduces the divergence difficulties in the radiation theory of the phenomenon. In general, however, it seems impossible to use the discrete energy method to investigate whether or not the divergencies in radiation theory are purely mathematical difficulties due to the neglect of higher-order processes.

537.291 2224

On Using the Energy of Electrons moving in H.F. Uniform Electric Fields—Neiman. (*See* 2407.)

537.525.3 2225

Positive and Negative Point to Plane Corona—L. B. Loeb and W. N. English. (*Phys. Rev.*, vol. 73, p. 532; March 1, 1948.) Summary of Amer. Phys. Soc. paper. A comparison of positive and negative corona in air in the pressure range 205 to 760 mm Hg. Minimum corona voltages are slightly higher for a negative point; the two cases differ slightly in the form of the variation with pressure. Preliminary experiments with nonmetallic points gave unexpected results.

537.527:536.2 2226

Influence of Cooling Conditions on High-Pressure Discharges—W. Elenbaas. (*Philips Res. Rep.*, vol. 2, pp. 161–170; June, 1947.) A detailed theoretical discussion for discharges in closed tubes and in free air. Experimental results for tube discharges confirm the theory.

GEOPHYSICAL AND EXTRATERRESTRIAL PHENOMENA

522/523 2227

Recent Developments in Astronomy—M. W. Ovenden. (*Jour. Brit. Interplanetary Soc.*, vol. 7, pp. 70–91; March, 1948.) A review of observation results and modern theories.

523.72.029.6:523.746:621.396.822 2228

Solar Radio Emissions and Sunspots—I. L. Thomsen: M. Ryle. (*Nature* (London), vol. 161, pp. 134–136; January 24, 1948.) The ratio of the equivalent source temperatures deduced from radiation measurements on 80 Mc and 175 Mc by Ryle and Vonberg (96 of February) is correlated with sunspot records. There is good general agreement and it is suggested that the emission from the undisturbed sun gives an equivalent temperature proportional to a lower power than λ^2.

Ryle points out that the mean ratio of minimum equivalent temperatures obtained for the above frequencies during the past 11 months is about 2.2. Further evidence of periodicities in the ratio T_{80}/T_{175} has been obtained.

523.72.029.6:621.396.822 2229

On the Conditions of Escape of Microwaves of Radio-Frequency Range from the Sun—M. N. Saha, B. K. Banerjea, and U. C. Guha. (*Indian Jour. Phys.*, vol. 21, pp. 199–221; October, 1947.) The conditions governing solar emission are discussed with reference to magneto-ionic theories of propagation of radio waves through an ionized atmosphere traversed by a magnetic field. The magnetic field of the spots enables the *e*-component of the waves to escape from deeper layers of the solar atmosphere, and this explains emission of rf energy by the spot regions themselves. The theories give a general explanation of circular polarization, and of the sudden intensification of emission with the occurrence of flares. See also 1626 of July (Denisse).

523.72.029.64:621.396.822 2230

Emission of Enhanced Microwave Solar Radiation—R. G. Giovanelli. (*Nature* (London), vol. 161, pp. 133–134; January 24, 1948.) Outline of a theory assuming emission of radiation by spiralling electrons. The theory is similar to that given by Kiepenheuer, but involves a different origin for the energy. The mechanism whereby electrons may gain energies much greater than thermal, and the resulting intensity of the radiation received at the earth, are considered.

523.746 2231

Alfven's Theory of Sunspots—T. G. Cowling. (*Mon. Not. R. Astr. Soc.*, vol. 106, no. 5, pp. 446–456; 1946.) Alfven's theory (3469 of 1945) is shown to be internally inconsistent, or inconsistent with observation, at many points. See also 3470 and 3796 of 1945 (Walén).

523.746.5:519.24 2232

New Statistical Method of predicting Sunspots aids Radio Propagation Forecasts—(*Jour. Frank. Inst.*, vol. 244, pp. 481–487; December, 1947.) The method is based on data for a number of previous 11-year cycles. The importance of sunspot number in radio propagation is emphasized. The method is probably applicable to a wide variety of cyclical phenomena, such as long-term weather variations and climatic changes.

537.591 2233

New Italian Cosmic Ray Laboratory—(*Nature* (London), vol. 161, p. 254; February 14, 1948.) Some details of the situation, equipment, and program of work of a laboratory at a height of 11,500 ft on the upper slopes of Monte Rosa. It was opened in January.

550.384:523.745 2234

Geomagnetic "Crochet" Occurrence at Abinger 1936–1946 and Allied Solar and Radio Data—H. W. Newton. (*Mon. Not. R. Astr. Soc., Geophys. Supplement*, vol. 5, pp. 200–215; January, 1948; summary in *Mon. Not. R. Astr. Soc.*, vol. 106, no. 5, p. 465; 1946.) An investigation of ultra-violet solar radiation effects and their relation to solar flares and radio fade-outs. Diurnal and seasonal factors are discussed. Some "crochets" in 1946 have coincided with reported bursts of solar noise.

550.384:523.746 2235

"Sudden Commencements" in the Greenwich Magnetic Records (1879–1944) and Related Sunspot Data—H. W. Newton. (*Mon. Not. R. Astr. Soc., Geophys. Supplement*, vol. 5, pp. 159–185; January, 1948; summary in *Mon. Not. R. Astr. Soc.*, vol. 106, no. 5, p. 464; 1946.) The average amplitude, direction of impulse, and hourly and monthly frequencies of "sudden commencements" (SC) are derived from 681 cases identified from the Greenwich magnetograms over six sunspot cycles.

The relation of SC occurrence to the sunspot cycle and to individual sunspots is investigated. A sample comparison of SC pulses on magnetograms for Abinger and Lerwick shows remarkable similarity between the records of these stations during inactive periods.

551.5:621.396 2236

Meteorology and Radio—A. Perlat. (*Onde Élec.*, vol. 28, pp. 44–54; February, 1948.) Applications of radio to modern meteorology are reviewed, well known types of apparatus being merely mentioned, and the effect of meteorological conditions in the lower atmosphere on usw propagation is discussed.

551.510.52:621.396.812.029.64 2237

Radar Reflections from the Lower Atmosphere—M. W. Baldwin, Jr. (Proc. I.R.E., vol. 36, p. 363; March, 1948.) Comment on 722 of March (Gould). See also 2769 of 1947 (Friis).

551.510.535 2238

Experimental Determination of the [ionospheric] Gyrofrequency—S. L. Seaton. (*Science*, vol. 106, pp. 496–497; November 21, 1947.) Vertical-incidence records made in Alaska indicate that both the longitudinal and transverse modes of propagation predicted by magneto-ionic theory occur simultaneously. The theory is used to calculate the gyrofrequency at heights corresponding to the sporadic-E, F_1, and F_2 regions. Above 200 km, agreement with the inverse-cube law is good for the longitudinal mode, but the transverse mode gives values consistently 0.08 Mc too high. Between 110 km and 160 km, the observed values of gyrofrequency are too high, and the change of gyrofrequency with height is greater than that predicted by the inverse-cube law. This is possibly due to currents flowing in the lowest 100 km.

551.510.535 2239

Variation of Electronic Density in the Ionosphere with Latitude—S. S. Banerjee and R. N. Singh. (*Sci. Culture*, vol. 13, p. 295; January, 1948.) Monthly ionospheric observations show that noon electron densities in the F_2 layer decrease near the equator; this is attributed in part to the increased temperature of the ionosphere in the equatorial region.

551.510.535 2240

An Improved Ionospheric Height Recorder—H. A. Thomas and R. G. Chalmers. (*Jour. IEE* (London), part III, vol. 95, pp. 7–13; January, 1948.) "The system comprises a transmitter, which scans a wide frequency range some twenty-five times per second, and a receiver which is tuned slowly through the same range. Each time the frequency of the transmitter passes through the pass-band of the receiver, a pulse is produced which is displayed on an oscillograph, together with other pulses produced by waves reflected from the ionosphere. The advantages of this technique are enumerated." A photographic record, covering the range 2 to 16 Mc, may be obtained automatically in a few seconds. The equipment has been designed with a view to duplication, and is semiportable.

551.510.535:523.5:621.396.11.029.62 2241

Reflections of Very-High-Frequency Radio Waves from Meteoric Ionization—Allen. (*See* 2328.)

551.510.535:551.594.5 2242

On the First Observations of the Night Ionized Layer lying above the F_2 Layer—A. N. Kazantsev. (*Compt. Rend. Acad. Sci.* (URSS), vol. 59, pp. 479–482; January 21, 1948. In Russian.) An ionospheric layer lying above the F_2 layer (at a height of 430 km) and possessing lower critical frequencies was observed in 1933 to 1937. See also 2243 below.

551.510.535:551.594.5 2243

Ionospheric Disturbances of a Special Type—N. V. Mednikova. (*Compt. Rend. Acad. Sci.* (URSS), vol. 59, pp. 475–478; January 21, 1948. In Russian.) An aurora was observed near Moscow on February 16, 1947, a comparatively rare phenomenon. Observations of the ionosphere carried out at the time showed the appearance of an additional layer above the F_2 layer. See also 2242 above.

551.594.5 2244

Interpretation of Radio Echoes from Polar Auroras—N. Herlofson. (*Nature* (London), vol. 160, pp. 867–868; December 20, 1947.) The results obtained by Lovell, Clegg, and Ellyett (422 of March) are used to show that the electron density of the aurora must lie between 2.6×10^7 and 4×10^4 electrons/cm^3 if the reflecting surface is assumed to be plane. For a diffuse boundary, the reflection coefficient is re-

duced by a factor of the order of 10^6 when the electron density is near the upper limit. The aurora boundaries would appear to be sharp to within a few meters.

551.594.5(47) 2245

Aurora in Southern Siberia—I. V. Zykov. (*Priroda*, no. 3, pp. 49–50; 1947. In Russian.) During the night of September 28–29, 1946, an aurora was clearly seen in the Kuzbass region (55°30′N, 87°40′E). Auroras are rarely observed at such low latitudes and, therefore, a detailed description is given. It was accompanied by a sharp change from rainy to anticyclonic weather, with high pressure and night temperature below freezing point.

LOCATION AND AIDS TO NAVIGATION

621.396.93:621.396.677 2246

Recent Advances in Aerial Balancing Technique and Radiogoniometer Design in Relation to High Frequency Direction Finders—S. A. W. Jolliffe and D. Watson. (*Marconi Rev.*, vol. 10, pp. 142–156; October to December, 1947.) The causes of instrumental error in the antenna and radiogoniometer system of an adcock df system are considered. A method of balancing such a system without introducing spurious effects due to bad siting of a local transmitter is described, and this method is compared with other techniques. A FM field transmitter for speedy and accurate calibration of wide-band hf direction finders is described. "Errors in the radiogoniometer due to the "vector sum" component of the voltage induced in the antennas are considered. Methods of reducing coupling law errors are given, and a balanced potentiometer which produces accurate voltage ratios for error curve tests is described."

621.396.932 2247

The Second International Meeting on Radio Aids to Marine Navigation: New York and New London—R. B. Michell. (*Jour. Inst. Nav.*, vol. 1, pp. 69–75; January, 1948.) A summary of the main items discussed at the conference. The conclusions and recommendations adopted are given in an appendix. First conference noted in 3135 of 1947 and 112 of February.

621.396.932 2248

Douglas Harbour Radar—(*Wireless World*, vol. 54, p. 130; April, 1948.) A 3-cm radar system for control of shipping passing in and out of the harbor has been installed. The transmitter and receiver are housed at the base of a 60-ft tower at the harbor mouth, and a PPI display unit is fitted in the harbor master's control room. Ranges of 3, 1.2, and 0.8 miles are provided. See also *Engineer* (London), vol. 185, p. 240; March 5, 1948.)

621.396.933 2249

New Radio Aids to Aerial Navigation—J. Fagot. (*Onde Élec.*, vol. 28, pp. 3–12 and 70–76; January and February, 1948.) A short description of the principles of operation of equipment demonstrated at Indianapolis in October, 1946, prior to the P.I.C.A.O. conference in Montreal.

621.396.933 2250

Radio Aids to Navigation—R. Watson-Watt. (*Jour. Inst. Nav.*, vol. 1, pp. 15–21; January, 1948.) A brief survey of the principles and limitations of existing aids and a discussion of probable future developments. It is suggested that the main impediment to the rapid development of satisfactory aids is lack of general understanding of their importance.

621.396.933:526.918.5 2251

Shoran for Surveying—W. F. Kroemmelbein. (*Electronics*, vol. 21, pp. 112–117; March, 1948.) Operation principles, circuits, and application techniques.

621.396.933.24 2252

Consol—A. H. Jessell. (*Jour. Inst. Nav.*, vol. 1, pp. 29–39; January, 1948.) A navigation aid consisting of a mf beacon radiating a pattern of dots and dashes which changes at regular intervals. By listening to this pattern, a navigator can determine his position to a much greater accuracy than with a df loop. The operation of the beacon and the method used to determine its bearing from the receiving point are described. For another account see 2912 of 1946.

621.396.96:531.381 2253

A Problem in Dynamic Balancing of Scanners for Radar—C. Fox. (*Phil. Mag.* vol. 37, pp. 830–842; December, 1946.) Theoretical conditions for dynamic balancing with spiral scanning are deduced, and practical arrangements fulfilling these conditions are described.

621.396.96:551.594.6 2254

Radar and Weather—B. A. Shlyamin. (*Priroda*, no. 3, pp. 50–52; 1947. In Russian.) The use of 3-cm radar for locating distant storms, warm and occluded fronts, and typhoons is considered. Typical images obtained and possible practical applications are discussed.

621.396.933 2255

Radar Aids to Navigation [Book Review]—J. S. Hall, L. A. Turner, and R. M. Whitmer (Eds). McGraw-Hill Book Co., New York, N. Y., 1947, 382 pp., $7.50. (Proc. I.R.E., vol. 36, p. 383; March, 1948.) "Should be of particular interest to those who deal with navigational problems and who were not closely associated with the wartime developments."

MATERIALS AND SUBSIDIARY TECHNIQUES

533.5 2256

Vacuum Pumping Equipment and Systems—H. M. Sullivan. (*Rev. Sci. Instr.*, vol. 19, pp. 1–15; January, 1948.) "Typical rotary mechanical pumps, booster pumps, and diffusion pumps available today are described. Fundamental operating principles are discussed. The method for making theoretical computations of the conductance and speed of evacuation of a vacuum system is outlined."

533.5 2257

Diffusion Pumps: A Critical Discussion of Existing Theories—D. G. Avery and R. Witty. (*Proc. Phys. Soc.*, vol. 59, pp. 1016–1030; November, 1947.) The original work of Gaede and others on diffusion pumps is described briefly. A critical discussion of this work leads to the formulation of a more complete theory of the action of the diffusion pump, and this is used to explain the practical characteristics of a simple form of modern diffusion pump.

535.37 2258

Fluorescence of Silicate Phosphors—K. H. Butler. (*Jour. Opt. Soc. Amer.*, vol. 37, pp. 566–571; July, 1947.) A theory relating this fluorescence to the energy levels of the activator ions, and giving a better explanation of several phenomena than the impurity theory.

535.37 2259

Luminescence of Solid Solutions of fhe System $CaMoO_4$–$PbMoO_4$ and of Some Other Systems—F. A. Kröger. (*Philips Res. Rep.*, vol. 2, pp. 183–189; June, 1947.) "The systems (Ca, Sr)WO_4, (Ca, Sr)MoO_4, and (Ca, Mg)WO_4 are shown to behave as (Zn, Mg)WO_4 whereas (Ca, Pb)MoO_4 behaves as (Ca, Pb)WO_4." See also 2260 below.

535.37:541.123.6 2260

Photoluminescence in the Quaternary System $MgWO_4$-$ZnWO_4$-$MgMoO_4$-$ZnMoO_4$—F. A. Kröger. (*Philips Res. Rep.*, vol. 2, pp.177–182; June, 1947.) Fluorescence and absorption are discussed for the four different crystal structures of this system. See also 2259 above.

535.37:621.327.43 2261

On the Quantum Output of the Luminescence of Certain Silicates, Tungstates, and Borates—D. A. Shklover. (*Zh. Tekh. Fiz.*, vol. 17, pp. 1239–1252; November, 1947. In Russian.) An experimental investigation to determine the luminescent outputs of the main luminophores used in fluorescent lamps. The luminophores were excited by the 2537-Å resonance line of mercury. The spectral energy distribution and the absorption coefficients for ultra-violet and visible radiations were also measured.

546.431.82:621.315.61 2262

The Effective Permittivity of Two-Phase Systems—D. F. Rushman and M. A. Strivens. (*Proc. Phys. Soc.*, vol. 59, pp. 1011–1016; November, 1947.) The preparation of samples of $BaTiO_3$ of varying porosity and measurements of their effective permittivity are discussed. The results are explained in terms of Wiener's mixture law.

546.431.82:621.317.3.011.5 2263

Dielectric Residual Effects in Titanates—J. R. Partington, G. V. Planer, and I. I. Boswell. (*Nature* (London.) vol. 160, pp. 877–878; December 20, 1947.) A study of the changes with time in permittivity and dielectric loss after a high, unidirectional electric field had been applied to $BaTiO_3$ and related compounds having structures of the perovskite type. Within the temperature range corresponding to a distorted perovskite structure, a temporary increase in permittivity was obtained, while in the region of a cubic structure, a temporary decrease was generally observed.

621.315.591† 2264

The Physics of Electronic Semiconductors—G. L. Pearson. (*Bell Sys. Tech. Publ. Monogr.*, B-1475, 6 pp.) The band theory of solids is used to explain the following properties of electronic semiconductors: (a) dependence of specific resistance on impurity content; (b) negative temperature coefficient; (c) sign of the Hall and thermoelectric effects; (d) direction of rectification. The specific resistivity and the Hall coefficient are measured for Si, and the results used to calculate the density, mobility, and mean free path of the electron carriers, as a function of temperature and impurity.

621.315.591.5† 2265

Semi-Conductors with Large Negative Temperature Coefficient of Resistance—E. J. W. Verwey, P. W. Haayman, and F. C. Romeyn. (*Philips Tech. Rev.*, vol. 9, no. 8, pp. 239–248; 1947 and 1948.) Crystals of Fe_3O_4 are mixed with certain substances of the same crystal structure. The advantages of the resulting semiconductors over the normal resistor materials are (a) better manufacturing tolerances; (b) much more stable electrical properties; (c) high absolute value of temperature coefficient, about 10 times that of a metal. Practical applications include limitation of surge current, voltage stabilization, and resistance thermometry. A detailed account is given of the physico-chemical theory underlying the development of these new materials.

621.315.591.5†:537.32 2266

Thermoelectric Power of Cadmium Oxide—J. P. Andrews. (*Proc. Phys. Soc.*, vol. 59, pp. 990–996; November, 1947. Discussion, pp. 996–998.) Discussion of measurements of the thermoelectric emf of the semiconductor CdO against Pt in the temperature range −110°C to 800°c.

621.315.616.:621.317.3.011.5 2267

The Intrinsic Electric Strength of Polythene and Its Variation with Temperature—W. G. Oakes (*Jour. IEE* (London), part 1, vol. 95, pp. 36–44; January, 1948.) A systematic investigation of all the factors affecting electric strength measurement, leading to the development of a method giving reliable re-

sults. The dc strength of polythene is substantially independent of specimen thickness up to 0.008 inch. At 25°C, the value is about 17 mv inch and changes only slightly down to −200°C. Above 25°C, it falls steadily to about 2 mv inch at the melting point (about 115°C). At power frequencies, the ac strength decreases with increasing specimen thickness and time of application of voltage, and ac values are lower than dc values at corresponding temperatures. Possible explanations of the differences between ac and dc breakdown are suggested.

621.383:535.215.1:546.682 2268
The Photoelectric Properties of Sulphides and Selenides of Indium—In 1388 of June please alter the first author's name to Kolomiets.

548:517.512.2 2269
Fourier Transforms and Structure Factors [Book Review]—D. Wrinch. Asxred Monograph No. 2, 1946, 96 pp., $4.00. (*Proc. Phys. Soc.*, vol. 59, pp. 1044–1045; November, 1947.) It is shown how the structure factor of a particular atomic grouping may be combined with those of other atomic groupings to determine the structure factor of a crystal as a whole. It is easier to work with the "transform" of a distribution (the structure factor per effective electron) than with the structure factor itself.

MATHEMATICS

517.512.2:621.396.67 2270
Fourier Transforms in Aerial Theory: Part 5—Fourier Approximation Curves—J. F. Ramsay. (*Marconi Rev.*, vol. 10, pp. 157–165; October to December, 1947.) A study is made of a beam having a sharp and a blunt side. This asymmetrical pattern is partitioned into its even and odd components, each of which have Fourier approximations. The aperture excitations, in the form of "mutilated" functions, are determined to provide these approximations. One aperture has a real even distribution, the other an imaginary (i.e., quadrature) odd distribution. These together provide the complex distribution necessary for the production of the asymmetrical pattern. The approximation pattern is also obtained without resorting to a partition by the use of the Dirichlet type of integral. A second example is concerned with the Fourier approximations to the "lobeless" Gaussian radiation pattern. Part 4, 1068 of May.

518.5:517.2/.3 2271
Errors of Electrical Differentiation and Integration Arrangements—O. Heymann. (*Frequenz*, vol. 2, pp. 1–5; January, 1948.) Discussion of simple circuits comprising resistors and capacitors which can be used for differentiation or integration. The conditions under which such circuits give accurate results are determined. A short derivation of the Fourier integral law and of the Laplace-Riemann transformation is given in an appendix.

518.5:621.318.572 2272
High-Speed *n*-Scale Counters—Sharpless. (*See* 2186.)

519.24:523.746.5 2273
New Statistical Method of Predicting Sunspots aids Radio Propagation Forecasts.—(*See* 2232.)

51(075):621.396 2274
Mathematics for Radio Engineers [Book Review]—L. Mautner. Pitman Publishing Corp., New York, N. Y., 1947, 327 pp., $5.00 (*Electronics*, vol. 21, pp. 266, 268; March, 1948.) "Penetrates further into the subject than books intended for technicians, but is less extensive than engineering texts used in colleges." See also 162 of 1947 (Colebrook).

517.564.3 2275
Applied Bessel Functions [Book Review]—F. E. Relton. Blackie and Son, London, England, 1947, 191 pp., 17s. 6d. (*Proc. Phys. Soc.*, vol. 59, pp. 1047–1048; November, 1947.) A book intended for the new physics or engineering graduate. "For anyone anxious to know something about Bessel functions there are books as good as this, but few could claim to be better."

MEASUREMENTS AND TEST GEAR

531.764.5 2276
Crystal Clock for Accurate Time Signals—V. E. Heaton. (*Instruments*, vol. 20, pp. 618–619; July, 1947.) A brief description of the continuous time-signal service provided by the National Bureau of Standards.

621.3.018.41 (083.74)+621.317.76 2277
A Standard of Frequency and Its Applications—C. F. Booth and F. J. M. Laver. (*Jour. IEE* (London), part III, vol. 95, p. 52; January, 1948.) Discussion on 2973 of 1946.

621.316.726:621.396.61 2278
Frequency Composition in Naval Communication Transmitters—J. J. Hupert. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 405–417; 1947.) A crystal-controlled frequency is mixed with the much lower frequency of an accurately calibrated variable oscillator. By using a variable oscillator covering a fixed range and by changing the crystal frequency in steps, a wide range of accurate resultant frequencies can be obtained. A full theory is given of the spurious frequencies which are inevitably produced by this system, and of methods by which they can be reduced. Application to standard naval equipment is described.

621.317.2:621.397.62 2279
Television Receiver Laboratory—F. R. Norton. (*Electronics*, vol. 21, pp. 86–89; March, 1948.) Description of test facilities and equipment, including power supplies and filtering, and the construction of a shielded room.

621.317.3.011.5:621.315.616 2280
The Intrinsic Electric Strength of Polythene and Its Variation with Temperature—Oakes. (*See* 2267.)

621.317.35:621.396.619.16:621.396.813 2281
PCM Distortion Analysis—A. G. Clavier, P. F. Panter, and D. D. Grieg. (*Elec. Eng.* vol. 66, pp. 1110–1122; November, 1947.) Long summary of A.I.E.E. paper. Information is transmitted by a coded pulse system. In the system analyzed, the modulating signal is divided into 31 levels, a five-unit binary numbering system being used to identify each amplitude.

In order to calculate the distortion introduced when pulse code modulation is applied to a sine wave, the signal may be replaced by a step function. The application of pulse AM to this step function will give the same modulated waveform and degree of distortion.

Fourier transformations are used in order to analyze pulse code modulation of signals consisting of continuous frequency bands, and to compute distortion and crosstalk. See also 1161 of May and back references.

621.317.372 2282
Measurement of High-*Q* Cavities at 10,000 Mc/s—R. W. Lange. (*Bell Sys. Tech. Publ. Monogr.*, B-1474, 6 pp.) Known methods of measuring Q in resonant cavities together with their sources of error are discussed. For low values of Q and frequency, bandwidth methods are more accurate than decrement methods. For values of Q above 30,000 at frequencies above 3000 Mc, the reverse is true. Design features and performance are discussed for wide-range heterodyne decrement apparatus in which the accuracy is improved by observing the decay over a relatively long interval.

621.317.7+621.38+621.396.69 2283
Physical Society's Exhibition—(*Wireless Eng.*, vol. 25, pp. 157–162; May, 1948. *Wireless World*, vol. 54, pp. 174–179; May, 1948.) Short descriptions of some of the exhibits.

621.317.725 2284
High-Frequency Crystal Voltmeter—B. F. Tyson. (*Electronics*, vol. 21, pp. 150, 154; March, 1948.) A probe type of instrument using a 1N28 crystal rectifier. Construction and circuit details are given.

621.317.725 2285
An Inverted Tetrode Voltmeter for High Negative Voltages—R. J. Schneeberger. (*Rev. Sci. Instr.*, vol. 19, pp. 40–42; January, 1948.) By interchanging the normal functions of the grid and anode of a conventional tube, large negative potentials applied to the anode can be measured as a function of grid current. A tetrode operated in this manner, in a circuit with a large amount of grid-cathode degeneration, makes possible an instrument possessing a high degree of stability and an imput impedance of the order of $10^{13}\Omega$. A nearly linear scale extends from zero to −30 kv.

621.317.726.029.63 2286
The Measurement of Peak Voltage at a Frequency of 600 Mc/s by means of a Modified Probe Circuit—G. W. Bowdler. (*Jour. IEE* (London), part III, vol. 95, pp. 25–30; January, 1948.) A cascade doubler circuit, using two Si-crystal rectifiers and a galvanometer, enables peak voltage to be measured, given the pulse repetition rate and wave form. When a tuned stub was used to compensate for the stray capacitance shunting the rectifiers, measurements were found to agree with values deduced from calorimetric methods.

621.317.729 2287
An Electric Field Meter for Use on Airplanes—R. C. Waddel. (*Rev. Sci. Instr.*, vol. 19, pp. 31–35; January; 1948.) An instrument for measuring the magnitude and polarity of electric fields encountered in flight. A control box, containing the amplifier, range switch, and voltmeter, is connected by a cable to a measuring head mounted on the surface of the selected part of the aircraft. Power is derived from the aircraft's 28-v dc supply. The range 5 to 4000 v/cm is covered in four steps with linear scales.

621.317.733 2288
A.C. Bridges—"Cathode Ray." (*Wireless World*, vol. 54, pp. 139–142; April, 1948.) A simple explanation, in terms of vectors, of basic principles.

621.317.738 2289
Capacitance Meter with Quartz [resonance] Indicator—P. G. Bordoni and D. Sette. (*Ricerca Sci.*, vol. 17, pp. 1122–1127; July and August, 1947.) Long summary. Essentially a substitution method, the unknown capacitance C_x being measured by the difference in the resonance readings of a standard capacitor without and with C_x in parallel. Full paper in *Elettronica*, vol. 2, pp. 171 ff.; 1947.

621.317.755:621.396.61.001.4 2290
Uses of the Cathode-Ray Oscillograph in Transmitter Adjustment and Control—S. H. Coombs. (*Oscillographer*, vol. 8, pp. 1–11; May and June, 1946.) Oscillographic patterns which can be used for transmitter adjustment are: (a) audio sine-wave, for testing the af channel, (b) rf unmodulated wave pattern, for adjusting the rf channel, (c) Lissajous pattern, for neutralization indication and adjustment of frequency multiplier stages, (d) modulated-wave pattern, for checking over-all performance and indicating sources of trouble, and (e) trapezoidal pattern, for monitoring modulation and checking performance. The theory of these patterns, circuits for producing them on the cro, and typical oscillograms due to common faults are discussed in detail. The Du Mont Type 213 A modulation monitor is also described briefly.

621.317.76:621.396.619.13 2291

F.M. Monitor has Pulse-Counter Discriminator—C. A. Cady. (*FM and Telev.*, vol. 7, pp. 18–21, 36; December, 1947.) Detailed description and circuit diagrams of a new General Radio frequency monitor for broadcasting and television sound transmitters, which gives continuous indications of the center-frequency deviation, modulation percentage, and over-modulation peaks. An imput of only 1 v into a high impedance is required.

621.317.761.029.4/.54 2292

A Direct-Reading Frequency Meter for the Audio and Supersonic Ranges—H. J. Reich and R. L. Ungvary. (*Rev. Sci. Instr.*, vol. 19, pp. 43–46; January, 1948.) An electronic meter operating in the range 20 cps to 200 kc. The accuracy of the instrument is determined by the accuracy of the indicating meter; the reading is independent of wave form as long as the input wave does not cross the zero axis more than twice in its fundamental period. With a sinusoidal input, the voltage required for reliable operation is less than 0.03 v rms between the frequencies 900 cps and 200 kc.

621.317.763 2293

Direct-Reading Wavemeter Design—G. E. Feiker and H. R. Meahl. (*Electronics*, vol. 21, pp. 103–107; March, 1948.) Construction details of wavemeters for the range 2 to 75 cm (15,000 to 400 Mc) are given. These precision wavemeters are suitable for field service. Comparison is made with some British developments. The theory involved in obtaining a linear law is given for cavity devices of the finger-contact and re-entrant line short-circuit types.

621.317.79:551.510.535 2294

High-Power Ionosphere-Measuring Equipment—P. G. Sulzer. (Proc. I.R.E., vol. 36, pp. 389–394; March, 1948.) The transmitter has a peak power output of 100 kw from 1 to 8 Mc, decreasing to 50 kw at 16 Mc. It gives pulses of duration 20 to 200 μs. Special features are described of a transformer which reverses the polarity of a 9-kv square-topped negative pulse and provides various output voltages up to 14 kv. The receiver is a Hammarlund Super-Pro, modified for pulse reception. Typical results obtained with the equipment are illustrated.

621.317.79:621.315.2 2295

Pulse Echo Measurements on Telephone and Television Facilities—L. G. Abraham, A. W. Lebert, J. B. Maggio, and J. T. Schott. (*Bell Sys. Tech. Publ. Monogr.*, B-1469, 8 pp.) A full description of two equipments. One is an improved pulse echo set for testing coaxial cables, with pulse rates of 56, 13, or 3.3 kc and pulse widths of 0.25 or 1.5 μs. The other is a device called the "Lookator," built to locate faults in spiral-four field cables and open-wire lines. A pulse repetition rate of 220 cps is used and the echo time can be measured to within about 5 μs. See also 429 of 1946 (Schott.)

621.317.79:[621.396.615+621.396.619 2296

Instrument for Intermodulation Measurements—G. Daniel. (*Electronics*, vol. 21, pp. 134, 150; March, 1948.) A signal generator in which two independent frequencies are produced and combined, and an analyzer in which the degree of modulation of the higher frequency by the lower is determined. A detailed circuit diagram is given.

621.317.79:621.396.615 2297

A New A.M.-F.M. Signal Generator—J. Najork. (*Radio News*, vol. 39, pp. 49–51, 114; February, 1948.) Complete circuit details are given of a portable instrument for the alignment of AM and FM receivers. The rf oscillator has a range from 100 kc to 150 Mc. The FM oscillator has center frequencies of 1, 20, and 50 Mc, with maximum frequency deviations of ± 20, ± 300, ± 700 kc respectively. In addition, there is a variable frequency audio oscillator, and a 1-Mc crystal calibrator.

621.317.79:621.396.621.54.001.4 2298

The Testing of Communication-Type Radio Receivers—W. J. Bray and W. R. H. Lowry. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 313–327; 1947.) Standard tests and equipment for determining the performance of superheterodyne receivers in the frequency range 30 kc to 30 Mc are described. Rf sensitivity, af linearity, and distortion measurements, the determination of the noise factor, cross modulation, blocking characteristics, and the performance of afc and agc systems are considered. The characteristics of a typical receiver are given to explain the method of presentation of the results.

621.317.79:621.396.812.3 2299

The Fading Machine, and Its Use for the Investigation of the Effects of Frequency-Selective Fading—W. J. Bray, H. G. Lillicrap, and F. C. Owen. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 283–297; 1947.) An equipment to simulate the frequency-selective fading characteristics of long-distance sw radio channels. Three transmission paths are incorporated; the group time-delay differences between these may be varied in steps from 0 to 2 ms. The phase differences between the paths may be varied manually or continuously at rates ranging from 0.1 to 10 fades/sec. Random noise may be included so as to synthesize a complete sw channel. The equipment may also be used to simulate diversity reception.

Examples are given of the use of the equipment to assess the relative merits of double-sideband, single-sideband, and FM transmission systems with telephony or telegraphy modulation, under conditions of severe selective fading and high noise level.

621.317.79.029.58 2300

Standing-Wave Indicator for 3–22 Mc/s—W. N. Baker. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 328–330; 1947.) A midget diode rectifies the rf current induced in a shielded loop coupled to the feeder lines. An adjustable portion of the dc voltage from the diode is fed through 100 ft of cable to a dc amplifier operating a milliammeter. The equipment is self-contained, battery-operated, and more robust than thermocouple or rectifier and microammeter types of instrument.

621.396.615.17 2301

A Portable Pulse Generator—E. W. Titterton and V. L. Fitch. (*Rev. Sci. Instr.*, vol. 18, pp. 639–643; September, 1947.) The generator provides either negative or positive rectangular pulses from a low-impedance source. The amplitude is variable from zero to 75 v at recurrence frequencies of 1 to 10,000 cps. The circuit may also be driven from an external signal generator.

OTHER APPLICATIONS OF RADIO AND ELECTRONICS

531.717.1:621.386 2302

X-Ray Thickness Gage—(*Electronics*, vol. 21, pp. 154, 168; March, 1948.)

533.15:537.534 2303

Electronics simulates Sense of Smell—W. C. White and J. J. Hickey. (*Electronics*, vol. 21, pp. 100–102; March, 1948.) The presence of halogen vapor compounds, such as freon or CCl_4, increases the positive-ion emission from a incandescent Pt wire in air. This effect is used in a simple and readily portable leak detector.

534.321.9.001.8 2304

Coupling Ultrasonic Energy to a Load—S. Y. White. (*Audio Eng.*, vol. 32, pp. 29–31, 43; March, 1948.) Discussion of problems involved in the practical application of ultrasonics.

537.531:620.179.1 2305

The Detection of Cracks by X-Rays and Gamma Rays—C. Croxson. (*Electronic Eng.* vol. 20, pp. 106–111; April, 1948.)

539.16.08 2306

Scale-of-Hundred Counting Unit.—J. Rotblat, E. A. Sayle and D. G. A. Thomas. (*Jour. Sci. Instr.*, vol. 25, pp. 33–37; February, 1948.) Two electronic scale-of-ten units, each having a resolving time of 3 μs, are followed by a mechanical counter; rates up to 1000 impulses/sec can thus be recorded from counters or ionization chambers. Each scale-of-ten unit consists of four scale-of-two units, the first three of which are standard. The fourth is modified so that it is triggered normally by the eighth pulse and then couples the first and second scale-of-two units until restored to its initial state by the tenth pulse.

539.16.08 2307

A Direct-Reading Indicator for Electronic Counters—G. T. Baker. (*Electronic Eng.*, vol. 20, pp. 112–113; April, 1948.) Uses 4 scale-of-2 circuits, with a small cr tube provided with a digit scale from 0 to 9. The working of the indicator is explained in detail.

539.16.08 2308

Some Discharge Characteristics of Geiger-Müller Counters—H. V. Neher. (*Phys. Rev.*, vol. 73, p. 533; March 1, 1948.) Summary of Amer. Phys. Soc. paper.

539.16.08:621.386 2309

Recovery Time of Geiger Counters for X-Ray Intensity Measurement—R. Pepinsky, P. Jarmotz, H. M. Long, and D. Sayre. (*Rev. Sci. Instr.*, vol. 19, pp. 51–52; January, 1948.) The response of a G-M counter to double bursts of X rays is displayed on a cro synchronized to the repetition rate of the double bursts. The time interval between the initial and the second burst of any pair can be varied from zero to 500 μs and both are observable on the cro as long as the second burst does not occur within the dead time of the counter.

621.317.39:620.178.3 2310

Airborne Engine Analyzer—V. C. Cetrone. (*Electronics*, vol. 21, pp. 90–95; March, 1948.) An electronic instrument for fault finding in aircraft engines is described. Diagrams are given of circuits which provide a cr display of ignition and vibration patterns regardless of engine speed. Typical ignition patterns for normal conditions, a fouled plug, an open secondary, magneto-point bounce, and the vibration pattern of a normal cylinder are illustrated.

621.319.339 2311

The Berkeley Four Million Volt Electrostatic Generator—C. Turner, B. Cork, J. Ballam, and H. Gordon. (*Phys. Rev.*, vol. 73: p. 534; March 1, 1948.) Summary of Amer. Phys. Soc. paper.

621.319.339 2312

Ion Source Equipment for Berkeley Electrostatic Generator—F. Fillmore, A. Hudgins, and M. Jeppson. (*Phys. Rev.*, vol. 73, p. 534; March 1, 1948.) Summary of Amer. Phys. Soc. paper.

621.38.001.8 2313

Industrial Electronic Apparatus—D. W. Thomasson. (*Wireless World*, vol. 54, pp. 88–91; March, 1948.) Discussion of design, stressing the importance of simple and robust construction, accessibility of components, and standardization of units.

621.384.6 2314

Development of the Frequency Modulated Cyclotron—J. R. Richardson, B. T. Wright, E. J. Lofgren, and B. Peters. (*Phys. Rev.*, vol. 73, pp. 424–436; March 1, 1948.) A theoretical and experimental investigation of some fundamental characteristics of the cyclotron, including the variation of ion current with modulation frequency, time of flight of ions, and beam focusing. Ion currents were measured by a probe method.

621.384.6 2315
Unidirectional Pulse Operation of a 22-Mev Betatron—H. W. Koch and C. S. Robinson. (*Rev. Sci. Instr.*, vol. 19, pp. 36–39; January, 1948.) Description of the circuit used. Minor changes are necessary in the methods of electron injection and orbit expansion used with continuous excitation.

621.384.6 2316
Physical Design of the Berkeley Linear Accelerator—H. Bradner, R. Crawford, H. Gordon, and J. R. Woodyard. (*Phys. Rev.*, vol. 73, pp. 534–535; March 1, 1948.) Summary of Amer. Phys. Soc. paper.

621.384.6 2317
Beam Dynamics in the Linear Accelerator—R. Serber. (*Phys. Rev.*, vol. 73, p. 535; March 1, 1948.) Summary of Amer. Phys. Soc. paper.

621.384.6 2318
Linear Accelerator Oscillator and Coupling System—W. R. Baker, J. V. Franck, and J. D. Gow. (*Phys. Rev.*, vol. 73, p. 535; March 1, 1948.) Summary of Amer. Phys. Soc. paper.

621.384.6 2319
Control of the Field Distribution in the Linear Accelerator Cavity—W. K. H. Panofsky, C. Richman, and F. Oppenheimer. (*Phys. Rev.*, vol. 73, p. 535; March 1, 1948.) Summary of Amer. Phys. Soc. paper.

621.384.6:621.3.011.4:537.52 2320
Use of Plasma Sheaths as Variable Capacitances—W. E. Parkins and J. M. Leffler. (*Phys. Rev.*, vol. 73, p. 538; March 1, 1948.) Summary of Amer. Phys. Soc. paper. A current-receiving electrode in a gas discharge has a capacitance which depends on the operating conditions, and specifically on the applied frequency. This capacitance variation may be useful in controlling the frequency of a FM cyclotron.

621.385.833 2321
Numerical Computation of the Constants of Magnetic Electron Lenses—M. V. Ments and J. B. Le Poole. (*Appl. Sci. Res.*, vol. B1, no. 1, pp. 3–17; 1947.) The paraxial quantities are computed using special methods of integration and field measurement. Spherical aberration constants are calculated in a number of cases. The distortion of projector lenses is determined experimentally. Graphs of practical results which have proved helpful in design are given.

621.385.833 2322
A New Experimental Electron Microscope—G. Liebmann. (*Jour. Sci. Instr.*, vol. 25, pp. 37–43; February, 1948.) The design of the specimen stage enables the vacuum to be restored to its working value 15 seconds after the specimen is changed. The lens coil currents may be varied to give a range of magnification from 100 to 13,500. By removing certain lens units, the instrument may be used for electron diffraction.

621.385.833:535.317.25 2323
The Limiting Resolving Power of the [magnetic] Electron Microscope—G. Liebmann. (*Phil. Mag.*, vol. 37, pp. 677–685; October, 1946.) Resolving power is limited by saturation in the em lens. Combining theoretical work by Ramberg (2198 of 1943) with measurements undertaken by the author "an estimate is obtained of the expected limit of resolving power that may be achieved in electron microscopes of the present type. This limiting resolving power has a slightly marked optimum for an accelerating voltage of 50 kv, and is of the order of 10 to 12 Å."

621.398:[623.746.48+623.451 2324
Guided Missiles—C. E. Chapel. (*Radio News*, vol. 39, pp. 39–41, 126; January, 1948.) Descriptions of a wide range of American guided missiles and pilotless aircraft, electronically controlled.

664.8:621.319.44 2325
Electronic Preservation of Food—W. Huber. (*Electronics*, vol. 31, pp. 74–79; March, 1948.) Foods of many kinds can be preserved for long periods by exposing them in their sealed containers to short pulses of cathode rays from a capacitron with a peak voltage of 3 mv. Similar treatment serves to sterilize drugs.

PROPAGATION OF WAVES

538.566.029.63:535.42 2326
Diffraction of Electromagnetic Waves—W. O. Tranter. (*Phys. Rev.*, vol. 73, p. 184; January 15, 1948.) Comparison of results obtained by Andrews (*ibid.*, vol. 71, p. 777; 1947) for an em wave that has passed through a circular aperture, with results for two vertical unactivated dipoles. The dipoles were supported by thin glass rods and separated in the H plane by successive distances λ, 2λ, and 3λ. Reradiation from the sides of the aperture must be considered in the diffraction of em waves. See also 3717 of 1946.

621.396:551.5 2327
Meteorology and Radio—Perlat. (*See* 2236.)

621.396.11.029.62:551.510.535:523.5 2328
Reflections of Very-High-Frequency Radio Waves from Meteoric Ionization—E. W. Allen, Jr. (PROC. I.R.E., vol. 36, pp. 346–352; March, 1948.) Continuous recordings of field-strengths of long-distance transmissions in the 42 to 84-Mc band reveal short "bursts" of varying intensity, which may be due to reflections from ionization in the upper atmosphere caused by meteor trails. The correlation between distributions of the bursts and theoretical and observed distributions of meteors is shown.

621.396.11.029.64 2329
Low-Level Atmospheric Ducts—J. S. McPetrie and B. J. Starnecki. (PROC. I.R.E., vol. 36, p. 363; March, 1948.) Field strength measurements for λ 3 cm and 9 cm over a 60-mile sea path in the Irish Channel showed good agreement with theoretical results when low-level atmospheric ducts were assumed present for about 70 per cent of the time. Later analysis of meteorological results proved the assumption correct, thus partly supporting the results reported in 521 of March (Katzin, Bauchman, and Binnian). It was not found, however, that high winds are associated with higher ducts, nor was there any critical difference between the propagation of 3-cm and 9-cm waves. See also 518 of 1947 (Megaw).

621.396.11.029.64:551.509.39 2330
Over-Water Microwave-Propagation Forecasting—J. R. Gerhardt and W. E. Gordon. (*Bull. Amer. Met. Soc.*, vol. 28, pp. 126–136; March, 1947.) The effect of weather on microwave propagation is discussed briefly and a method of forecasting radar ranges is developed. Results based on meteorological soundings of the atmosphere over Massachusetts Bay in the summer of 1944 indicate that the method can provide reasonably accurate forecasts of maximum radar ranges up to 24 hours in advance.

621.396.812.029.64 2331
Experimental Studies of the Propagation of Very Short Radio Waves—E. C. S. Megaw. (*Jour. IEE* (London), part III, vol. 95, pp. 51–52; January, 1948.) Discussion on 518 of 1947.

621.396.812.029.64:551.578.1 2332
Rainfall Intensities and Attenuation of Centimeter Electromagnetic Waves—R. Wexler and J. Weinstein. (PROC. I.R.E., vol. 36, pp. 353–355; March, 1948.) Analysis of rainfall intensities and radar coverage at four selected stations in the United States indicates that the use of high-power 3-cm radar for storm detection is not seriously limited by attenuation due to rain.

621.396.812.3:621.317.79 2333
The Fading Machine, and its Use for the Investigation of the Effects of Frequency-Selective Fading—Bray, Lillicrap, and Owen. (*See* 2299.)

RECEPTION

621.396.61/.62 2334
The Design of Transmitter Drives and Receivers for Single-Sideband Systems—Bray, Lillicrap and Lowry. (*See* 2395.)

621.396.621 2335
The Receiver R.C.A. AR-88D—P. A. Boursault. (*T.S.F. Pour Tous*, vol. 23, pp. 267–273; December, 1947.) Complete circuit details, description and performance. The frequency range from 535 kc to 32 Mc is covered in 5 bands.

621.396.621 2336
A High-Fidelity Receiver—L. Chrétien. (*T.S.F. Pour Tous*, vol. 23, pp. 258–261; December, 1947; and vol. 24, pp. 3–5, 35–36, and 61–64; January to March, 1948.) The first four of a series of articles on a receiver suitable for feeding the high-fidelity amplifier previously described (1047 of 1947). The advantages and disadvantages of a hf stage are discussed fully.

621.396.621 2337
This Radio services Itself—R. Brenta. (*Radio Craft*, vol. 19, pp. 26–27; December, 1947.) To avoid the trouble and inconvenience involved in servicing the standard type of receiver, the "Cosmo Compo" receiver is designed so that each major stage of the circuit—oscillator, rf amplifiers, if amplifiers, etc.—is contained in a separate can which is plugged into the chassis. There are no components fixed directly to the chassis and the only wiring is that connecting the various plug points. When a fault develops, the faulty unit is removed and a sound one substituted.

621.396.621 2338
Ekco Model A52—(*Wireless World*, vol. 54, pp. 94–96; March, 1948.) Test report of a 5-tube superheterodyne for ac supply, with rotary program-selector switch, three-position tone control, and bandspread tuning on short waves. Wave ranges are 11 to 14 m, 16 to 20 m, 24 to 50 m, 200 to 550 m, and 1000 to 2000 m.

621.396.621 2339
The FreModyne F.M. Detector—(*Radio News*, vol. 39, pp. 48, 156; February, 1948.) A double triode is used in this inexpensive unit which can be added to an existing AM receiver for FM reception. One triode is used as a superheterodyne oscillator; the other acts as a superheterodyne converter, a superregenerative if amplifier, a FM/AM converter and an af detector. See also 1731 of July and 2065 of August (A. A. McK.).

621.396.621:621.396.619.11 2340
The Synchrodyne—E. Langberg. (*Electronic Eng.* (London), vol. 20, p. 132; April, 1948.) A short account of the application of the synchrodyne principle to the construction of a communication receiver. See also 1139 of May and back references.

621.396.621.029.63:621.396.931 2341
Receiver for the Citizens Radio Service: Part 2—W. C. Hollis. (*Electronics*, vol. 21, pp. 80–85; March, 1948.) Circuit operation and construction details for a 465-Mc superheterodyne receiver for operation in conjunction with the transmitter noted in 855 of April. The if is 15 Mc and bandwidth 250 kc. The measured noise figure is 20 db above the theoretical value. This small portable unit has grounded-grid input, cavity resonators, and crystals in limiter and discriminator.

621.396.621.54:621.396.611.21 2342
Few Crystals control Many Channels—W. R. Hedeman, Jr. (*Electronics*, vol. 21, pp. 118–121; March, 1948.) The local oscillator of a multichannel superheterodyne receiver for aircraft communication is controlled on 120 channels by only 10 crystals. Conditions for

minimum number of crystals, frequency spacing, and stability are discussed.

621.396.622:621.396.619.13 2343
Concerning the Detection of F.M. Waves—E. Labin. (*Onde Élec.*, vol. 28, pp. 60–69; February, 1948.) The output signal is calculated for a FM detection system when the incident wave has both FM and AM. According to some views, the output signal should only depend slightly, or not at all, on the AM of the input signal, since the detector system is of the differential type. Discussion shows that this idea is not well founded, though the symmetrical detector may give a higher signal-to-noise ratio and improve the linearity of the characteristic.

621.396.622.7 2344
An Answer to N.F.M. Reception—L. H. Allen. (*QST*, vol. 32, pp. 28–29; February, 1948.) Discusses the advantages of the ratio detector as a simple means of providing either AM or FM reception and gives the modifications necessary for alternative reception on a communication receiver.

621.396.622.7 2345
F.M. Reception: Comparison Tests between Phase Discriminator and Ratio Detectors—D. Maurice and R. J. H. Slaughter. (*Wireless World*, vol. 54, pp. 103–106; March, 1948.) The ratio detector is about 2 db noisier, except for very small signals, with no appreciable difference in quality of reproduction for a given input signal. The relative merits of the two systems are discussed.

621.396.622.71 2346
Ratio Detectors for F.M. Receivers—S. W. Seeley. (*FM and Telev.*, vol. 7, pp. 26–27, 47; December, 1947.) Principles and performance characteristics. See also 3643 of 1947 (Seeley and Avins), 248 of February and 1476 of June (Hayes).

621.396.662 2347
The C.A.P. F.M.-A.M. Tuner—W. H. Collins. (*FM and Telev.*, vol. 7, pp. 18–20; November, 1947.) The FM section, operating at 88 to 108 Mc, comprises an Armstrong circuit followed by 3 if stages, a cascade resistance-coupled limiter and a duo-diode discriminator. The AM section consists of 2 rf band-pass stages, a buffer stage, and a duo-diode detector. The output of either section can be switched into an af triode amplifier.

621.396.82 2348
Mutual Radio Interference in H.M. Ships—C. Matthews and R. L. A. Borrow. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 418–426; 1947.) The principles involved in avoiding mutual interference which were enforced before 1939 could not be maintained during the war, because of the necessity of fitting commercial equipment, and especially with the coming of radar. Radar equipment constituted the main source of interference in communication reception, though insufficiently screened receivers and spurious frequencies in transmitters decreased efficient reception. Radio interference suppression units, which muted the receiver during the transmission of each radar pulse, were fitted to reduce interference when the offending equipment could not be modified. By using receiving antenna filters and limiter circuits, interference was further reduced. A code of practice has been formulated for the installation of equipment so as to reduce interference as much as possible.

621.396.822 2349
Measurements of "Inherent" Noise in Radio Receivers.—M. K. Dasgupta, J. Ray, and S. R. Khastgir. (*Indian Jour. Phys.*, vol. 21, pp. 239–258; October, 1947.) The inherent noise was measured in terms of the equivalent noise voltage at the input of the receiver. Measurements were made on 6 ac/dc commercial receivers at frequencies between 1 Mc and 20 Mc and for carrier voltages from 0.02 μv to 1 mv. The equivalent noise voltages were found to be about 1 μv for a carrier input of 1 μv, and except for mf channels, there was a linear relation between input and output up to a certain level. Above this level, there was an increase in noise level due to avc action, and the rate of increase of equivalent noise voltage decreased. There was, in general, an increase in this voltage for higher frequency channels. Results for a receiver operated (a) from the mains, and (b) from batteries, show the effect of imperfect filtering of line noise.

621.396.822 2350
On a Non-Linear Noise Problem—F. L. H. M. Stumpers. (*Philips Res. Rep.*, vol. 2, pp. 241–259; August, 1947.) Noise from a normal source is passed through a filter with a rectangular amplitude versus frequency characteristic. The output is applied to a tube with a nonlinear characteristic. The resulting energy frequency spectrum is considered with special reference to the work of Fränz (3026 of 1941 and 443 of 1943). The partial spectra arising from multifolds of the regional central frequency have different forms; they can be distinguished by their order. A formula is derived from which all partial spectra can be computed directly if the characteristic is given as a polynomial or power series. The effect of carrier waves is also considered.

621.396.822:621.396.813.015.33 2351
The Effect of Fluctuation Noise Interference on Pulse Distortion—P. J. Hilton. (*Phil. Mag.*, vol. 37, pp. 685–693; October, 1946.) Theoretical discussion, resulting in a table which gives the probability that the total distortion will exceed the maximum distortion for the signal pulse alone, as a function of noise amplitude and rms voltage.

621.396.828 2352
Radiation from Receivers—G. J. McDonald and D. A. Thorn. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 427–436; 1947.) "Details are given of the Statutory Rules and Orders which were promulgated to control the level of radiation permissible from ships' receivers and the problem of developing a technique suitable for checking individual installations for compliance with the limits laid down in these regulations is discussed. Examples of practical testing installations are given together with details of new receiver designs and modifications to existing equipments which ensured a sufficiently low and safe level of radiation."

STATIONS AND COMMUNICATION SYSTEMS

621.391.64:621.327.44 2353
Modulation of the Resonance Lines in a Cesium Arc—J. M. Frank, W. S. Huxford, and W. R. Wilson. (*Jour. Opt. Soc. Amer.*, vol. 37, pp. 718–725; September, 1947.) Full paper. Summary abstracted in 4023 of January.

621.394.441 2354
Carrier-Frequency-Shift Telegraphy—R. Ruddlesden, E. Forster and Z. Jelonek. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 379–388; 1947.) The use of pulled-crystal and mixing-oscillator frequency-shift exciters is discussed. Pulled-crystal exciters, by virtue of their simplicity, are used primarily for mobile stations. Methods of detection, by band-pass filter selection or by a discriminator, are described. Practical advantages of frequency-shift operation are considered. The threshold value of the signal and of the time shift produced by random noise are anlayzed in an appendix. Although the discriminator method of detection has no theoretical advantage over keyed-carrier systems, in a typical example, post-discriminator filtration gave a 14-db improvement and band-pass filter selection a 10-db improvement in threshold value.

621.395.44:621.315.052.63:621.316.9 2355
Power Line Treatment for the M1 Carrier Telephone System—J. M. Dunham. (*Bell Lab. Rec.*, vol. 26, pp. 2–5; January, 1948.) Precautionary measures observed in carrier telephony over hv ac transmission lines are described, with details of some of the components used in such systems.

621.395.44:621.396.619.2 2356
A 48-Channel Carrier Telephone System—G. H. Bast, D. Goedhart, and J. F. Schouten. (*Philips Tech. Rev.*, vol. 9, no. 6, pp. 161–170; 1947.) Design considerations for a system using the available frequency range of 200 kc of existing multiconductor cables in Holland. Each audio channel undergoes three successive modulations in such a way that narrow-band filters are needed only at low frequencies. The volume of the coils in these filters has been reduced to a fifth of its former value by using "Ferroxcube" (3155 of 1947).

621.396.1 2357
Atlantic City. Summary of the Findings of the International Telecommunication Conferences—(*Wireless World*, vol. 54, pp. 148–150; April, 1948.) For another account see 1749 of July (A.H.M.).

621.396.3 2358
Some Developments in Commercial Point-to-Point Radiotelegraphy—J. A. Smale. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 345–367; 1947.) A survey of developments concerned chiefly with ensuring continuous communication on difficult circuits. Discussion of: (a) frequency shift keying, which is compared with on-off keying; (b) the introduction of relay stations in long difficult circuits such as London-Australia; (c) the use of ph.m. for sw transmitters on long-distance facsimile circuits; (d) the development of horizontal dipoles to augment vertical arrays; (e) wide-band coupling units for coaxial feeders to balanced antennas (f) telegraph printing systems; (g) a variable-frequency FM system for the control of radio stations; (h) a high-speed photoelectric morse transmitter; (i) an electronic keying device.

621.396.41:621.385.832 2359
Pulse-Modulation Multiplex Communication System—D. E. Manfredi. (*Elettronica*, vol. 2, pp. 335–338; November, 1947.) A system developed between 1935 and 1946 in Italy and very similar in principle to that described in 883 of April (Greig, Glauber, and Moskowitz). The commutator is a cr tube with a number of small anodes, arranged in line or in a circle, instead of a fluorescent screen. Each anode is connected to the suppressor grid of a tube whose first grid is controlled by the microphone current from one of the programs. The electron beam scans each anode in turn, giving it momentarily the electric charge necessary to make the suppressor grid positive. Thus each tube in succession is open to the passage of a very short signal conveying a small fraction of the corresponding program. The successive pulses are amplified and transmitted in order. At the receiving station another commutator tube, synchronized with the first, separates the signals, distributing them via the appropriate anode, to the separate channels.

621.396.61/.62 2360
The Admiralty Type 612 Transportable Radio Equipment—D. Hamilton. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 401–404; 1947.) The equipment is of robust unit construction, is watertight and floats in water. Designed primarily as a large transportable outfit, it is also used in ships as an emergency transmitter/receiver for the 1.5 to 13-Mc band. An additional receiver for use in 4 bands from 15 to 500 kc is included. The circuit design follows conventional practice, but the high stability oscillators and frequency-setting facilities are of interest.

621.396.61/.62 2361
152- to 162-Mc Mobile Equipment—B. J.

Cosman. (*FM and Telev.*, vol. 7, pp. 36–40; November, 1947.) A detailed description and circuits of emergency equipment. The transmitter unit is a crystal-controlled ph.m. type with an output of 25 to 30 w. The receiver unit uses miniature tubes in 2 rf stages, a van der Bijl detector mixer, 4 if stages at 5.3 Mc followed by a Foster-Seeley discriminator using crystal rectifiers, and an af amplifier with an output of about 1 w.

621.396.619.15 2362

Frequency Shift Telegraphy—Radio and Wire Applications—J. R. Davey and A. L. Matte. (*Bell Sys. Tech. Publ. Monogr.*, B-1490, 15 pp.) Comparison is made with AM methods. Bandwidth problems, demodulation, noise effects, signal-level variation, and frequency instability are considered. The outstanding characteristic of the frequency-shift method is its ability to accept large and rapid changes in signal amplitude. The signal-to-noise ratio is considerably higher than with AM telegraphy. Diversity reception, multipath propagation, and the use of superimposed ph.m. for reducing the effects of fading, are also discussed.

621.396.619.16 2363

PCM Equipment—H. S. Black and J. O. Edson. (*Elec. Eng.*, vol. 66, pp. 1123–1125; November, 1947.) Long summary of A.I.E.E. paper. The advantages of pulse code modulation include: (a) ease of multiplex operation; (b) nonlinearity does not introduce cross talk; (c) high signal-to-noise ratios are attainable; (d) flexibility of transmission standards. The modulation process is based on the principles of sampling and quantizing. The sampled and filtered AM voice signal pulses are first converted to length-modulated pulses and finally to pulse-code-modulated signals. See also 1161 of May and back references.

621.396.619.16:621.317.35:621.396.813 2364

PCM Distortion Analysis—Clavier, Panter, and Greig. (*See* 2281.)

621.396.619.16:621.385.1 2365

Electron Beam Deflection Tube for Pulse Code Modulation—Sears. (*See* 2413.)

621.396.619.16:621.395.43 2366

An Experimental Multichannel Pulse Code Modulation System of Toll Quality—L. A. Meacham and E. Peterson. (*Bell Sys. Tech. Jour.*, vol. 27, pp. 1–43; January, 1948.) A full account of an experimental 24-channel multiplex system in which coding is effected by means of a new electron-beam tube [2413 below] and decoding by pulse excitation of a reactive network. Functional problems are discussed in general terms, and the complete experimental layout and some novel circuit techniques are described. Good speech quality is obtained. See also 545 of March (Goodall).

621.396.65 2367

Some Studies on Emergency Mine Communications—E. J. Coggeshall, E. W. Felegy and L. H. Harrison. (*United States Bureau of Mines*, Report R.I. 4135, 44 pp.; January, 1948.) Experiments on the transmission of signals at af and rf from various locations in a mine to the surface are described. Af tests were conducted with telephone, ground-telegraphy, and voice-amplification systems, and in the rf range, vhf "walkie-talkie" and lf 33 to 120-kc systems were used. Except in the case of the vfh system, which had an antenna, the transmitter and receiver were connected either to ground or to the track rails. Communication was practicable only by conduction currents and the optimum frequency depended on the type of conductor used. With ground conduction, this frequency was above 120 kc but probably below 500 kc; with metallic conduction it was between 40 and 90 kc.

621.396.65 2368

Multi-Channel Communication System—(*Wireless World*, vol. 54, p. 93; March, 1948.) Details of a vhf wide-band FM link, for use in conjunction with the projected oil pipe line from the Persian Gulf to the Mediterranean. The link will provide seven voice-frequency channels in each direction.

621.396.65 2369

Railway Communications for the Royal Tour—M. W. Manson. (*Trans. S. Afr. Inst. Elec. Eng.*, vol. 38, part 11, pp. 305–342; November, 1947. Discussion, pp. 343–348.) A full description of the communication and other equipment installed on the royal and pilot trains, with details of the results achieved in long-distance communication. A vhf FM radio telephone link operated between the trains; long-distance communication was maintained by means of a 500-w transmitter fitted in the pilot train and operating on any one of 5 frequencies in the range 4 to 12 Mc.

621.396.65.029.64:621.397.743 2370

FM Relay Out-Performs Coax for Television—M. B. Sleeper. (*FM and Telev.*, vol. 7, pp. 15–17, 35; December, 1947.) Discussion of the results of a comparison of video and af modulated radio relay signals from Boston to New York with signals by coaxial cable from Washington. The FM radio relay gave definitely superior picture quality. See also 1755 of July (Durkee) and 1756 of July.

621.396.712:621.396.61 2371

The Radio Transmitting Station at Criggion—W. West, A. Cook, L. L. Hall, and H. E. Sturgess. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 269–282; 1947.) The station has a long-wave transmitter which is essentially similar to the GBR transmitter at Rugby, two lf transmitters and fourteen hf transmitters. The construction of the station and the events leading to its establishment are discussed. Various units of the internal and external plant are described, with special reference to a recently developed single-sideband transmitter, power and water supplies, and line-termination equipment.

SUBSIDIARY APPARATUS

621.314.65 2372

The Steel Bulb Mercury Arc Rectifier—H. J. H. Nethersole and L. L. Brinkworth. (*Trans. S. Afr. Inst. Elec. Eng.*, vol. 38, part 12, pp. 351–364; December, 1947. Discussion, pp. 365–369.) Theory, construction, methods of ignition and excitation, and applications of various types of such rectifiers are discussed, with particular reference to the evolution of the steel-bulb type, its properties and its advantages over earlier types.

621.315.66:621.396.65.029.64 2373

Portable Microwave Tower—(*Bell Lab. Rec.*, vol. 26, pp. 6–8; January, 1948.) The sectionalized tower may be erected quickly and safely without the assistance of professional riggers. Assembly and erection are carried out by progressively raising the tower in an erection frame and inserting further sections at the base. A height of 200 ft may be attained.

621.318.5 2374

Mercury Contact Relays—J. T. L. Brown and C. E. Pollard. (*Elec. Eng.*, vol. 66, pp. 1106–1109; November, 1947.) Long summary of paper presented at the National Electronics Conference. Metal contacts are kept wetted with mercury by means of a capillary connection to a reservoir in the base of the relay. The light-weight armature supporting the capillary also carries the moving contacts. See also 562 of March (McCormick).

621.318.5:621.315.212 2375

Glass-Enclosed Reed Relay—W. B. Ellwood. (*Elec. Eng.*, vol. 66, pp. 1104–1106; November, 1947.) Long summary of paper presented at the National Electronics Conference. A high-speed compact relay, one form of which can be used inside coaxial cables. The relay comprises a coil and a magnetically operated switch. The switch is used as the core of the coil which provides the operating magnetomotive force. Terminals are provided for the switch and coil on an octal base. Design and performance are fully discussed. See also 1583 of 1946 (Ellwood) and 562 of March (McCormick).

621.318.572 2376

Electronic Switching for the Ham Antenna—T. Gootée. (*Radio News*, vol. 39, pp. 50–51, 116; January, 1948.) Government surplus stocks of transmit-receive radar tubes are available and may be used without modification for automatic transmit-receive switching of amateur installations. Circuits and operational details are given.

621.396.68:621.397.6 2377

Television E.H.T. Supply: Parts 1 and 2.—Walker. (*See* 2390.)

621.396.681 2378

Emergency Supply Systems with Accumulator Batteries—H. A. W. Klinkhamer. (*Philips Tech. Rev.*, vol. 9, no. 8, pp. 231–238; 1947 and 1948.

621.396.681 2379

Servicing Vibrator Packs—(*Radio and Electronics*, Wellington, N.Z., vol. 2, pp. 25–27, 31; January 1, 1948.) The principal features of synchronous and nonsynchronous vibrators are described and practical servicing procedures are given.

621.396.682:621.316.722.1 2380

Controlled Power Sources for Heavy Direct Currents—G. Houck. (*Electronic Ind.*, vol. 1, pp. 12–13; December, 1947.) A general description of a commercial unit, the "nobatron" which supplies 50 to 350 amp dc at 28 v from a 3-phase supply. Voltage regulation is better than 0.1 per cent for a load change from 50 to 300 amp. See also 284 of January.

621.396.69:621.315 2381

F.M. and TV Transmission Line Installation Problems: Part 2—J. S. Brown. (*Communications*, vol. 28, pp. 20–21; January, 1948.) Illustrations and some details of equipment at WBNS-WELD, Columbus. Part 1, 1176 of May.

621.385.832:778 2382

Photographic Recording of Cathode-Ray Tube Traces [Book Review]—R. J. Hercock. Ilford Ltd., London, 1947, 60 pp., 5s. (*Proc. Phys. Soc.*, vol. 59, p. 1048; November, 1947.) No. 1 of a series of Ilford technical monographs; its object is to provide those familiar with the use of cr tubes with an insight into the photographic technique necessary to obtain useful records of traces.

TELEVISION AND PHOTOTELEGRAPHY

621.397.26 2383

The New Radio Picture Service—(*Radio and Electronics*, Wellington N.Z., vol. 2, pp. 4–5; January 1, 1948.) A brief description of the crystal-controlled equipment used for the service recently inaugurated between London, Australia, and New Zealand.

621.397.26 2384

A Method of Transmitting Sound on the Vision Carrier of a Television Service—D. I. Lawson, A. V. Lord, and S. R. Kharbanda. (*Jour. IEE* (London), part III, vol. 95, pp. 13–16; January, 1948.) Discussion on 3091 of 1946.

621.397.331.2 2385

Limiting Resolution in an Image-Orthicon-Type Pickup Tube—H. B. DeVore. (Proc. I.R.E., vol. 36, pp. 335–345; March, 1948.) The distribution of charge spreading across alternate illuminated and dark strips is found and the potential due to the varying charge is expressed as a modulation, which is a function of the target thickness. For a given resolution, the target thickness required is proportional to the target resistivity. Target area and charge

density are discussed by calculating electron transit times and displacements from the true focus. Variation of the degree of modulation with the wavelength of the incident light is investigated.

621.397.335 2386
Frame Time Base Synchronisation in Television Receivers—A. W. Keen. (*Jour. Telev. Soc.*, vol. 5, pp. 49–64; June, 1947.) A detailed discussion of the characteristics of framing-signal separators using (a) frequency selective networks; (b) shaping networks; (c) delay networks; (d) combination with a locally generated signal; (e) automatic frequency and phase control.

621.397.5:535.88 2387
Home Projection Television: Parts 1–3—H. Rinia, J. de Gier, and P. M. van Alphen; G. J. Siezen and F. Kerkhof; J. Haantjes and F. Kerkhof. (PROC. I.R.E., vol. 36, pp. 395–411; March, 1948.)

Part 1. Cathode-Ray Tube and Optical System—The 2.5-inch tube has a very small spot size, narrow neck, face plate ground to meet optical requirements, and metal-backed screen. A modified Schmidt projection system is used, with aspherical correction plates prepared by a simple method from gelatine solution.

Part 2. Pulse-Type High-Voltage Supply—A compact 25-kv unit with automatic voltage control. A voltage tripling circuit with miniature rectifiers is used. Resonant circuit losses are kept low by means of a low-loss magnetic ferrite material.

Part 3. Deflection Circuits—Description of magnetic deflection circuits adaptable to either projection or direct-viewing receivers, and an interlace method using the first serration in the vertical synchronizing signal. The horizontal deflection output stage includes a power tube and also a diode which improves power economy and sweep linearity and suppresses spurious oscillations. See also 2408 below.

621.397.6 2388
Simplified Signal and Pattern Generators—G. T. Clack. (*Jour. Telev. Soc.*, vol. 5, pp. 75–80; June, 1947.) Construction and circuit details of two instruments for amateur use.

621.397.6 2389
Television Field Equipment—J. H. Roe and N. S. Bean. (*FM and Telev.*, vol. 7, pp. 31–35; November, 1947.) Illustrated description of camera, camera control, and power-supply units.

621.397.6:621.396.68 2390
Television E.H.T. Supply: Parts 1 and 2—A. H. B. Walker. (*Wireless World*, vol. 54, pp. 120–125 and 169–173; April and May, 1948.) The need for a well-regulated supply is stressed and performance requirements are discussed. The relative merits of three alternative systems are considered: (a) hv mains transformer and half-wave rectifier; (b) hf oscillator and rectifier; (c) use of line fly-back pulses. A method of voltage multiplication using combinations of rectifiers and capacitors is also described. Data for suitable rectifiers are tabulated.

621.397.62 2391
Considerations in Design of Home Constructed Receivers: Part 1—W. I. Flach. (*Jour. Telev. Soc.*, vol. 5, pp. 10–12; March, 1947.) Abstracted with part 2, 2392 below.

621.397.62 2392
A Television Receiver for the Home Constructor: Part 2—W. I. Flach. (*Jour. Telev. Soc.*, vol. 5, pp. 65–73, 80; June, 1947.) Full circuit details of the receiver noted in 577 of March, with discussion of the considerations leading to the choice of components and circuits. Part 1, 2391 above.

621.397.62:621.317.2 2393
Television Receiver Laboratory—Norton. (*See* 2279.)

621.397.743:621.396.65.029.64 2394
FM Relay Out-Performs Coax for Television—(*See* 2370.)

TRANSMISSION

621.396.61/.62 2395
The Design of Transmitter Drives and Receivers for Single-Sideband Systems—W. J. Bray, H. G. Lillicrap, and W. R. H. Lowry. (*Jour. IEE* (London), part IIIA, vol. 94, no. 12, pp. 298–312; 1947.) The present system of single-sideband transmission incorporates two channels each 6 kc wide, on either side of a reduced-level pilot carrier 26 db below the sideband peak level.

The following equipment to provide this service is described: (a) a low-power drive stage; (b) a monitor receiver which enables either channel of the rf signal to be demodulated for tests of quality and distortion; (c) single-sideband receivers for single-antenna and triple-diversity-spaced-antenna operation at the receiving end of a radio link. The design, layout, and performance of a typical receiver are discussed.

621.396.61:621.316.726 2396
Frequency Composition in Naval Communication Transmitters—Hupert. (*See* 2278.)

621.396.61:621.396.932 2397
The Design of Marine Transmitter Equipments Type "Trader" and "Oceanspan"—D. F. Bowers and E. F. Cranston. (*Marconi Rev.*, vol. 10, pp. 133–141; October to December, 1947.) Circuit details and principal features. The "Trader" equipment covers only the mf band 375 to 500 kc and on cw delivers 100 w to the antenna; on mcw the output is 120 w. The "Oceanspan" covers both the mf and hf bands (3.5 to 23 Mc). The hf oscillator is quartz-controlled, 10 crystals being used, with suitable frequency multipliers and switching arrangements.

621.396.61.001.4:621.317.755 2398
Uses of the Cathode-Ray Oscillograph in Transmitter Adjustment and Control—Coombs (*See* 2290.)

621.396.61.029.58:621.396.712 2399
International Broadcast H.F. Transmitter with Continuously Tunable Plate System—D. A. Miller. (*Communications*, vol. 28, pp. 10–13, 28; January, 1948.) The mechanical and electrical design of the 20-kw output stage. The frequency range is 2.85 to 22.5 Mc. The anode coil is made from Ag-plated Cu tube and has mycalex supports. A short-circuiting spider is geared to a tuning handle on the front panel. Mycalex supports are also used for the plates of the anode-tuning capacitor. Frequency drift is low. Temperature rise is only 5°C after several hours' operation.

621.396.61.029.62 2400
A Mobile Midget for 144 Mc—C. V. Chambers. (*QST*, vol. 32, pp. 21–27; February, 1948.) Circuit and construction details. The crystal-controlled transmitter has an output of 5 w. One 6J6 is used as a 48-Mc oscillator and frequency tripler, followed by a second 6J6 as push-pull amplifier; a 6AQ5 is used as modulator. Current is measured by switching to 100-Ω resistors in various leads.

621.396.611.33:621.396.671 2401
Determination of Matching Ranges [plages d'adaptation] of Transmitters in the Case of Indirect Inductive Coupling to the Aerial—V. Familier. (*Onde Élec.*, vol. 28, pp. 108–114; March, 1948.) The antenna is coupled to an oscillatory anode circuit (L_1C). Simple constructions for the matching surfaces are given for various combinations of conditions in which C, L_1, L_2 and K are variable. L_2 is the self-inductance of the antenna coupling coil and K the coupling coefficient for L_1 and L_2. See also 1958 of 1947 (Glazer and Familier).

621.396.615 2402
F.M. Master Oscillator of U.S.W. Transmitter for High-Fidelity Broadcasting—M. G. Margolin. (*Radiotekhnika* (Moscow), vol. 2, pp. 19–33; May and June, 1947. In Russian, with English summary.) Circuits and design theory of an experimental 1-kw equipment at Moscow. A zero-beat discriminator is used for stabilizing the center frequency.

621.396.619.11/.13 2403
Generalized Theory of Multitone Amplitude and Frequency Modulation—L. J. Giacoletto. (PROC. I.R.E., vol. 36, pp. 240–243; February, 1948.) Discussion on 3700 of 1947.

621.396.619.13:621.317.76 2404
FM Monitor has Pulse-Counter Discriminator—Cady. (*See* 2291.)

621.396.619.15 2405
Frequency-Shift Radio Transmission—L. E. Hatfield. (PROC. I.R.E., vol. 36, pp. 116–120; January, 1948.) Various methods of obtaining carrier-frequency shift in communications transmitters are discussed, and a satisfactory reactance-type frequency-shift transmitter keyer is fully described. See also 275 of 1947 (Buff).

621.396.619.23 2406
Frequency Modulators with Reactance Valves—I. S. Gonorovski. (*Radiotekhnika* (Moscow), vol. 2, pp. 3–18; May and June, 1947. In Russian, with English summary.) Design theory is given for a modulator using a reactance tube for frequency control of a self-oscillator. Parasitic AM is considered and various methods of coupling the reactance tube to the oscillator are investigated. Tests on an experimental 1-kw FM usw broadcast transmitter at Moscow are discussed.

VACUUM TUBES AND THERMIONICS

537.291 2407
On Using the Energy of Electrons moving in H.F. Uniform Electric Fields—M. S. Neiman. (*Radiotekhnika* (Moscow), vol. 3, pp. 3–21; January and February, 1948. In Russian.) Discussion of the movement of an electron between two parallel grids to which a hf voltage is applied, with particular reference to (a) conditions for complete and partial retardation of the electron when the transit time τ is large, (b) more general cases when τ is not an odd number of half cycles and the electron enters the space between the grids at any phase, (c) the current induced by the electron in the external circuit setting up the field, (d) a passive circuit where the voltage between the grids is due entirely to the energy liberated by the retardation of electrons, (e) the motion of an electron beam between the grids, and the beam retardation, when τ is large, (f) the effect of superimposing a constant voltage upon the alternating voltage.

537.291+538.691]:621.396.615:518.5 2408
Tracing of Electron Trajectories using the Differential Analyzer: Introduction and Parts 1–3—J. P. Blewett; G. Kron, F. J. Maginnis, and H. A. Peterson; W. C. Hahn and J. P. Blewett; J. R. Whinnery and H. W. Jamieson. (PROC. I.R.E., vol. 36, pp. 69–83; January, 1948.)

Introduction—The differential analyzer is used to obtain electron trajectories for magnetron and triode oscillators with dc and rf anode potential. A dc space charge is approximately allowed for in the case of the magnetron; otherwise space charge is neglected.

Part 1—Differential Analyzer Representation.

Part 2—Electron Paths in Magnetrons. The work hitherto done has been largely exploratory; although the differential analyzer solutions are rather sensitive to errors in flux plotting, they give promise of yielding much valuable information about magnetron behavior.

Part 3—Study of Transit-Time Effects in Disk-Seal Power-Amplifier Triodes. The 2C39 power amplifier is discussed. Anode efficiency of about 50 per cent and power gain of 8 or 9

should be obtained at 3000 Mc. Improvement in cavity and plunger design increased the anode efficiency from about 10 to 20 per cent, which is much less than the 50 per cent predicted. The largest loss of dc power comes from anode dissipation; in a triode there is a fundamental conflict between the desire for small transit times and that for low anode impact velocity.

621.385.029.63/.64 2409

Experimental Determination of Helical-Wave Properties—C. C. Cutler. (Proc. I.R.E., vol. 36, pp. 230–233; February, 1948.) The properties of the wave propagated along the helix of a traveling-wave amplifier are discussed. Measurements of axial field strength, of field distribution around the helix, and of the velocity of propagation are discussed. The actual field in the helix is slightly weaker than that predicted by Pierce (2284 of 1947) for a hypothetical helical surface.

621.385.1 2410

British Sub-Miniature Valves—(*Wireless World*, vol. 54, pp. 80–81; March, 1948.) A new Mullard series, with 25-ma oxide-coated tungsten filaments of extremely small diameter. Operating data are tabulated for a voltage amplifying pentode DF70, and for output pentodes DL71, DL72, as used in hearing-aid circuits. Gain is practically independent of filament voltage over a wide range. Over-all dimensions compare favorably with those of corresponding American types.

621.385.1:621.396.619.16 2411

Electron Beam Deflection Tube for Pulse Code Modulation—R. W. Sears. (*Bell Sys. Tech. Jour.*, vol. 27, pp. 44–57; January, 1948.) An experimental 7-digit tube is described. Rapid coding is achieved by a system which compares signal amplitude with a set of scaled levels and generates a group of on-off pulses identifying the appropriate level. A perforated code masking plate, fitted in front of the output plate, modulates the electron beam to give the pulses required. A parallel-wire grid provides feedback to lock the beam to each discrete amplitude level. See also 2366 above.

621.385.1.029.6+621.396.694 2412

Valves and Circuits for Ultra-Short Waves—R. Suart. (*Radio Franç.*, pp. 17–23; February, 1948.) A concise description of the construction of disk-seal and lighthouse tubes, with electrical data for British tubes CV257 and CV273, American 2C40, French CX103, CX104, and CX105, and details of tuning methods using coaxial lines or cavity resonators. All the tubes mentioned can be used for pulse generation, in which case the peak power is about 500 times that quoted for cw operation.

621.385.1.029.6 2413

The Behaviour of Receiving Valves at U.H.F. and Related Technical Problems—L. Piatti. (*Alta Frequenza*, vol. 13, pp. 67–94; June, 1944.) In Italian, with English, French, and German summaries.) Reasons are discussed for the different behavior of receiving tubes at very short and at broadcasting wavelengths. The effects of the finite electron transit time between the electrodes on the four admittances of a tube, and the similar effects of the inductances of electrode leads, are analyzed as functions of frequency. Noise is considered and its importance at uhf stressed. The ideal features of a receiving tube for uhf are derived and the physical characteristics of some special tubes are described.

621.385.1.032.216 2414

The i/V Characteristic of the Coating of Oxide Cathodes during Short-Time Thermionic Emission—R. Loosjes and H. J. Vink. (*Philips Res. Rep.*, vol. 2, pp. 190–204; June, 1947.) The potential differences existing across an oxide coating during short-time emission were investigated by subjecting experimental indirectly heated diodes with sliding anodes to a recurring capacitor discharge. The time constant was 10^{-4} sec and the pulse recurrence frequency 25 to 50 cps.

Using a specially developed oscillograph technique, i/V characteristics were plotted for varying anode versus cathode distances; the value of the potential difference across the oxide coating was found by extrapolating to zero distance.

At normal working temperatures in the range 900 to 1100°K and for current densities of 5 to 10 a/cm², potential differences of 50 to 200 v existed across the oxide coating. It is shown theoretically that the i/V curves obtained by this method are a good approximation.

621.385.1.032.216 2415

On the Activation of Oxide-Coated Cathodes—H. C. Hamaker, H. Bruining, and A. H. W. Aten, Jr. (*Philips Res. Rep.*, vol. 2, pp. 171–176; June, 1947.) Glass heated to 400°C during degassing can emit a small amount of HCl; chloride deposits are thus formed on anodes and grids. Under electron bombardment, these chlorides decompose and poison the cathode emission.

621.385.832 2416

The Design and Construction of Cathode Ray Tubes—R. H. Craig. (*Nickel Bull.*, vol. 21, pp. 30–33; March, 1948.) General design considerations, with special reference to the materials used for the electrodes and deflector systems, and to construction details for accurate alignment of the electrode assembly and for good spot quality.

621.396.615.141.2 2417

Methods of Tuning Multiple-Cavity Magnetrons—R. B. Nelson. (Proc. I.R.E., vol. 36, pp. 53–56; January, 1948.) The most successful method of tuning involves simultaneous variation of both the inductance and the capacitance of all the resonant cavities by a single tuning motion. Tuning ranges of better than 1–4 to 1 have thus been obtained with good efficiency. A magnetron is described which can deliver over 2 kw cw power at any frequency between 760 and 1160 Mc.

621.396.822 2418

A Unified Theory of Spontaneous Electrical Fluctuations in Thermionic Valves—R. Fürth. (*Nature* (London), vol. 160, pp. 832–833; December 13, 1947.) It is asserted that thermal fluctuation and shot effect are, in general, only two aspects of one and the same statistical fluctuation process, which can be theoretically treated by the application of the principles of statistical thermodynamics. An approximate formula for the shot effect in diodes, which covers the whole range of the characteristic, may readily be derived. Certain anomalous data presented by MacDonald (953 of 1947) are explained by the theory.

621.396.822 2419

On the Theory of Electrical Fluctuations—R. Fürth. (*Proc. Roy. Soc. A*, vol. 192, pp. 593–615; March 18, 1948.) A critical analysis of the fundamental principles of the theory of thermal fluctuations of electricity, with a new derivation of Nyquist's theorem. Examination of the methods of derivation of the formulas for "shot" fluctuations suggests that they are, in fact, identical with thermal fluctuations. On this basis, a general formula for the shot effect in diodes is given which should be valid throughout the whole range of the characteristic.

621.396.822 2420

Valve Noise and Transit Time—N. R. Campbell, V. J. Francis, and E. G. James. (*Wireless Eng.*, vol. 25, pp. 148–157; May, 1948.) Earlier treatments of tube noise in diodes and triodes are extended to allow for the transit time of the electrons. The reaction of the tube to a sinusoidal input is analyzed and modifications of the formula are then made to allow for the finite duration of the noise events, involving a discussion of the charge induced on the grid by an electron as it approaches and recedes. Discussion of measurement results indicates that the assumptions made in previous analyses by Bakker (*Physica*, vol. 8, p. 23; 1947) and by Benham (*Wireless Eng.*, 1928 abstracts, p. 288 and 1931 abstracts, p. 212) are of doubtful applicability to tubes of practical importance. See also 1191 and 1471 of 1946.

621.396.822 2421

Noise in Resistances and Electron Streams—Pierce. (*See* 2220.)

621.392+621.385 2422

Electronic Circuits and Tubes [Book Review]—War Training Staff of the Cruft Laboratory, Harvard University. (*See* 2222.)

MISCELLANEOUS

002:778.1 2423

Proposed Central Publication of Scientific Papers—W. Davis. (*Nature* (London), vol. 161, p. 896; June 5, 1948.) Description of a method of auxiliary publication used in the United States since 1936. Original scientific papers can be deposited by journal editors with the American Documentation Institute, Washington, D. C.; a catalogue of documents deposited is available. Journals can publish a summary (long or short) of the article, and insert the number of the document deposited, and the price of a microfilm or enlarged photoprint. Only the original document and the microfilm negative need be stored. Copies are ordered directly by readers from the American Institute of Documentation. Neither the journal editor nor the author incurs any financial liability.

371.3:621.3 2424

The Practical Training of Professional Electrical Engineers—(*Jour. IEE* (London), part I, vol. 94, pp. 437–446; October, 1947.) Report of a committee appointed by the Councils of the British Electrical and Allied Manufacturers' Association, the Radio Industry, and the Institution of Electrical Engineers. Recommendations are made for two types of apprenticeship: a graduate apprenticeship lasting 2 years, and a student apprenticeship, with technical education by part-time study, lasting 4 years.

522.1 (42) Greenwich 2425

The Royal Observatory, Greenwich—H. Spencer Jones. (*Endeavour*, vol. 7, pp. 9–14; January, 1948.) A short historical account of the observatory and its equipment. Removal to Herstmonceaux Castle, Sussex, is in progress.

621.3(083.74) 2426

On the Use of Standard Terms and Abbreviations—G. W. O. H. (*Wireless Eng.*, vol. 25, pp. 67–69; March, 1948.) General discussion, including a suggestion that "ampere" should be shortened to "amp."

621.3(083.74) 2427

Standard Terms and Abbreviations—M. G. Scroggie. (*Wireless Eng.*, vol. 25, pp. 130–131; April, 1948.) Favors the adoption of "amp" for the unit of current and also the practice of using lower-case letters for initial abbreviations of words which would begin with them if written in full, such as if, agc, and ac.

621.3(083.74) 2428

Standard Terms and Abbreviations—E. D. Hart; L. F. Odell. (*Wireless Eng.*, vol. 25, pp. 164–165; May, 1948.) Comment on 2427 and 2428 above. Reasons are given in favor of the contraction of "ampere" to "amp," the use of "K" instead of "k" for "Kilo," and of "dB" instead of "db" for "decibel." The use of lower-case letters in abbreviations is also considered.

621.396 2429

The Inventor of Radio Telegraphy—G. W.-O. H. (*Wireless Eng.*, vol. 25, pp. 135–137; May, 1948.) A discussion of the evidence regarding the earliest experiments in wireless telegraphy. Recent claims that Popov had a working system earlier than that of Marconi do not appear to be supported by the evidence. See also 1842 of July and back references.